A TREATISE

ON

PHOTOGRAPHIC OPTICS

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BY

R. S. COLE, M.A.

LATE SCHOLAR OF EMMANUEL COLLEGE, CAMBRIDGE,
ASSISTANT-MASTER, MARLBOROUGH COLLEGE

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PREFACE

THE object of this treatise is to provide an account of the principles of Optics, so far as they apply to Photography, in a form which is of scientific value, while not of too abstruse a nature to place it beyond the reach of all but the professional mathematician or physicist.

I have attempted to steer a middle course between giving too much mathematics and giving none at all; the former course would restrict the book to a few, while the latter would deprive it of all real value.

To make the mathematics as intelligible as possible, most of the results have been illustrated by worked numerical examples, and symbolical results have been expressed in words.

The chapter on aberration necessarily contains a certain amount of algebraical formulæ, too many perhaps for many readers, but care has been taken to explain clearly, by means of diagrams, the principles which underlie the formulæ.

I have to thank Captain Abney for giving me permission to use many of his diagrams, Major Darwin for allowing me to quote from his paper on lens testing at Kew Observatory, and the publishers of *Photography Annual* for allowing me to copy the tables contained in §§ 127, 151, and 156; also I owe it to the kindness of the Sandell Plate Company that I am able to give an example of a "test film" for the speed of plates.

My thanks are also due to the Rev. W. F. H. Curtis, who first suggested to me to write, and has given me many valuable suggestions and much help.

Marlborough,
October 1898.

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CORRIGENDA

p. 128, last line but two, *for* $\left(\frac{u f}{v + f}\right)^2$ in second part of the expression *read* $\left(\frac{u f}{u + f}\right)^2$

p. 133, line 16, *for* $\left(\frac{u \mathrm{F}}{v + \mathrm{F}}\right)^2$ *read* $\left(\frac{u \mathrm{F}}{u + \mathrm{F}}\right)^2$

p. 135, line 12, last half, *for* B - &c. *read* B = &c.

pp. 139, 140, Figs. 47*a* (4) (5) (6) should be reversed end for end.

p. 150, line 26, *for* refractory *read* refracting

p. 157, line 2, *for* the *read* two

p 172, line 13, *for* K X *read* K Y

p. 188, last line but one, *for* white *read* violet.

p. 273, line 30, *read* in 1·2/360 seconds

p. 317, line 5 should *read*

$$\text{hence } \frac{(1+n)f - l}{\mathrm{F}} = \frac{y - z}{x} \text{ or } \mathrm{F} = \frac{x}{y - z}\left\{(1+n)f - l\right\}$$

p. 317, lines 13 and 14, *for* u *read* U

PHOTOGRAPHIC OPTICS

INTRODUCTION

MANY books have been written on optics, and of these not a few are designed to meet the wants of photographers; but they seem to have done little as yet to drive away the clouds of mystery which hang about a lens. Very few people who practise photography know much about the instrument which they use.

The reason of this ignorance and indifference is not hard to find; it arises from three causes: First, the photographer is dazzled by the results of his work, and his main idea is to turn out each picture more perfect than the preceding one. Secondly, writers on optics have for some occult reason deemed it necessary to surround the simplest optical matters with a cloud of symbols repulsive to those who are not mathematically minded, or to leave out all the mathematics, and with it all definiteness and accuracy of expression, which causes even worse confusion than symbols. Thirdly, a lens is far too complicated to be made by an amateur; the photographer buys his lens ready for use, and it is nearly impossible to find out its construction, and so as long as it works well no questions are asked.

But there are some photographers who are not so much engrossed in picture-making that they can attend

to nothing else, but they take an interest in all matters connected with photography—the chemistry, optics, and mechanics of it.

It is for such photographers that this book is written, and it is intended as an attempt to place the main ideas of optics within the reach of those who care to acquire them, without an undue amount of mathematics or sacrifice of precision.

The reader is presumed to possess some elementary mathematical knowledge, Euclid, Algebra, and the simplest parts of Trigonometry; the use of Trigonometry might possibly have been avoided, but it would have led to a great deal of circumlocution. The mathematical working will be kept within the narrowest possible limits, consistent with giving a satisfactory account; to have reduced it further would have been to sacrifice some of the most useful results.

Those who experience a difficulty in reading through a particular piece of work will find it a great assistance to use a pencil and piece of paper as they read, roughly jotting down and working out calculations, and sketching the diagrams. Besides this, the various experiments described should, when possible, be performed; in many cases a lens, a few pieces of cardboard, and a candle or two will be all that is required.

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CHAPTER I

ON LIGHT

1. Since photography is the outcome of the chemical action of light, a thorough knowledge of the subject cannot be gained without some acquaintance with the theories which have been proposed to account for optical phenomena, and specially with the wave theory, which is now generally considered to be the one nearest to the truth. Detailed explanations would, of course, be out of place here, and must be looked for in special treatises, but a rapid sketch will serve to show what is now supposed to be the nature of the machinery at work, and to explain many points which would otherwise remain obscure.

2. It will be well at the outset to get a clear idea of the problem with which we have to deal when we inquire as to the nature of light, for all our actions are so dependent on sight that it is not easy to grasp what it is that needs explanation. Sight has been called that sense which compensates for the want of ubiquity by giving information about objects at a distance; this information is brought to us through the agency of light.

We may then describe light as some kind of information which is conveyed to us from illuminated bodies; we are convinced by experience that it is not merely a subjective phenomenon, but that, in the space between us and the distant object, something does actually take place.

The problem, then, is to discover in what manner

this information is conveyed: to do this we must take into account the known properties of light. The most important properties are as follows:—Light is propagated through air and homogeneous media of uniform density in straight lines: it is reflected if it meets a smooth opaque body: it is bent or refracted, and also reflected, when it passes from one transparent medium to another, *e. g.* from glass to water: it is propagated through air or space with the definite, though enormous, velocity of 188,000 miles per second—this fact was discovered by Roemer from the study of the times of eclipse of Jupiter's satellites, and has since been amply verified by the experiments of Fizeau and others.

Let us consider now the methods which can be used to convey information from one place to another. The simplest way is to send a messenger from one of the places to the other; but the information may also be conveyed by handing it on from person to person placed in suitable positions. In the first case some material substance must be transferred from the one place to the other, but in the second it need not. These two methods illustrate the two chief theories that have been proposed to explain light—the emission theory and the wave theory.

3. In the emission theory proposed by Sir Isaac Newton it is supposed that sources of light shoot out very small particles in all directions with great velocity. These would rebound on striking any body, and reflection would thus be produced, and in passing from one medium to another they would be deflected, and refraction would result. As far as reflection and refraction are concerned, the theory was fairly satisfactory; but when attempts were made to explain other phenomena, those of polarization,[1] for instance, very serious difficulties were encountered, and to surmount these many additional hypotheses were required. After a time the theory became so unwieldy that, to say the least, it

[1] For these see any treatise on optics.

must have required a wide stretch of imagination to regard it as representing the real state of affairs. Besides this, there was the difficulty of accounting for the fact that a dead-black surface which absorbs nearly all the light falling on it receives no increase of weight on that account. And lastly, the emission theory has failed to stand the test of a crucial experiment devised by Arago.[1]

4. Newton, however, before he proposed the emission theory, had entertained the idea of the wave theory, but had been unable to reconcile it with the rectilinear propagation of light.[2] For while sound, which is due to waves of condensation and rarefaction in air, is able, to a great extent, to bend round obstacles, so that they offer little impediment to it, on the other hand, light had, so far as Newton could observe, no similar power of bending, anything between the object and the eye effectually concealing the object. But the difficulty, being to some extent based on a false analogy, is not so great as it at first sight appears to be; for a sound is not altered in character by any reflection it may undergo, *e. g.* at the walls of a room, and thus after such a reflection we recognize it as the original sound, though diminished in intensity. But if light enter a room through a small opening, and is reflected and scattered by the walls, we cannot thereby see the object outside, though light may penetrate to all parts of the room.

At the beginning of the century this difficulty about the rectilinear propagation of light was finally cleared up by the independent labours of Dr. Thomas Young in England and of Augustin Fresnel in France. This great obstruction being then removed, the wave theory made rapid progress, and in a few years was accepted by the majority of scientific men.

[1] Glazebrook's *Physical Optics*, p. 121.

[2] Glazebrook, Presidential Address to Section A, British Association, 1893, reported in *Nature*, September 14, 1893.

The theory has stood many very severe tests, both of experiment and calculation. Fresnel applied it to explain the phenomena of diffraction and of double refraction, verifying most of his results by careful measurement; since his time it has constantly been the subject of investigation, so that at the present day, though there still remain points (such as astronomical aberration) to be cleared up, there is very little doubt of its truth. Recently it has received still further confirmation from the discovery, by Clerk-Maxwell and Hertz, of the intimate connection between optical and electrical phenomena.

5. The second method of conveying information is that of handing it on from place to place without the transference of any material substance; we have now to see how wave motion satisfies this condition and fits in with the other points enumerated.

To those who live by the seaside there will be little difficulty in understanding how waves can travel for considerable distances and preserve their identity, and most people are familiar with the effect of throwing a stone into a still pond, thus causing waves to travel outwards to great distances from the disturbance. That a wave travels does not mean that water is carried with it, but merely that an up-and-down motion is handed on from particle to particle; if this were not so there would be a tendency for water to accumulate on any shore on which waves were breaking. It is shown in special treatises that the rectilineal propagation of light, its reflection, refraction, polarization, etc., can all be accounted for in a natural and simple manner from the properties of wave motion.[1]

Reflection of waves is not at all difficult to observe. For example, at any long sea-wall, such as that over which the Great Western Railway runs between Dawlish and Teignmouth, there, when the sea is not too rough, the incoming waves and the outgoing

[1] Glazebrook's *Physical Optics*, chap. iii.

reflected waves can be clearly seen passing over each other without stopping each other's motion. Also, in addition to the long waves, other small waves and ripples, crossing and recrossing in all directions, will be seen, illustrating how different beams of light can pass quite independently across the same space. A short time devoted to the study of waves under these conditions will convey clearer and more tangible ideas of wave motion than any description.

We must now see to what properties of wave motion the intensity and colour of light correspond; it seems reasonable to expect that the former should depend on the height to which the wave rises, or, in other words, on the amount of disturbance to which the particles of water are subjected. To state this more exactly, call the length of the excursion of the particles the *amplitude* of their motion, then the intensity is proportional to the square of the amplitude; this has been verified by the agreement of calculation with observation.

Colour depends on the *frequency* of vibration of the wave, by which we mean the time taken by a particle to oscillate up and down and then return to its original position; violet light, which is the most photographically active, is composed of the waves of shortest frequency, while red is composed of slower waves. As long as we keep to one medium the frequency of the wave is proportional to its length, or to the distance between crest and crest. In what follows we shall restrict ourselves to light waves in air unless it is otherwise stated.

We can therefore substitute wave-length for frequency, the violet light having a shorter wave-length than red light. Experiment has shown that there are light waves both too short and also too long to affect the eye, that, in fact, the eye is sensitive only to a comparatively small part of the waves which come from the sun.

Light of shorter wave-length than the violet, though

invisible, yet exerts a chemical action, while rays of greater length than the red are perceived as heat. Thus the radiation we receive from the sun may roughly be divided into three parts—the chemical, the luminous, and the calorific; but there is no sharp line of division between these parts, they overlap each other, the waves of each being of precisely the same nature, differing only in length.

6. Some figures in connection with the wave theory mayserve to render ideas more concrete. As stated above, the velocity of light in space has been found to be about 180,000 miles, or 300,000,000 metres, per second, a velocity which would enable it to travel in one second three times round the earth.

The unit by which wave-lengths are usually measured is the tenth metre, which is 10^{-10}, or one ten thousand millionth part of a metre; the wave-length of the shortest visible violet ray is about 3,900 tenth metres, that of the yellow light obtained by putting common salt in the flame of a spirit lamp is 5,890 tenth metres, and that of the red rays about 7,640 tenth metres. The length of the largest visible wave is thus barely twice that of the shortest.

Knowing the length of a wave in air and its velocity, we can find how many waves enter the eye in one second, for they will be all the waves comprised within the length of three hundred millions of metres. If we take the mean wave-length given above, we get the enormous number of sixty billions in round numbers. This seems incredible, but the magnitude of the number is no *primâ facie* evidence against its accuracy, for our idea of time is only relative.

Prof. C. V. Boys[1] has recently photographed a flying rifle bullet illuminated by an electric spark which he found to last only for one or two millionths of a second. It might be thought that this is too short a time for the light to produce any effect on a photographic plate, but

[1] *Nature*, vol. xlviii. pp. 415, 440, March 2 and 9, 1893.

from what has been said above, something like sixty millions of waves impinge on the plate during each millionth of a second, which gives the matter a very different aspect.

7. **The Ether.**—If light consists of wave motion there must be some medium through which it is propagated. It is very unlikely that air is this medium, for light passes freely through the best vacuum that has been produced. Besides this, we cannot suppose that all space is filled with an atmosphere such as that we breathe, but yet we receive light from the sun and stars. So we are driven to the conclusion that there must be some medium which is the vehicle of light, but which we cannot directly perceive. This medium is called the *luminiferous ether*, and the evidence for its existence, though indirect, is very strong. Various suggestions have been made as to its nature, and though some of them account for a great deal, none meet all the observed phenomena. One great difficulty is, that in order to transmit light at its enormous velocity of 180,000 miles a second, the ether must possess extreme rigidity; but that in spite of this it offers an inappreciable resistance to the motions of the planets.

8. Some difficulty may be felt in connecting theory with experience, and in realizing that the sensation of light is really produced by waves entering the eye; but it should be remembered that all interpretations of sensations are the results of education. We learn by experience that a certain sensation is the result of certain external circumstances, and after a time we are so much accustomed to this association that the sensation becomes involuntary, and we lose sight of the intermediate steps. For instance, the picture formed on the retina of the eye is inverted, yet we feel no inconvenience, and do not recognize the fact.

9. **Physical and Geometrical Optics.**—For complete explanations of optical phenomena we must turn to the wave theory, from which, provided our mathematical

machinery is powerful enough, we can in most cases get what we want; this has been done in the case of phenomena called diffraction, observed under certain conditions when light passes through small holes and slits. But this method would often be very tedious, particularly in the theory of lenses, and a different one, which is accurate enough in most cases, can be employed.

The former method is called physical optics, and the latter method geometrical optics. In geometrical optics certain fundamental laws derived from and confirmed

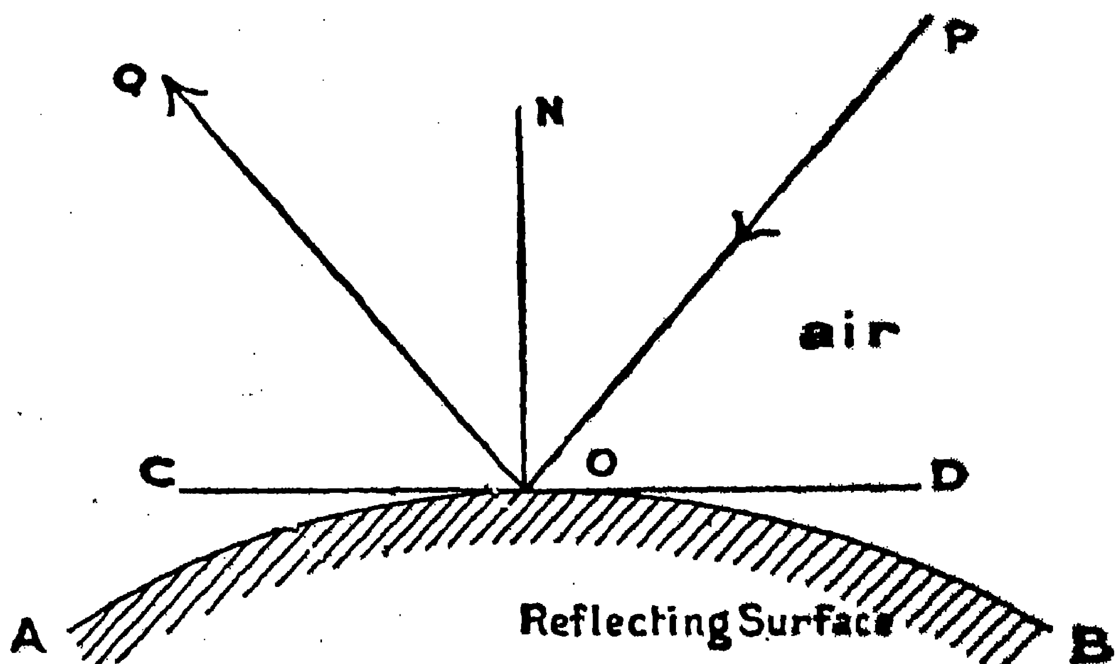

FIG. 1.

by experiment are taken for granted, and are used as the starting-point for investigation, no inquiry being made as to the nature of light; instead of waves, rays of light are considered, these being straight in a homogeneous medium, but reflected or refracted according to definite laws when meeting an obstacle or another medium.

10. Reflection and Refraction.—When light meets a transparent obstacle, part of it is reflected and part refracted in a definite nanner, but, besides this, there is an irregular reflection or scattering of the light in all

directions, due to slight roughness of the surface; if the obstacle is polished a comparatively small part is scattered, and the amount reflected depends on the angle at which the light is incident. All bodies, even if they do not reflect or refract regularly, scatter light, and it is through this that we see them. We must now state the laws of regular reflection and refraction. If a ray of light P O meet the surface A B (Fig. 1), draw the plane C D to touch the surface at O, and O N the normal to

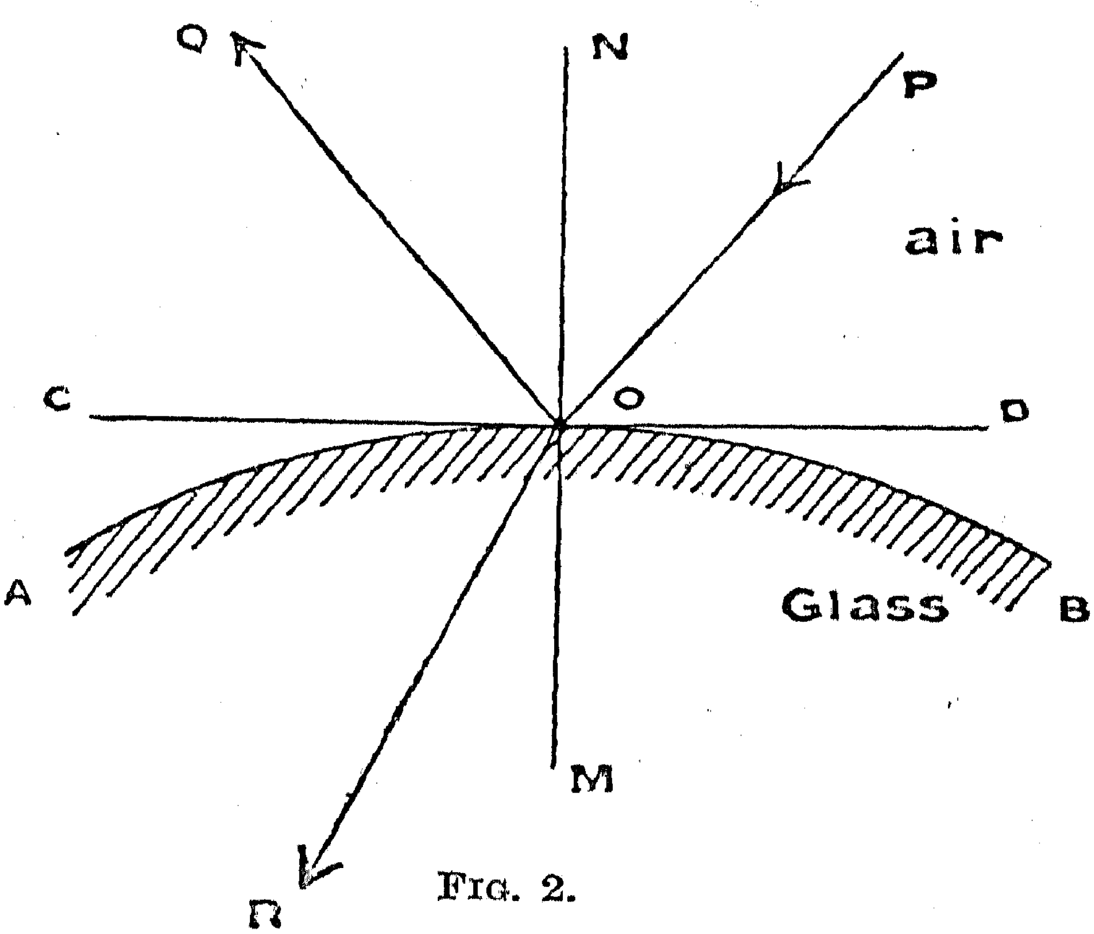

FIG. 2.

the surface perpendicular to C D; then if O Q be the reflected ray, the lines O N, O P, O Q will all be on one plane, and the angles P O N, Q O N will be equal.

The angle P O N which the incident ray makes with the normal to the surface at the point of incidence is called the *angle of incidence*, and similarly the angle Q O N is called the *angle of reflection*.

Thus we may state the law of reflection:

The incident and reflected rays are in the same plane

with the normal to the surface at the point of incidence, and the angles of incidence and reflection are equal.

Next, in the case of refraction, let A B (Fig. 2) be the boundary surface between the two media, air and glass for example, and let the ray P O be incident at O; draw the tangent plane C D to the surface, and M O N the normal. Let O Q be the reflected ray, and O R the refracted ray; the angle R O M is called the angle of refraction. Denote the angles P O N, R O M of

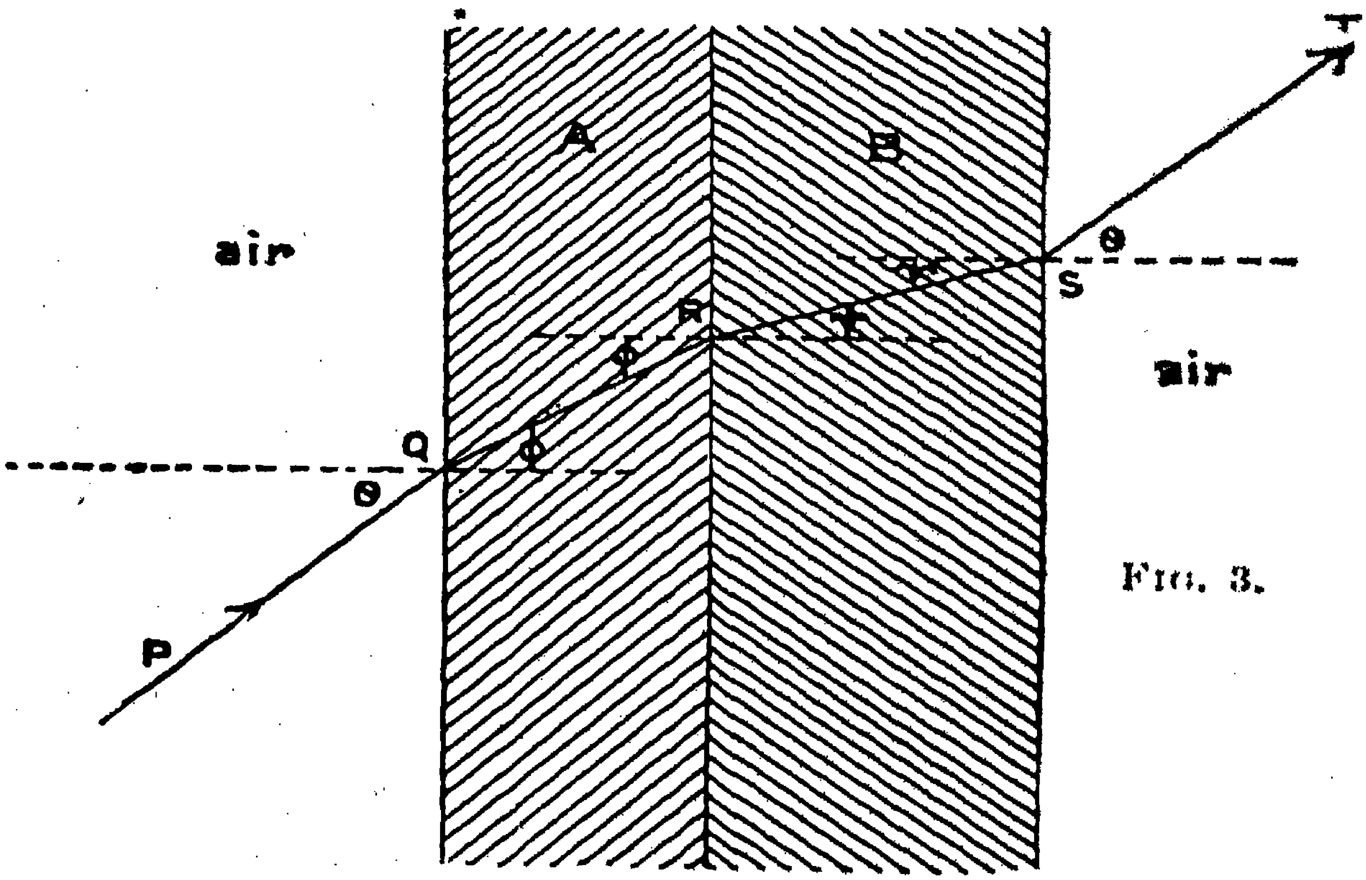

FIG. 3.

incidence and refraction by θ and ϕ. Experiment shows that the relation between the angles of incidence and refraction is given by

$$\frac{\sin\theta}{\sin\phi} = \text{a constant quantity}$$

as long as we keep to the same substances; and also that O P, M O N, and O R are all in one plane.

Thus we may state the law of refraction:

The incident and refracted rays are in the same plane with the normal to the surface at the point of incidence,

and make with it angles whose sines are in a constant ratio as long as the media are unchanged.

When the ray passes from air or vacuum (which, for our purpose, are optically indistinguishable), the constant ratio is denoted by μ, and called the *refractive index* of the medium into which the ray passes, so that

$$\sin\theta = \mu \sin\phi.$$

11. — Refraction through two Media. — We must now see what the ratio becomes when a ray passes from one homogeneous medium to another, and neither of them is air; a case which occurs with cemented lenses.

Let A and B (Fig. 3) be plates of two transparent media whose refractive indices are μ_1 μ_2, the faces of the plates being parallel, and let a ray of light P Q R S T traverse the two plates and emerge again into the air; it can be shown experimentally that the emergent ray S T is parallel to the incident ray P Q.

It will be useful further on to remember that if a ray of light passes through a plate of any medium with parallel faces, and out again into the original medium, it will emerge parallel to its old direction, but will be laterally displaced; this is an experimental fact, and can be shown to agree with the law of refraction. Let the angles which the different portions of the ray make with the normals to the surfaces be as marked in the figure, the first and last angles being of course the same. Then at incidence and emergence

$$\frac{\sin\theta}{\sin\phi} = \mu_1 \qquad \frac{\sin\theta}{\sin\psi} = \mu_2$$

At R between the two media

$$\frac{\sin\phi}{\sin\psi} = const = \mu_{AB} \text{ say.}$$

Then

$$\mu_{AB} = \frac{\sin\phi}{\sin\psi} = \frac{\sin\phi}{\sin\theta} \cdot \frac{\sin\theta}{\sin\psi} = \frac{\mu_2}{\mu_1}$$

Or, when light passes from one medium to another, and

neither is air, the constant ratio of the sines of the angles of incidence and refraction can be got by dividing the refractive index of the second medium by that of the first medium, and this is often called the refractive index between the two substances. If $\mu_2 > \mu_1$ or the second medium is more refracting than the first, then $\phi > \psi$ or the refracted ray is nearer to the normal than the incident ray, and *vice versâ*.

12. Total Internal Reflection.—When light passes from a rare to a dense medium, the refractive index between the two is greater than unity; for air and flint glass it is about 1·6, thus:

$$\sin\theta = 1{\cdot}6 \sin\phi.$$

We have then θ the angle of incidence given, and require to find ϕ, which we must do from the relation, $\sin\phi = \frac{1}{1\cdot6}\sin\theta$; this is always possible whatever the value of θ, for $\sin\theta$ cannot be greater than unity, and $\sin\theta/1{\cdot}6$ is therefore always less than unity, and an angle ϕ can be found corresponding to all the values of $\sin\phi$.

But, on the other hand, when light passes out from the dense medium, we know ϕ, and have to find θ from the relation

$$\sin\theta = 1{\cdot}6 \sin\phi.$$

It is obvious if $\sin\phi$ is greater than $1/1{\cdot}6$, which is quite possible, that $1{\cdot}6 \sin\phi$ is greater than unity, and then no value of θ can be found to correspond, for the sine of an angle cannot be greater than unity. This means that if the angle of incidence is greater than that given by $\sin\phi = 1/1{\cdot}6$, called the critical angle for the substance, the ray of light cannot emerge, and is found to be totally reflected. The critical angle in this case is about $38° 41'$. This phenomenon of the stoppage of light does not occur suddenly when the critical angle is reached, but some of the light is reflected at all angles of incidence, the amount of it increasing rapidly as the critical angle is approached.

We see then that lenses should be arranged so that the angle of incidence of the light on all the surfaces may be as small as possible, to avoid loss of light by internal reflection.

Total internal reflection can easily be observed by hanging a string into water in a vessel with transparent sides, and examining the portion in the water from below; it will in most cases be found impossible to see the portion above the surface, what appears to be the continuation of the string being only the reflection of that in the water, as an attempt to touch it will show.

We have now given as much about the simple laws of reflection and refraction as concerns us; a more detailed account must be looked for in such books as Deschanel's or Ganot's *Physics*.

13. Measurement of Light. — We must now consider the question of the measurement of quantity of light and intensity of illumination, concerning which we must have clear ideas when we come to deal with relative exposures with various stops and lenses.

Light cannot be measured with the same facility as length and weight, but it is none the less a definite and useful quantity. We cannot exactly define what we mean by a quantity of light, but the conception presents no practical difficulty; we can get some idea of the quantity of light which has fallen on a sensitive plate from the density of the deposit produced on development, and provided the plate was not over-exposed, the density is roughly proportional to the quantity of light, per unit area, that has fallen on the plate during its exposure. But we cannot use this as an absolute measure of a quantity of light, for we cannot estimate density except by the quantity of light which the film intercepts.

We must first choose a unit source of light which can be used as a standard with which to compare other sources; this is generally taken to be the *standard candle* of the Board of Trade, used in testing the

illuminating power of gas. In place of the standard candle other sources of light have been proposed as standards, such as the Harcourt Pentane lamp. The absolute intensity of the standard is not important, but it should be easily and exactly reproducible.

We shall not be very much concerned with absolute values of sources of light, though it may at first sight appear that this would be the case, for the photographic effect of light depends not only on its intensity, but on its colour composition, which is very difficult to estimate. In most cases the intensity is judged from experience, or if an instrument is used it is one whose action depends on the photographic power of the light; this will be treated further on when we come to the question of exposure.

In the following paragraphs on photometry, the sources of light are taken to be all of the same colour; the comparison of sources of different colours being a very difficult matter.

We now need two definitions:

The *unit quantity* of light is that quantity which falls per second on unit area of the surface of a sphere of unit radius at whose centre a unit source of light (one candle) is placed; the source of light being small compared with the radius of the sphere.

The *intensity of illumination* of a surface is the quantity of light which falls per second on unit area of the surface.

The alteration of the intensity of illumination of a surface due to a change of its distance from the source of light can be found from the fact that light travels in straight lines (Fig. 4). Let the source of light be a candle L; take a piece of cardboard A and place it at unit distance from L, take a second piece B and place it at twice the unit distance from L, making it of such a size that it is just covered by the shadow of A. B must therefore be four times the size of A.

A certain quantity of light falls per second on A,

and if this screen be removed the same quantity of light will fall on B, which is four times the size; hence the illumination of B will be only one-quarter that of A. In a similar manner it can be shown that the illumination of a screen C at three times the unit distance from A will be only one-ninth that of A, and we could thus find the illumination at any distance from L in terms of that of A. The law connecting the intensities of illumination at different distances is usually given by saying that the illumination varies inversely as the square of the distance from the source of light. It must, of course, be understood that the source must be so small that we may without serious

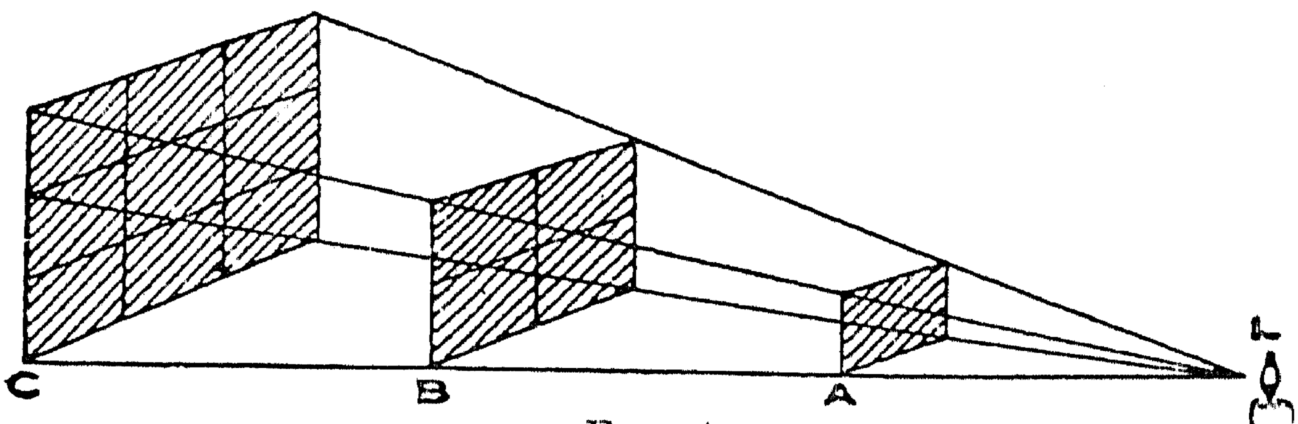

FIG. 4.

error regard it as practically a point compared with the distances to be measured. If the source is not small we must then imagine it divided up into several small portions, the effects of these found separately, and then added together. We can put results of this article in a symbolical form, which may be easier to realize.

14. The unit quantity of light has been defined to be that which falls per second on unit area of a sphere of unit radius with one candle at the centre. If there be L candles at the centre, L units of light per second will fall on each unit area of the sphere.

The surface of a sphere of radius R feet is [1] $4\pi R^2$

[1] π is used to denote the ratio of the circumference of a circle to its diameter, and is for most purposes given accurately enough by 22/7. So in any subsequent calculations π will simply be an abbreviation for 22/7.

square feet, and the surface of a sphere of unit radius is 4π square feet; hence the light falling on the whole surface is $4\pi L$ units per second. So that the whole quantity of light emitted by a source of candle power L is $4\pi L$ units per second, and the intensity of illumination of the surface of the sphere is L. We have taken the surfaces to be part of spheres, but, if the surface is small compared with its distance from the source, we can, with enough accuracy, replace it by a plane surface; but it must be remembered that if the plane is large compared with its distance from the object the illumination will not be uniform all over it.

Let us now find the illumination at a point Q, distant r feet from L, and let P be distant one foot from L; then by the law of the inverse squares—

$$\frac{\text{Illumination at Q}}{\text{Illumination at P}} = \frac{\text{L P}^2}{\text{L Q}^2} = \frac{1}{r^2}.$$

$$\therefore \text{Illumination at Q} = \frac{1}{r^2} \times \text{illumination at P} = \frac{\text{L}}{r^2}$$

Example.—It is found when printing on bromide paper by contact that the proper exposure is twenty-five seconds if the printing frame be held at a distance of three feet from a twenty candle-power gas flame. Find the necessary exposure at a distance of three and a half feet from a fifteen candle-power gas flame.

The total quantity of light per unit area which falls on the exposed negative in both cases must be the same.

From the relation given above, the illumination in the first case (L/r^2) is 20/9, and hence the total quantity of light which falls in unit time on unit area of the surface in twenty-five seconds is—

$$\frac{20}{9} \times 25 \text{ units.}$$

Now let t seconds be the proper exposure in the

second case, then the total quantity of light incident will be $\frac{15}{(3\frac{1}{2})^2} \times t$ units.

$$\text{Hence} \qquad \frac{15}{(3\frac{1}{2})^2} \times t = \frac{20}{9} \times 25$$

$$\therefore t = \frac{20 \times 25 \times 49}{9 \times 4 \times 15} = 46 \text{ seconds, nearly.}$$

14*a*. Illumination of an Area oblique to the Incident Light.—In the cases considered above it is assumed

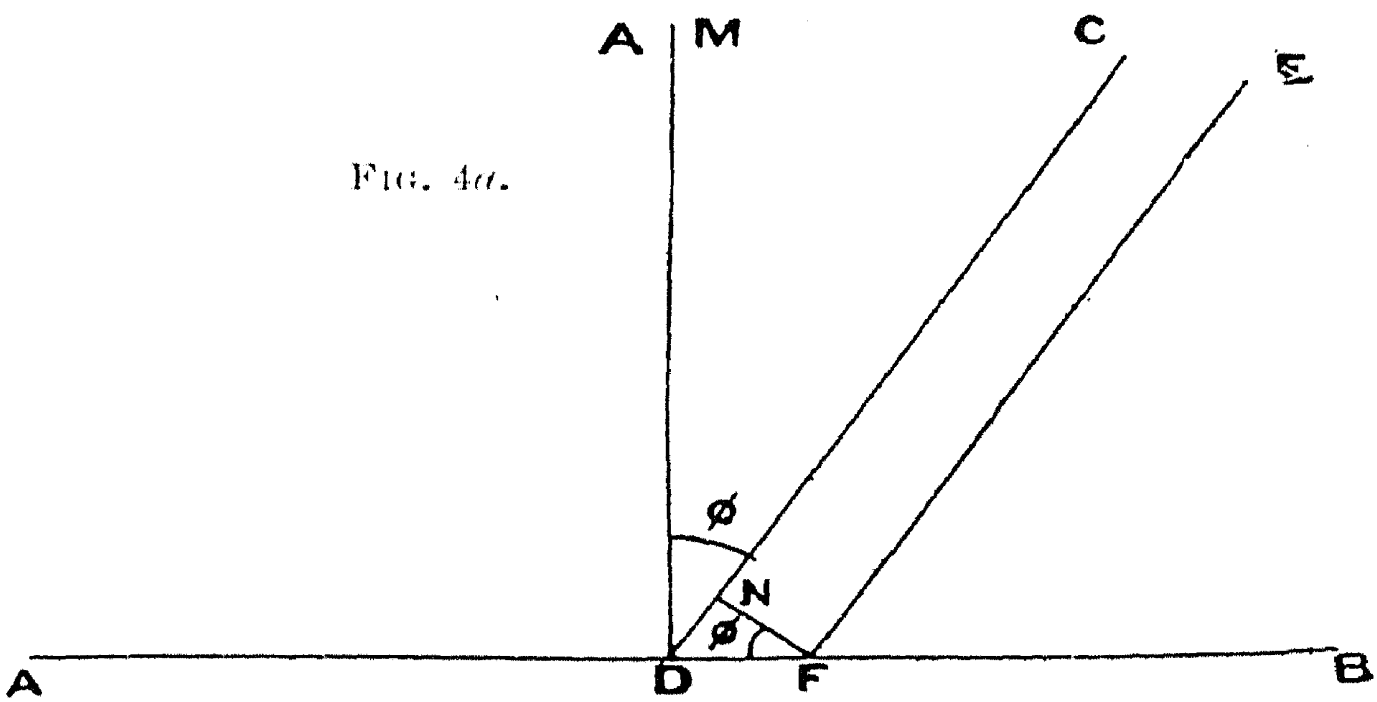

Fig. 4*a*.

that the surfaces are all at right angles to the incident light; if they are not so the illumination will not be the same as that which we have found.

Let light coming from a fairly distant source fall on a surface at an angle of incidence ϕ; let the plane of the paper (Fig. 4*a*) be the plane of incidence. Consider the light inside a small circular cylinder bounded by incident rays, of which C D, E F are the section; draw F N perpendicular to C D.

Let I be the illumination of the area D F, and I′ the illumination that an area placed in the position F N would receive.

The cylinder being small, the areas F N and F D are practically at the same distance from the source of light, and hence their illumination will be proportional to the quantity of light received per unit area.

Now the two areas would clearly receive the same total quantity of light, since either would receive all the light inside the cylinder, hence the quantity of light per unit area received by each will be inversely proportional to its area.

But the area F N is the projection of the area F D on a plane inclined to it at an angle ϕ; hence

$$\text{Area F N} = \text{area F D} \times \cos \phi,$$

and therefore

$$\frac{I}{I'} = \frac{\text{area F N}}{\text{area F D}} = \cos \phi \text{ or } I = I' \cos \phi$$

Hence we see that the illumination of an inclined area is got from that of an area perpendicular to the incident light by multiplying by the cosine of the angle of incidence. It should be noted that the above result is very approximately true even if the incident rays C D, E F are not parallel, for the cylinder can be taken as small as we like, and the rays C D, E F in consequence as near as we like without altering the reasoning.

15. Photometry.—The law of the inverse square supplies the means of estimating the relative intensities of the different sources of light; descriptions of the various instruments employed can be found in text-books on light. Although for economy of space descriptions of photometers are not here given, yet their study is strongly recommended, and as the apparatus required is in most cases very simple and easily constructed, some practical acquaintance with their working should be acquired.

16. Dispersion.—If a ray of white light fall on a prism of glass or other refracting substance, its direction, as we have seen, is altered, and experiment shows that the emergent beam is no longer white, but coloured.

Sir Isaac Newton made the experiment by allowing a beam of sunlight entering a darkened room through a small hole in the shutter to fall on a prism with its edge horizontal, and refracting angle turned downwards; he received the refracted ray on a screen behind, and obtained a coloured vertical band, red at its lower end, and passing through orange yellow, green to blue, indigo and violet at the upper.

This shows that white light is composed of light of

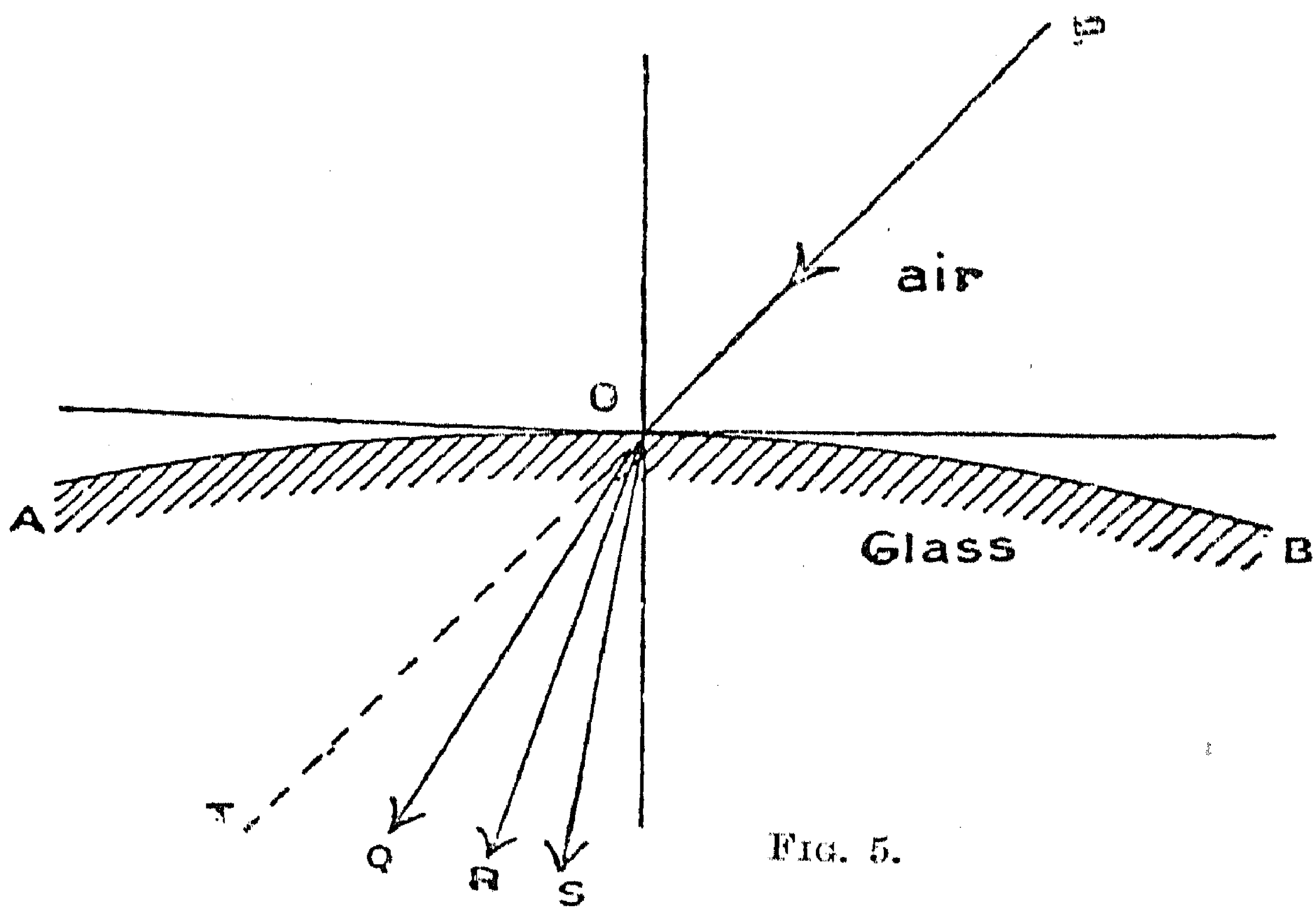

FIG. 5.

various colours, and also that different coloured lights are differently refracted. This phenomenon can be shown to take place not only with a prism, but with a single refracting surface; let a ray P O of white light strike the surface A B at O (Fig. 5), the refracted beam will consist of rays of various colours such as O Q, O R, O S in various directions.

Produce P O to T, then the angles T O Q, T O R, T O S through which the rays are bent from their

original direction P O are called their *deviations.* If some particular ray such as O Q be taken as the standard of reference, then the angles R O Q, S O R which the directions of the other refracted rays make with it are called their *dispersions.* This term is very appropriate, as it conveys vividly the idea of the spreading out which the various rays undergo on refraction. The result of this dispersion evidently is that rays of different colours have different refractive indices. As a general rule the red rays are the least bent, and have therefore the refractive index nearest to unity, and the violet rays are the most bent, and have the greatest index, or the red rays are the least and the violet the most refrangible. The dispersion is by no means the same in all substances or in all kinds of glass, and in some substances it is very irregular indeed.[1]

Dispersion is of vital importance in connection with photographic lenses, and must, therefore, be carefully studied. The dispersion produced by one refraction is comparatively small, but it can be increased by making the rays undergo two or more refractions at suitably arranged surfaces. The usual arrangement is the prism, which consists of a piece of glass, bounded by two plane surfaces, inclined at a suitable angle. If a prism be held close to the eye, and a candle be examined through it, both the deviation and dispersion will be very evident, for in order to see the candle through the prism it must be looked for in a direction considerably different from its actual direction, and also its edges will be vividly tinted with colour, one side being violet and the other red.

To make this clear, let a ray of light C P (Fig. 6) strike a prism at P, and let P Q and P R be the paths of the violet and red rays respectively after the first refraction, their dispersion will then be the angle Q P R. After a second refraction, let Q S and R T be their

[1] See Glazebrook's *Physical Optics*, chap. viii.

directions, which, owing to the increase of deviation, will be more inclined to each other than before.

To an eye receiving the rays Q S and R T, the red rays will show the object C as if it were in the direction S Q, and the violet rays as if in direction T R; therefore, if either ray could be stopped completely, the object would appear to be either all red or all violet, or, in other words, each colour gives rise to a separate image of the object C. If C is of an appreciable size, then the images of the different colours overlap, and

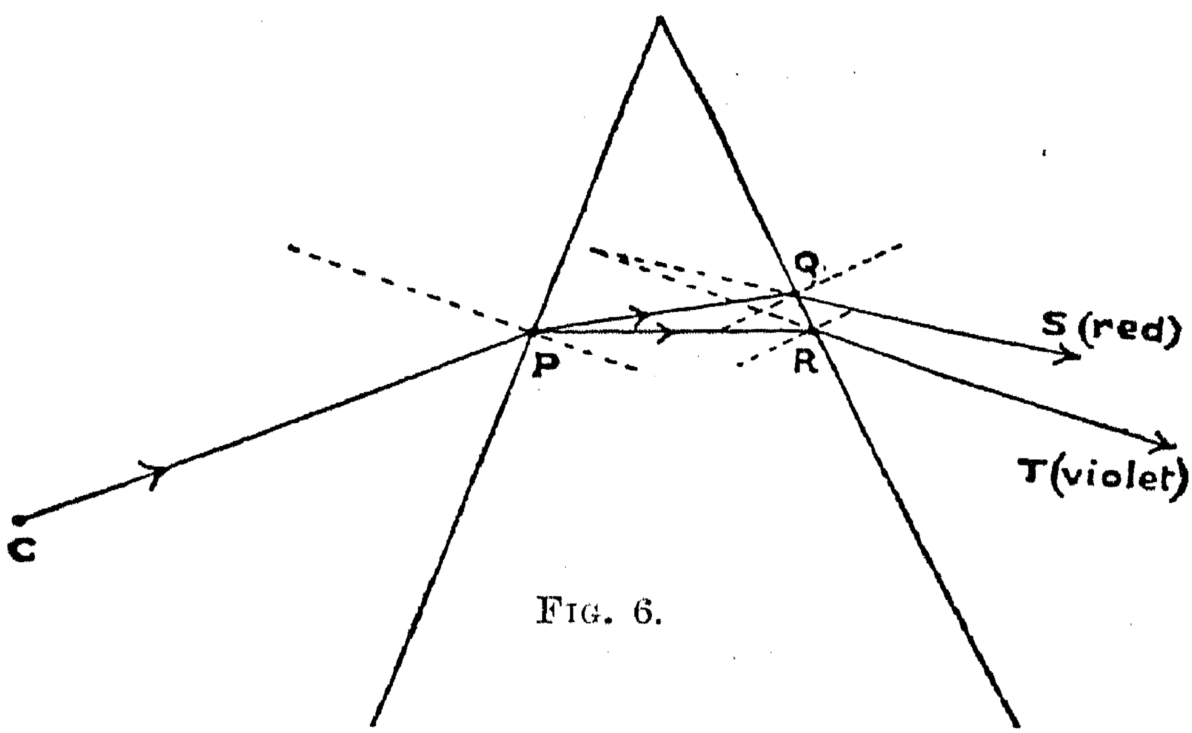

FIG. 6.

the result is that in the overlapping portion the colours re-combine and give the original colour of the object, while there are fringes of red and violet on either side. In this description two colours only, red and violet, have been mentioned for the sake of simplicity, but there will actually be images of the object C due to all shades of colour in the light. The image thus produced is a jumble of all the colours present, white at the middle and coloured at the edges; hence, if a prism is to be used to study the nature of light, some means must be found to separate the images due to the various colours.

17. The Spectroscope.—The instrument which is used for the separation of colours is called the spectroscope. The following are its main outlines (Fig. 7): A slit A is formed between two straight edges of metal placed parallel to the edge of the prism, one of the straight edges being movable, enabling the breadth of the slit to be altered at pleasure; L M is a lens so placed that the slit is at its principal focus, and thus, as will be shown further on, it will cause the rays to issue in a parallel pencil. After the refraction the rays of the same colour are all parallel, but the pencils for different colours are in different directions (red and violet are shown). These rays are now received by a telescope converged to a focus and viewed through an eye-piece, the telescope being mounted so that it can be rotated to take in all the parallel pencils of rays in turn.

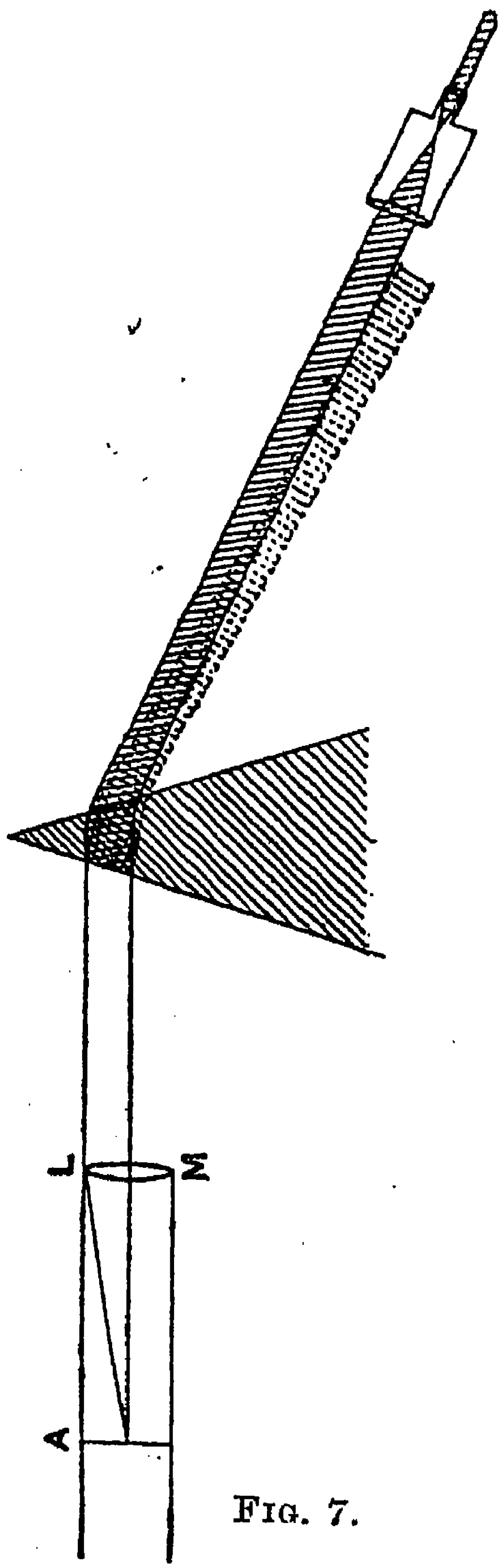

Fig. 7.

What is seen is an assemblage of images of the slit, one due to each of the colours present in the light. With one prism the separation is not very perfect, but more prisms can be used if desired, the rays passing through them in succession, but the description given is enough for our purpose.

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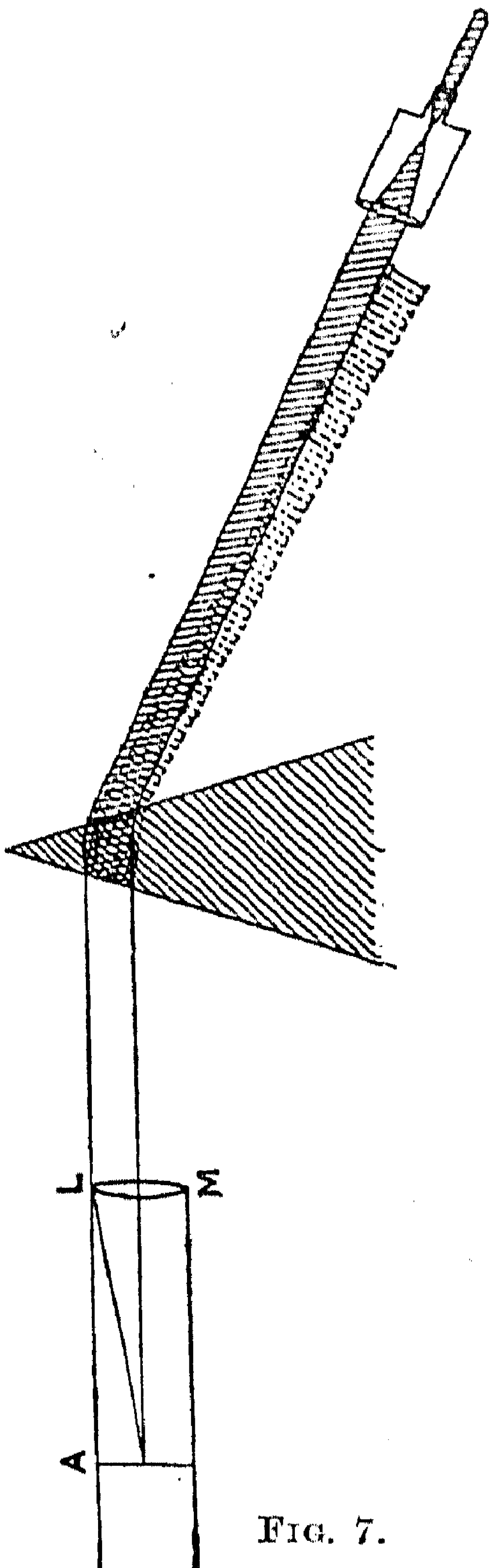


FIG. 7.

What is seen is an assemblage of images of the slit, one due to each of the colours present in the light. With one prism the separation is not very perfect, but more prisms can be used if desired, the rays passing through them in succession, but the description given is enough for our purpose.

If the light from a highly incandescent solid, such as platinum wire raised to a white heat by the passage

of an electric current, fall on the slit, and the appearance be examined through the telescope, it will be found to consist of a continuous band of light, the colour varying through every shade from violet to red, showing that an incandescent solid sends out light of every visible shade. This band of colour is called a spectrum, and we have just seen that when the light comes from a heated solid it is continuous, but this is not generally the case. Instead of the heated solid,

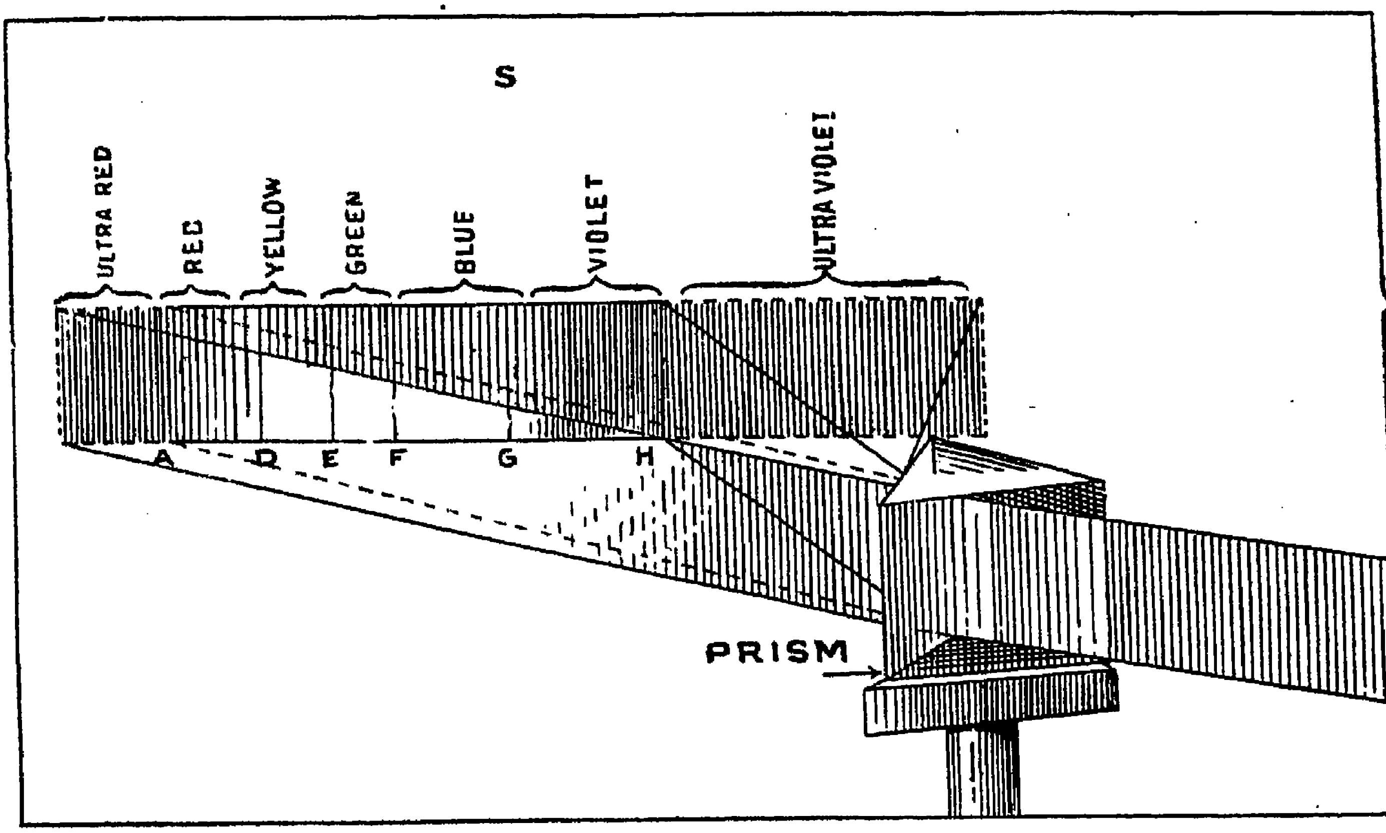


FIG. 8.

let the light from various metallic salts, placed in the flame of a spirit lamp or Bunsen gas-burner, be examined; we now get only a few bright lines—for example, potassium gives two reddish lines, and one in the violet which is difficult to see; lithium gives red and yellowish lines; strontium (the chief ingredient of red fire) gives a group of red lines and one blue line, while sodium gives one yellow line, which can be separated by powerful instruments into two close lines. But if sunlight be used, quite a different sight is

presented (Fig. 8), a band of [illegible] to violet, but crossed by [illegible] have been found by careful [illegible] dent with the bright lines [illegible] dark lines are called Fraunhofer [illegible] coverer, and are of [illegible] scope, enabling the presence [illegible] of many of our elements [illegible] lines do not concern us, except [illegible]

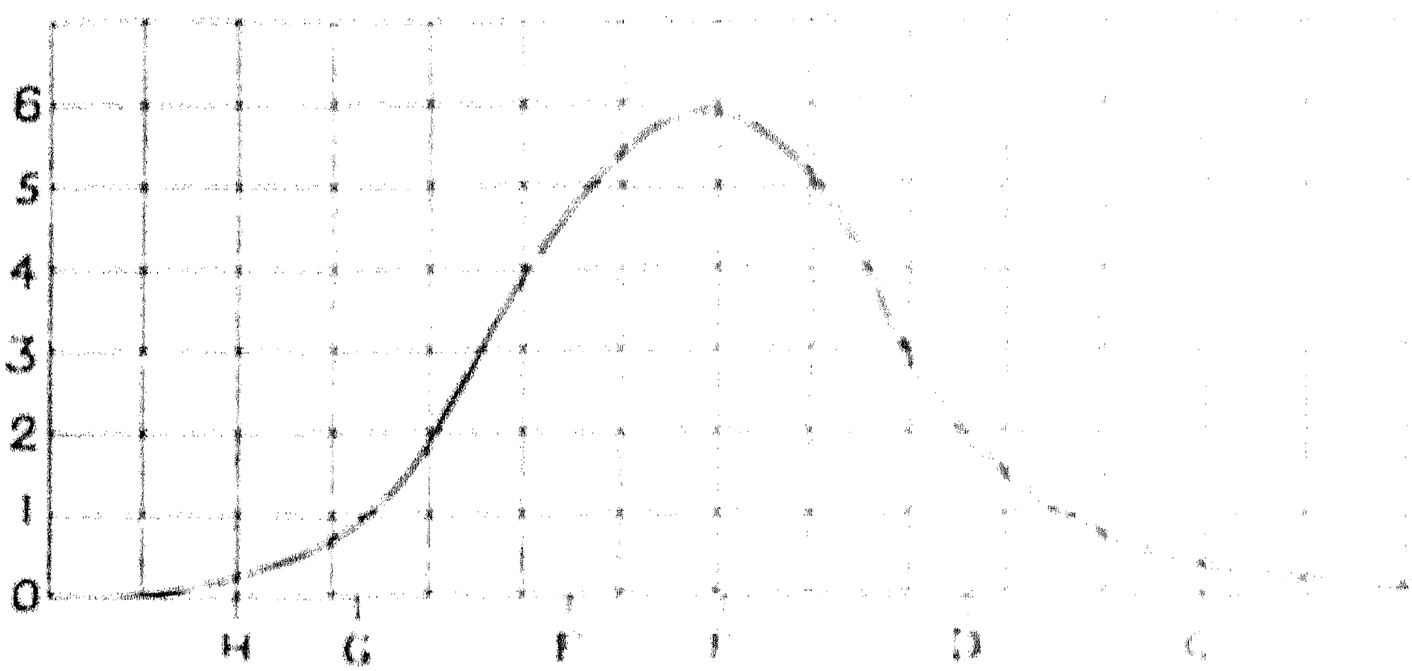


Fig. [illegible]

are denoted by letters, and are used [illegible] different parts of spectra.

18. Visual Intensity of Different Parts of the Solar Spectrum.—We must now compare the effect of light from various parts of the solar spectrum on the eye and on sensitive plates and papers. The curve in Fig. 9, given by Langley,[1] shows the [illegible] of light, as judged by the eye, along the [illegible] is shown the arrangement of the spectrum, the [illegible]

[1] S. P. Langley, On the [illegible] 1886, vol. [illegible], p. [illegible]

presented (Fig. 8), a band of colour stretching from red to violet, but crossed at intervals by *dark* lines, which have been found by careful measurement to be coincident with the bright lines due to many metals. These dark lines are called Fraunhöfer lines, from their discoverer, and are of the utmost importance in spectroscopy, enabling the presence, in the sun and other bodies, of many of our elements to be detected. These dark lines do not concern us, except that the principal ones

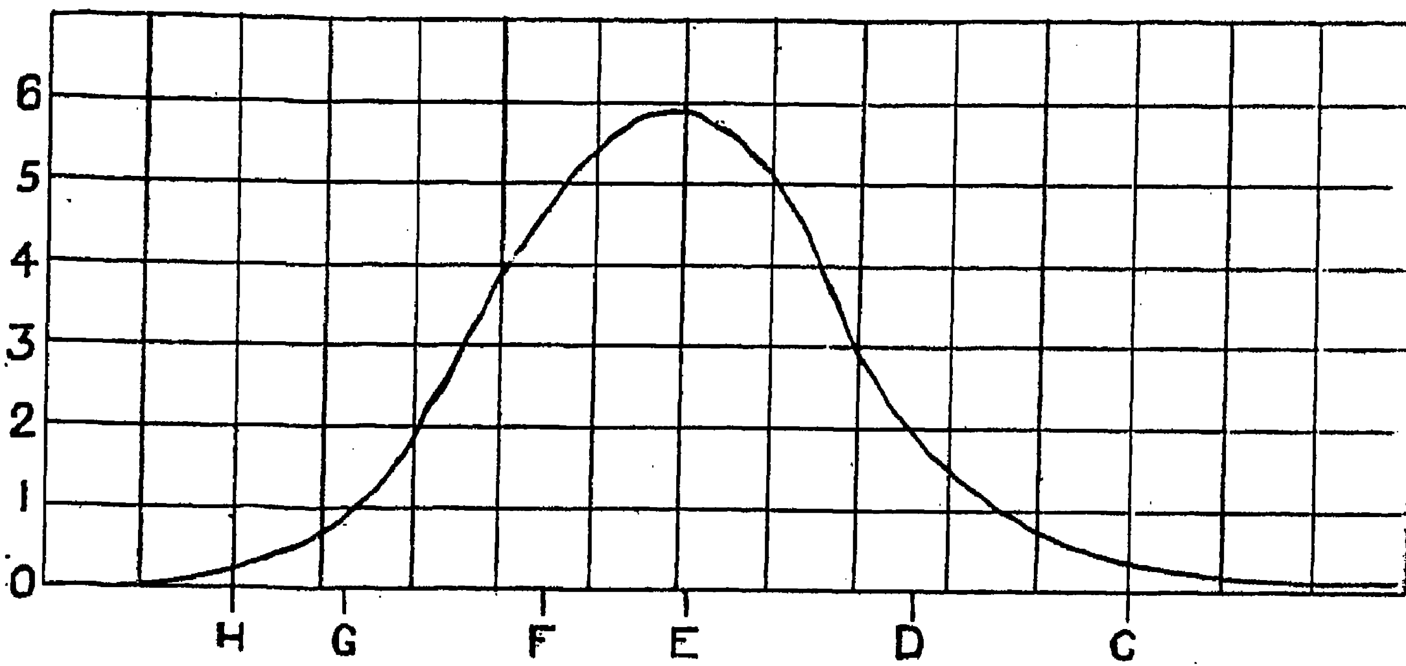


Fig. 9.

are denoted by letters, and are used to indicate the different parts of spectra.

18. Visual Intensity of Different Parts of the Solar Spectrum.—We must now compare the effect of light from various parts of the solar spectrum on the eye and on sensitive plates and papers. The curve in Fig. 9, given by Langley,[1] shows the distribution of light, as judged by the eye; along the horizontal line is shown the arrangement of the spectrum, the intensity

[1] S. P. Langley. On the cheapest form of light, *Phil. Mag.*, 1890, vol. xxx. p. 270.

at any point being given by the height of the vertical line at that point, drawn to meet the curve.

In this and in following diagrams, the relative intensity only is shown, and no two diagrams can be compared as regards absolute intensity. We thus see, from the diagram, that the greatest visual intensity is about the line E in the green, and before the line G, and after the line D, the intensities are comparatively small. We shall see below that the maximum of photographic action does not coincide with this maximum of visual effect.

19. Measurement of Density.[1]—To estimate the photographic effect of various parts of the solar spectrum we must be able to measure the density of deposit, in a plate, caused by the light, for when the plate is not over-exposed we may consider the density as very approximately proportional to the total quantity of the actinic rays that have fallen on it.

The following methods and results are those of Abney: By the density of a deposit we mean the proportion of the incident light which it stops; thus if half the light is stopped the density would be one-half, and perfect opacity is denoted by unity. The following arrangement will determine the quantity of light transmitted, and we can then reckon the quantity stopped, and thus get the density: A piece of ferrotype plate or blackened cardboard (Fig. 10), about the size of a half-plate, is taken, and a square aperture A is made in it; a square B equal to A is marked on the plate, but not cut out; both A and B are then covered with white translucent paper. If now a candle or lamp be placed in front of A (Fig. 11), both A and B will be illuminated, but if a rod of suitable size be placed vertically in front of A at the proper distance it will prevent any light

[1] "Photo-Chemical Investigations," by Hurter and Driffield, *Journal of the Society of Chemical Industry*, May 31, 1890, No. 5, vol. ix.

from the front falling on A. Now place a lamp behind the screen, then A is illuminated by the light which comes through from behind; the distances of the two lights can be arranged so that A and B appear equally bright. If now we place close behind A a piece of the negative to be examined it will shut off part of the light; to render A and B again of the same brightness the lamp behind must be moved up nearer to the screen. If we measure the distance of the lamp behind from

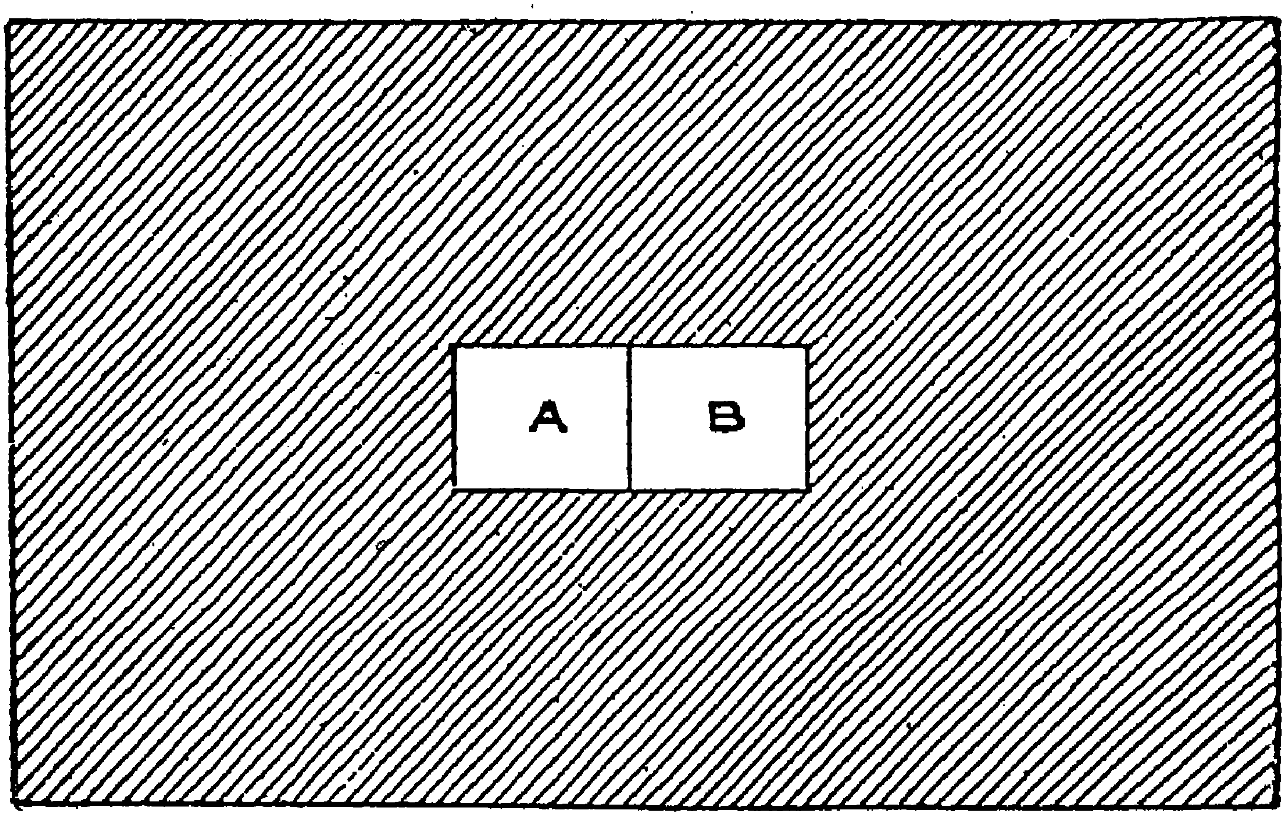


FIG. 10.

the screen in both cases we can at once calculate the fraction of the whole amount of light which the negative transmits. For example, suppose that in the first case the light was thirty-six inches away from the screen, and that in the second case it was nine inches away (one-quarter of the former distance), then the illumination of the negative was sixteen times as great as that of the bare paper in the first case; hence the negative transmits only one-sixteenth of the incident light, and its density is fifteen-sixteenths.

some cases the light coming through the negative

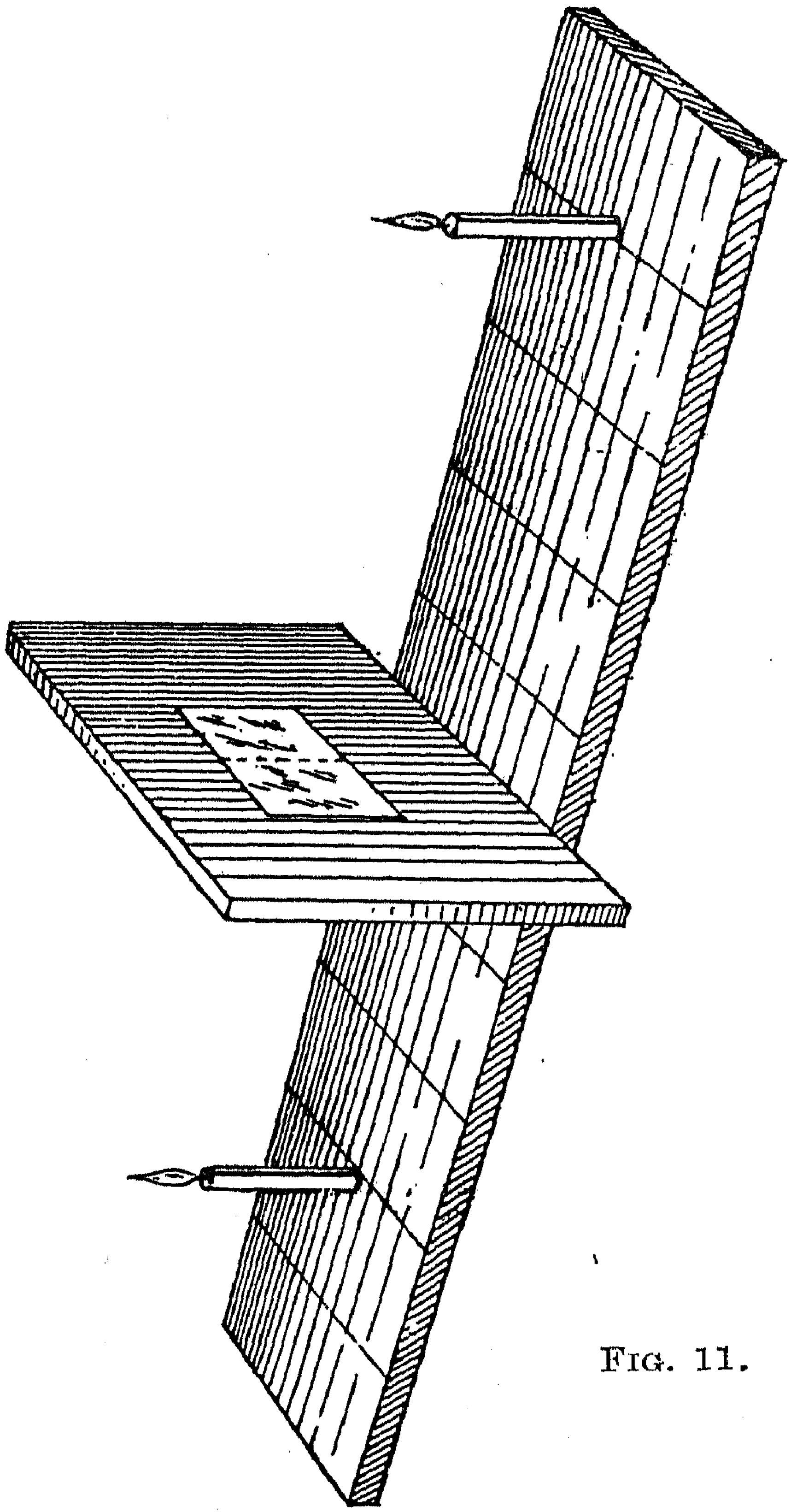


Fig. 11.

ghtly tinted, which makes it difficult to compare brightness of A and B, but this may be avoided

either by dyeing the paper a light coffee colour or by observing through pale canary-coloured glass. To avoid all extraneous light, the experiment should be conducted in a darkened room, or if this cannot be done the whole apparatus may be placed in a large box painted dead black inside, and the observations of A and B made through a small hole; in all cases care should be taken that light is not reflected by surrounding objects on to A or B. The actual size of A and B does not matter, but it is important that they should be equal, as it is found that correct estimates cannot be made unless the whole quantities of light coming from the two squares are equal.

In estimating the density of photographs of the solar spectrum a direct measurement at the different points cannot easily be made on account of the rapid variation from point to point; an indirect estimation must therefore be made. A scale of density is prepared by exposing small squares along one side of the plate, for equal times at different distances from a lamp; the distances being chosen to give a regular scale, and the absolute density of one of these squares is found as already described. The densities at different parts of the spectrum are then found by comparing them with the density of the squares on the scale.

20. Photographic Effect of the Solar Spectrum.—We must now examine some diagrams given by Abney. Fig. 12 shows the effect of the solar spectrum on silver bromide. As before, the vertical ordinates show the intensity of the effect at any point, and both in this and other diagrams the principal Fraunhöfer lines are marked to enable the portion of the spectrum in question to be easily recognized. We notice in this figure that the maximum effect is due to the portion near the line G. On referring to Fig. 9 we see that the whole of the part of the spectrum which acts on the bromide produces only a feeble effect on the eye. Again, for a mixture of silver bromide and silver iodide (Fig. 13)

the maximum effect is midway between the lines F and G. In both these cases, then, which give a fair idea of the capabilities of plates which are not isochromatic, the portion of the spectrum which produces the photographic effect is considerably different from that which most powerfully affects the eye; hence the resulting monochromatic rendering of coloured objects is not a true one. Violet and blue are represented on the print

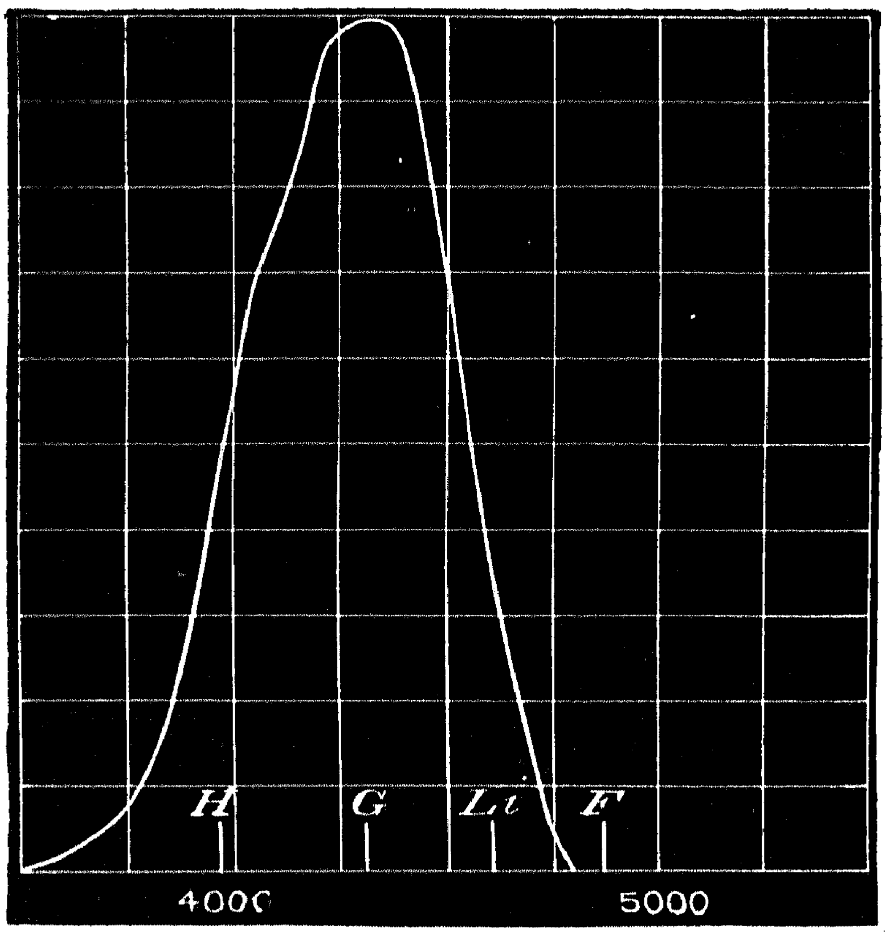


FIG. 12.

as light-coloured, while green, such as grass, is much more prominent, and red, such as that of a soldier's coat, comes out quite black. This explains many things familiar to the practical photographer—for instance, sky and light clouds both sending out light rich in violet rays are photographically very much alike, though different to the eye, and hence little difference between them is produced in a negative; or

in the evening, when the sun is low down, and the light has to pass through a considerable portion of the atmosphere usually laden with vapour, the violet rays are greatly intercepted, and the light is, in consequence, very red, so that although to the eye there is plenty of light, yet to the sensitive plate it may be nearly dark.

21. Orthochromatism.—To obviate these defects in the rendering of colour, it is necessary to make the

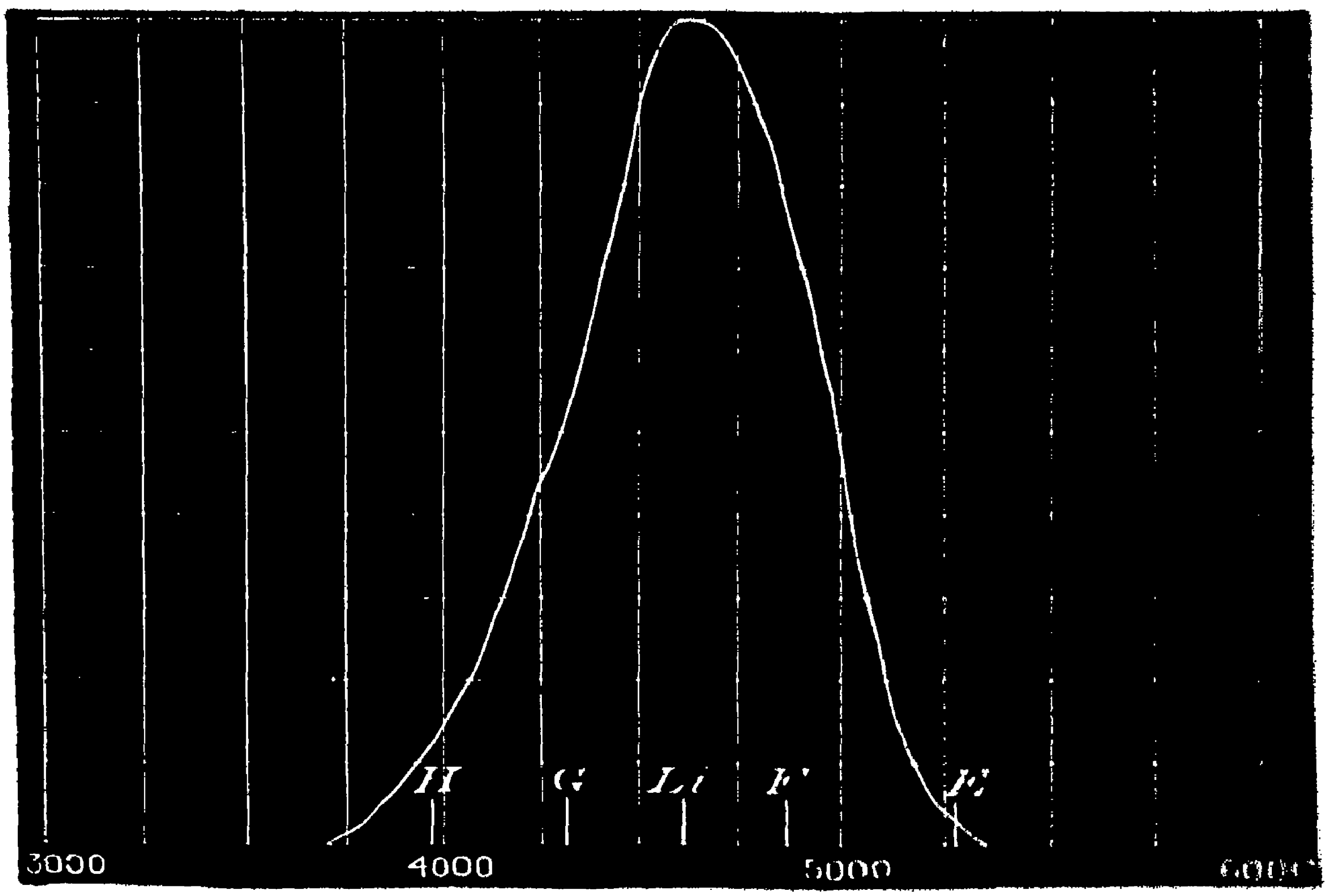


FIG. 13.

plate sensitive to that part of the spectrum which affects the eye. This has been done to a certain extent by staining the film with an organic compound, which increases its absorptive power for yellow and green rays, and in some manner not perfectly understood acts as a go-between, and enables the light to affect the silver salts; cyanin blue and erythrosin are such compounds. In Fig. 14 are two curves given by Abney. No. I. is

the intensity curve for an ordinary mixture of silver bromide and silver iodide, and No. II. is the intensity curve for the same mixture when dyed with erythrosin. The contrast is very marked; in No. I. the maximum is about the G line in the weakly luminous portion, while in No. II. the maximum has shifted to midway between the D and F lines, well within the luminous portion,

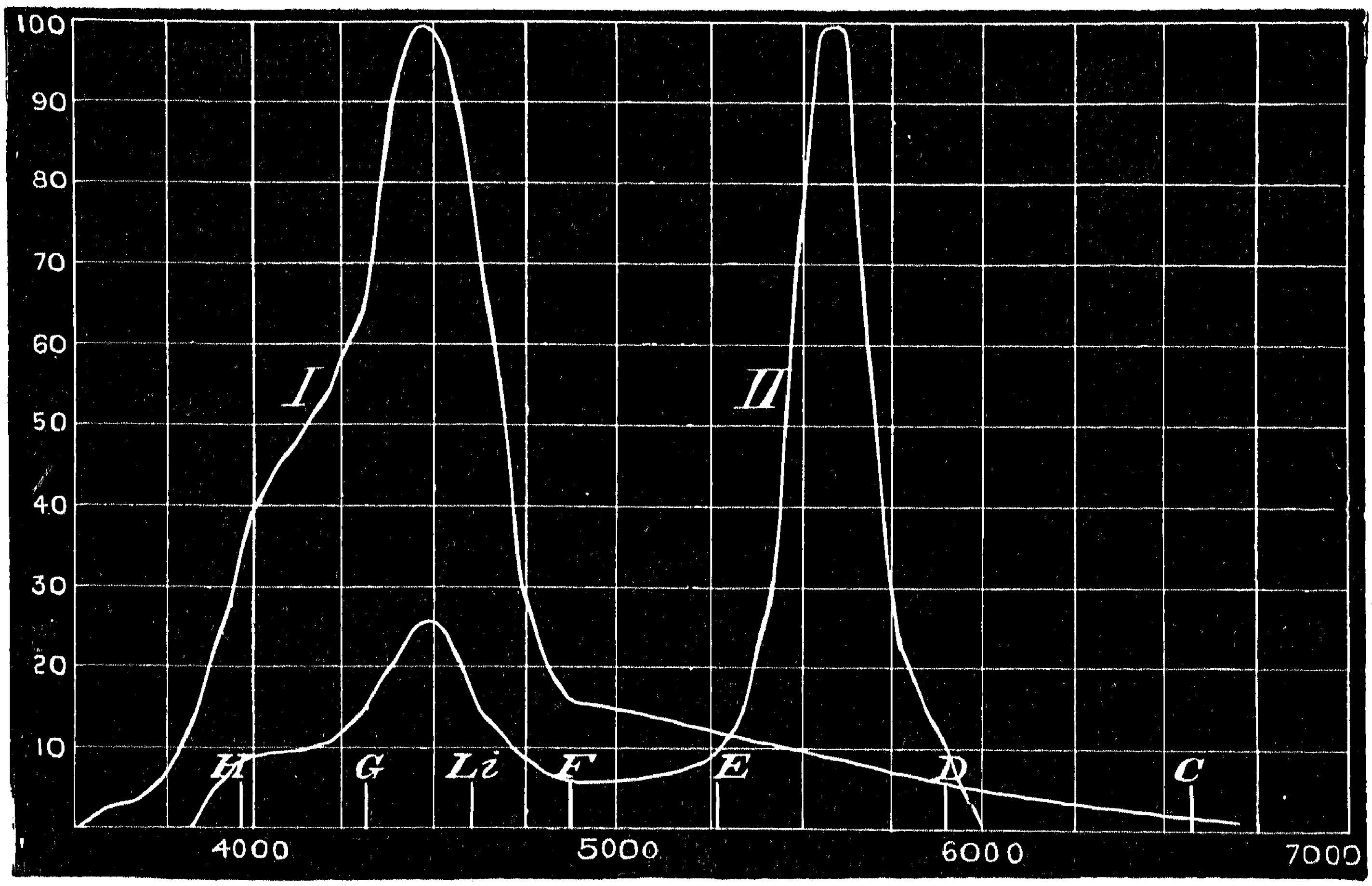


FIG. 14.

though a secondary maximum still remains near the G line.

These examples are enough to show the general nature of the phenomena; for more detailed information the reader is referred to Abney's original papers, or to his book,[1] where extensive information is given about both plates and papers, or to Vogel's *Das Licht im Dienste der Photographie* (Berlin, 1894).

[1] *Photography.* Text-books of Science Series.

22. Absorption.—Whenever light passes through any medium some part of it is absorbed, even though a thin layer of the medium may appear transparent. In inferior lenses the glass is sometimes of a yellowish tinge, and though this may not seem to the eye to cut off much light, yet it intercepts a considerable quantity from the violet end of the spectrum, and this makes the lens slow in action.

On the other hand, absorption by means of yellow glass is made of use in orthochromatic photography to improve the monochromatic rendering at the expense of speed. We have seen (Fig. 14, II.) that with a plate dyed with erythrosin there is a secondary maximum of effect near the G line, and this if uncorrected will produce an undue prominence of bluish objects; to obviate this, a yellow glass is interposed either before or behind the lens, which cuts off all rays beyond the F line. Thus the secondary maximum is excluded, and the rays are limited to those between the lines D and E, which are well within the visual portion of the spectrum. It is found that even clear glass absorbs a great portion of the violet rays, and hence, if extreme rapidity is required, some other substance should be used. Prof. Boys employed a quartz or pebble lens when photographing a flying rifle bullet.

To allow the process of development to be observed, it is necessary that the light in the dark-room should be such as lies well outside the photographically active portion of the spectrum, and yet within the visual portion. The only way to ensure that the light, transmitted by coloured glass or medium, is of a safe nature is to examine its spectrum, and compare it with the foregoing diagrams. As the result of such an examination, Abney recommends a combination of stained red and ruby glass for absolute safety, though stained red glass by itself is safe enough with ordinary care. A very good medium is the common orange paper which is used for packing purposes; three thicknesses of it

are safe when sunlight falls on the window. Canary medium should be used with caution, as it allows too much green light to pass.

When artificial light is required, a suitable lamp can easily be found among those in the market, the chief point then being to find one in which the combustion is good, and which does not get too hot. If electric light is available, a lamp made of red glass for this purpose can be obtained, thus avoiding the bother of screens.

The subject of absorption is important in considering the sensitiveness of plates, for the principle of energy shows that a plate is acted on by the light which it absorbs, so that the sensitizing dye should, if possible, be of the same colour as the rays which it is required to utilize. To render a plate sensitive to all rays, it would therefore be necessary to use a black dye if one could be found to act, though in this case the negative would be useless unless the dye could afterwards be bleached.

23. Photography in Colours.—Attempts have constantly been made to produce a photograph showing the natural colours of objects; we do not, of course, mean the commercial coloured photograph, which is merely painted, but one in which all the colours are produced by purely photographic processes. Even in the early days of photography some colour effects were obtained. Sir John Herschel exposed a sensitive paper to the solar spectrum, and obtained coloured prints, but could not fix them. Becquerel, a little later, produced coloured photographs of the spectrum by using silver chloride on a (polished ?) silver plate, and this is very interesting in view of recent results. M. Vallot, in 1890, obtained coloured prints on paper sensitized by a special process, but could not fix them. Prof. Vogel, Mr. R. E. Ives, and others have obtained coloured photographs by taking three or four negatives with light passing through coloured screens, and super-imposing the prints

from these in their respective colours. "The method consists in taking a red, a yellow, and a blue negative of objects on plates specially sensitized for colours. The three negatives are then printed on to one and the same paper by means of complementary coloured rollers on stones. In order to obtain the colours exactly complementary to those of the negatives, the colours used for printing were either the coloured substances themselves or some substance whose equivalence to these had been determined spectroscopically."[1] In this manner good results have been obtained, though the process is necessarily costly.

One curious case which was the result of accident is worthy of mention; at a meeting of the Manchester Philosophical Society of April 8, 1857, Mr. Sidebotham communicated the following: "In the ordinary collodion negatives on glass we occasionally meet with examples of partial natural colouring, such, for instance, as a green tint on the foliage. I have had one in which the green and red in a photograph of some scarlet geraniums were tolerably bright, and I have here on the table a landscape with trees, and a red-brick house taken in bright sunshine, and you will see the green foliage and red house are tolerably well marked in colour."[2] This photograph was fixed, and after thirty-five years the colours still remained vivid, but all attempts to reproduce the effects failed. A great step has recently been made by Lippman, who has succeeded in taking coloured photographs on plates; his method was rather a surprise, for it depends on the physical properties of light, whereas progress was looked for mainly on the chemical side by the discovery of suitable sensitive compounds. A brief description of the method and theory may be interesting.

24. Lippman's Coloured Photographs.—These are produced by taking the photographs in the ordinary

[1] *Nature*, vol. xlvi., p. 263.
[2] *Liverpool and Manchester Journal*, April 15, 1857.

way on plates backed with some reflector such as a clean mercury surface; wet plates are used, as the grain

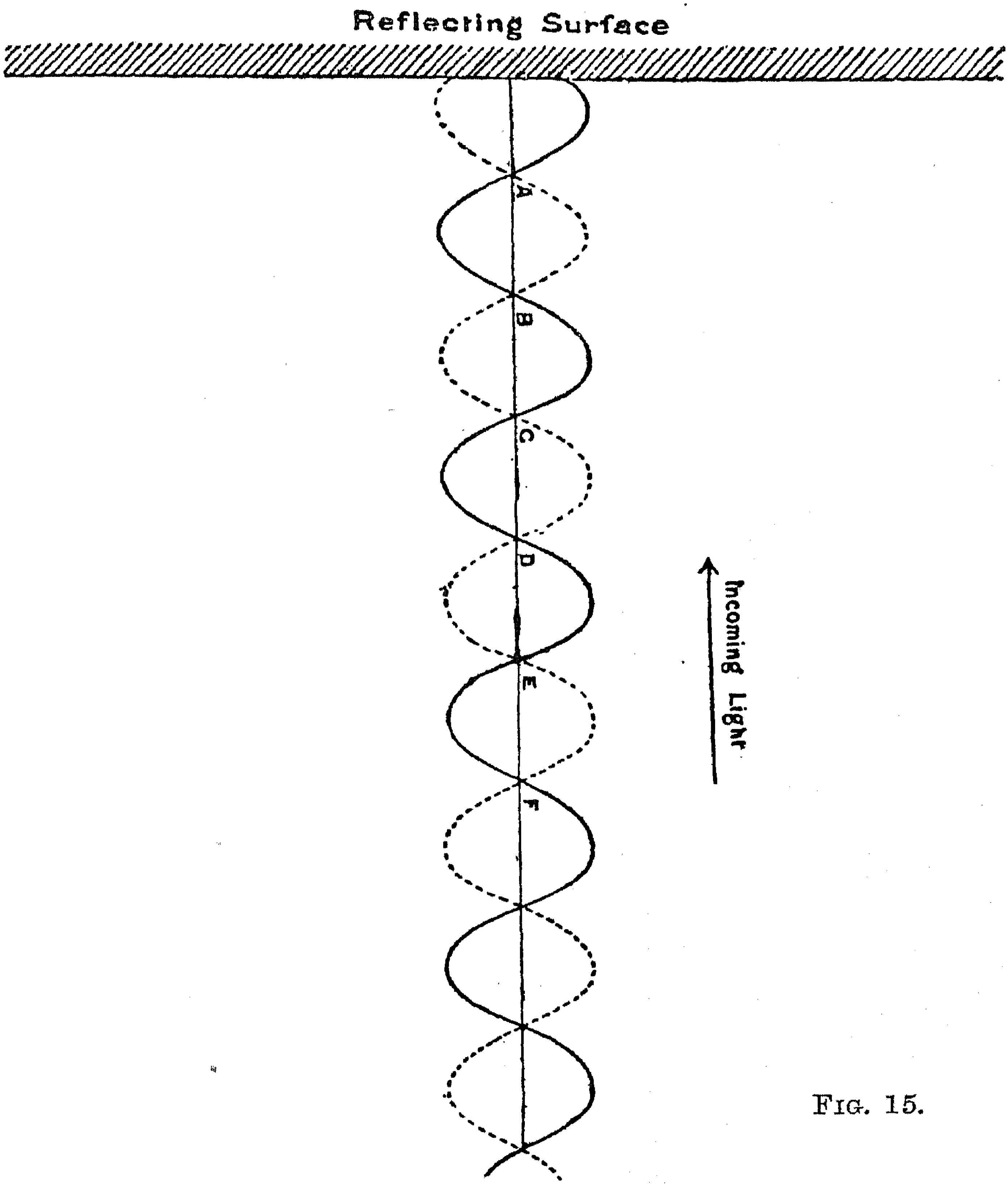


FIG. 15.

of dry plates is too coarse. The photographs are developed in the ordinary way, and the colours then

appear ; the peculiarity is that no coloured pigment is produced, the colour being due to the peculiar arrangement of the deposit.

The outline of the theory is as follows : When waves are incident directly on a reflecting wall, the incident waves interfere with those reflected, producing what are called stationary waves, in which the vibration disturbance, instead of advancing, remains at rest. The effect produced is very much like the vibration of the string of a musical instrument.

This is diagrammatically represented in Fig. 15 : at a series of points A, B, C, D, etc., called nodes, half a wave-length apart, there is rest, while between them there is vibration, most intense at points midway between them. The wave-length of light being very small, a great number of these nodes occur in the thickness of the film, and at them no effect is produced on the sensitive plate, but midway between them chemical action takes place. Hence when the plate is developed the silver is deposited, not uniformly throughout the thickness, but in layers at regular intervals ; the distances between the layers, being half the wave-length of the light, are different for light of different colours. When the negative is viewed by reflected light the photograph is seen in natural colours. There does not seem to be much hope of making this process commercial, as great care is required in the manipulation, and it is obviously very difficult to produce anything like a negative from which prints can be obtained.

CHAPTER II

ELEMENTARY THEORY OF LENSES

25. Definition of an Image.—We have now learned something of the nature of light, and must direct our attention to the production of pictures. We want to be able to throw a picture of the object to be photographed on a sensitive plate, and we must clearly understand what is meant by this; a little consideration will show that what we have to do is to make the illumination at each point on the plate depend solely on the illumination of some one point of the object, and independent of that at other points. In fact, each point of the image must correspond to, and depend solely on, some point of the object, and it is to obtain this result that lenses are employed. We can, however, produce pictures without the aid of any lens at all by means of a small aperture, or pin-hole, and this we shall consider first, as being the simplest case, and giving the opportunity of introducing some points of great importance.

26. Pin-hole Photography.—It is well known that if a hole be made in the shutter of a darkened room, and a sheet of paper be held near it, a picture of external objects is thrown on the paper; the phenomenon is not uncommon, and imperfect images are often cast when the light shines through cracks or slits—for instance, most people must have noticed, when lying awake in the morning with the blinds down, the confused motion

of the masses of light and shade on the ceiling caused by anything passing outside the window.

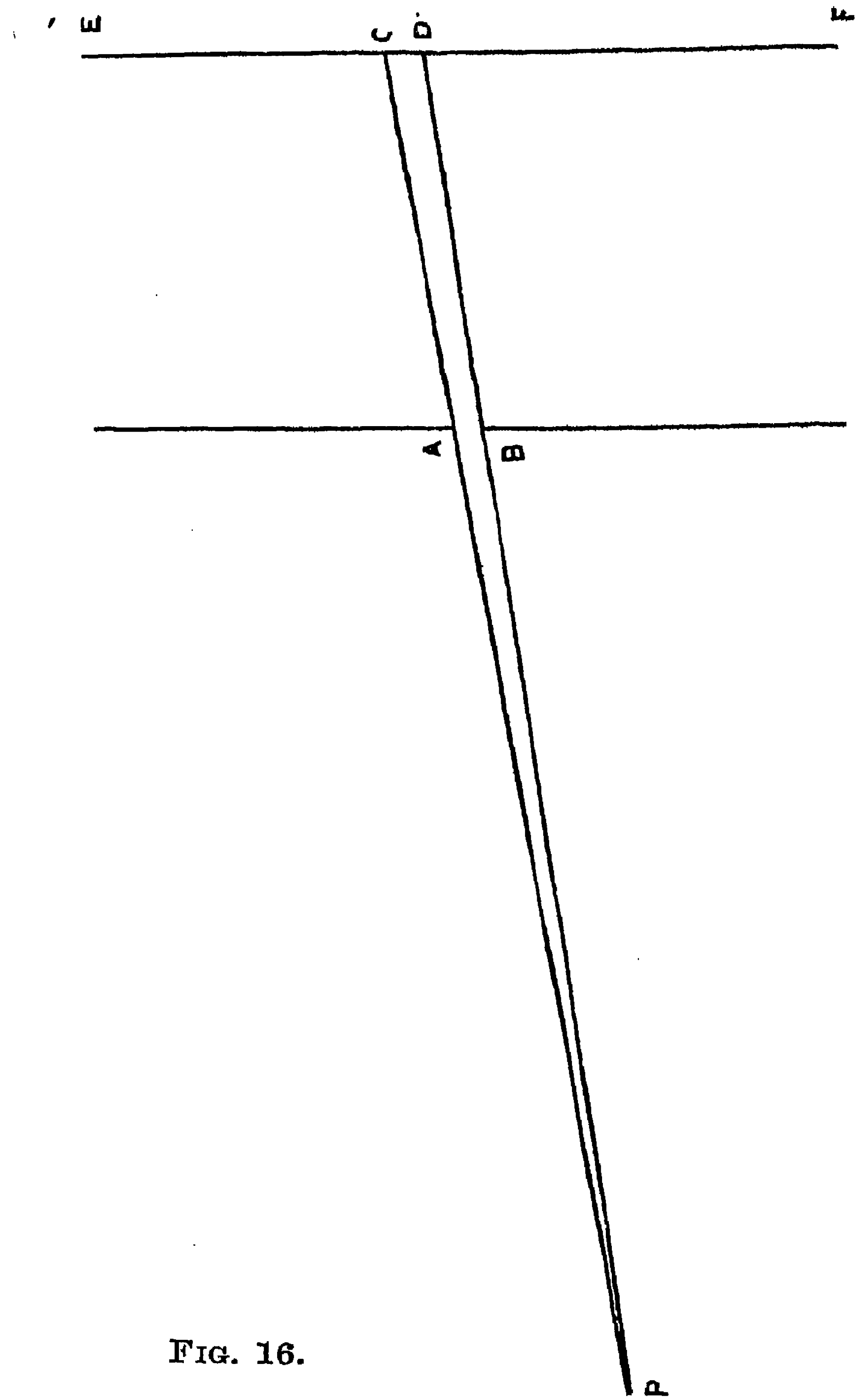


FIG. 16.

The elementary explanation is not difficult. Let P (Fig. 16) be a luminous point, and A B a small hole in

a screen (the shape of the hole is not of much consequence, but a circular one is generally used); the only rays of light admitted are those lying between P A and P B, forming a small cone of light—this will strike the screen E F behind, and produce a small patch of light C D. If the hole is small enough the patch C D will not appear to the eye to differ appreciably from a point of light, and the illumination at each point of the screen will depend solely on that of the corresponding point of the object, and a picture will therefore be formed.

27. Sharpness of the Image.—As far as we have seen yet, it does not much matter at what distance behind the hole the screen E F is placed, as the picture will always be in focus; but there is another thing to be considered, the separating or defining power of the arrangement, on which the sharpness will depend.

When a picture smaller than the object is produced, there must naturally be some suppression of detail; if the optical instrument were perfect, it would reproduce every detail exactly, so that if the picture were magnified everything could be seen. But this is never the case. We have already seen that with a pin-hole a point of light gives rise to a patch, not a point, of light in the picture. If, then, two points of light in the object be near together, the patches in the picture produced by them may be so close as to be mixed up. The eye is no exception to the general rule, as we know from experience, for we cannot distinguish close objects at a considerable distance. In practice, this imperfection of optical instruments is not of any great disadvantage if it can be kept within limits, so that the picture produced is, at least, as good as that shown us by the eye. This separating power of any instrument can be studied by piercing two holes in a cardboard screen, placing a lamp behind, and finding how far off they can be viewed through the instrument, and yet appear distinct from each other.

The defining power of an instrument is measured by

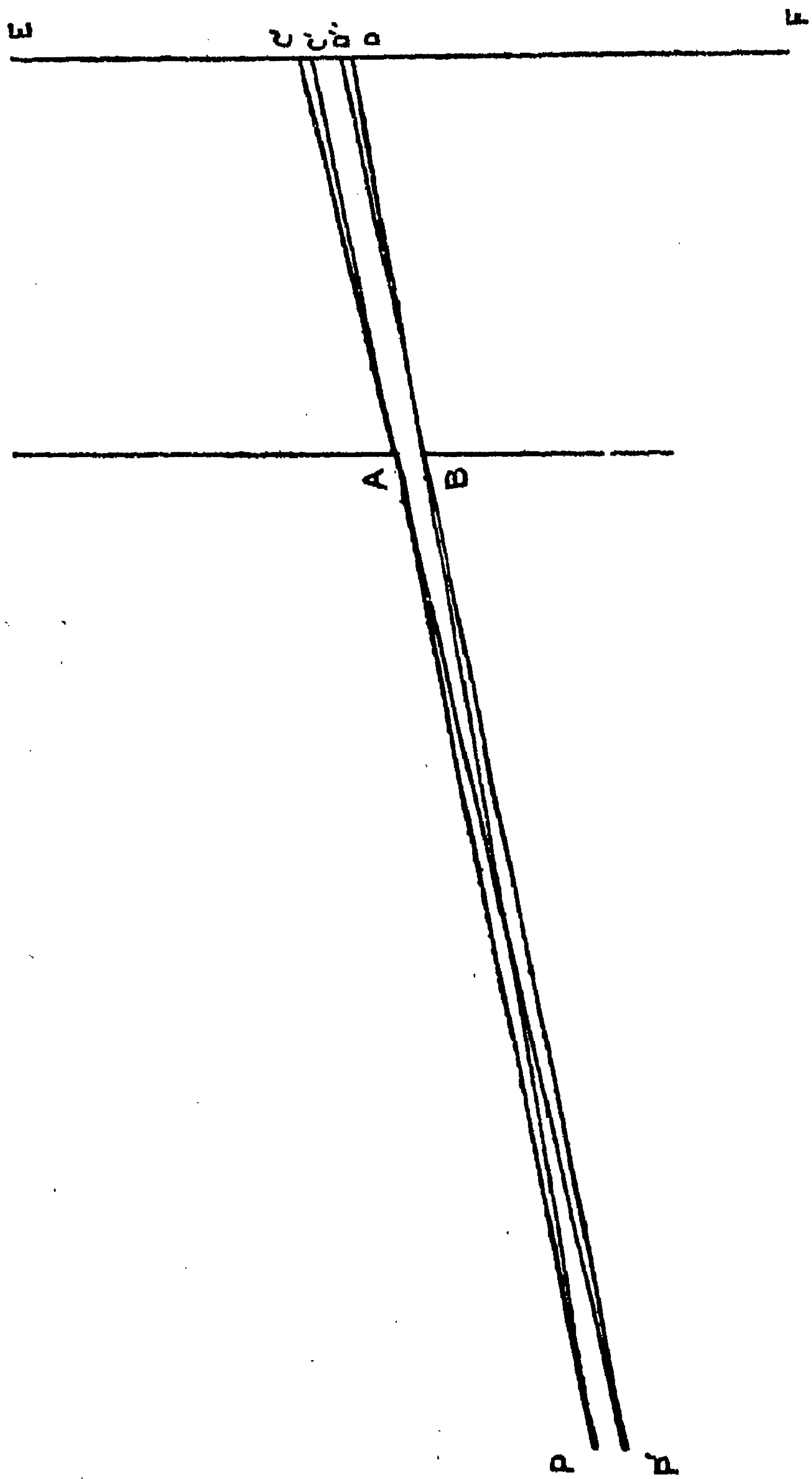


Fig. 17.

the angle subtended at the instrument by the distance

between the closest pair of objects that can be separated. In the case of the eye objects must subtend an angle of one minute, which is about the angle subtended by a length of eighteen inches set upright at the distance of a mile.

Obviously, if points are to appear distinct in the picture, the patches which represent them must appear to the eye to be separated when the picture is at the distance of distinct vision, and hence the smaller these patches the better will be the definition of the instrument. Wallon takes the diameter of the smallest permissible patch to be ·01 cm. or ·004 inch.

28. If, in the case of a pin-hole, we have two objects $P\,P'$ near together (Fig. 17), then the corresponding patches $C\,D$, $C'\,D'$ on the screen $E\,F$ will overlap, and the two images are not separated; this will be to some extent remedied if the screen is moved further back, for then $C\,D$ and $C'\,D'$ will be more separated, but at the same time the sizes of the patches of light will be increased, and the advantage gained is doubtful. When the objects $P\,P'$ are at a great distance from the hole the cones of light become cylinders, and their sections will be of the same size wherever they are cut by $E\,F$, and hence in this case, if we move the screen further back, the patches will move further apart, but not increase in size (Fig. 18); so that although according to the rough theory the position of the screen does not matter, yet we shall improve the definition by putting $E\,F$ at some definite distance.

The problem of pin-hole photography has been worked out by Lord Rayleigh from the considerations of physical optics,[1] and he shows that the patches of light are not sharply marked, but that the light fades away gradually as we proceed from the centre to the edge, the rate of fading depending on the distance of the screen $E\,F$ and the diameter of the hole. The more rapidly the light fades, the nearer together can we bring

[1] *Phil. Mag.*, 1891, vol. xxxi. p. 87.

two patches without confusion; thus the definition

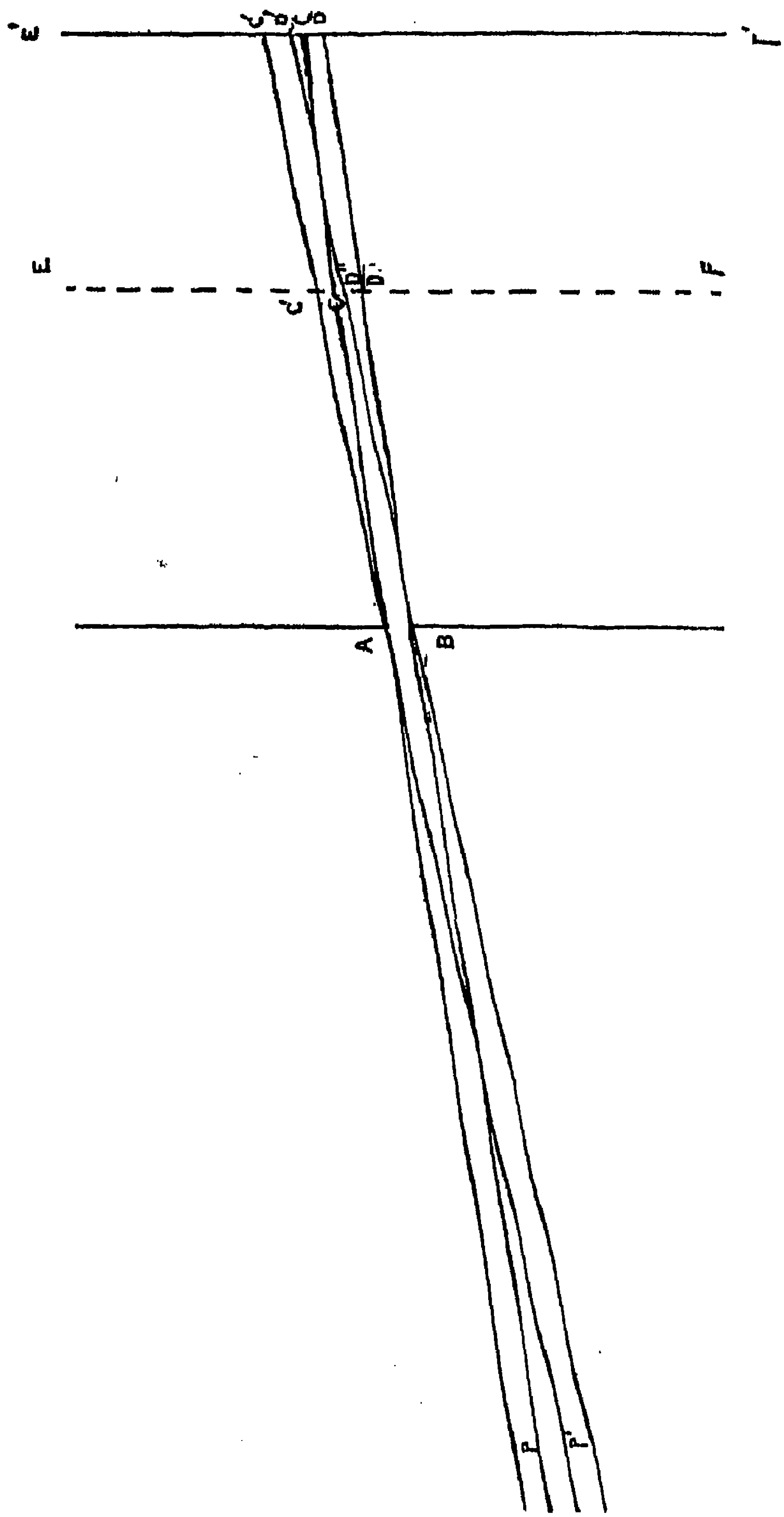


FIG. 18.

depends on the size of the hole and the distance of the

screen. As the result of both theory and experiment, Lord Rayleigh has given a relation between these quantities for obtaining the best results.

If d be the diameter of the hole, and f the distance of the screen E F from it, the relation is

$$d^2/f = 10^{-5} \times 6\cdot 0 \text{ inches} = 10^{-4} \times 1\cdot 5 \text{ cm.}$$

d and f being measured in the first case in inches, in the second in centimetres; the light being taken as that coming from the most photographically active portion of the spectrum.

M. Colson[1] has written a pamphlet on pin-hole photography, the results in which do not agree with those given above, but his analysis does not appear to have been as thorough, and besides this, his comparison of a pin-hole with a lens, as regards distortion, is faulty.

29. Disadvantages of Pin-hole Photography.—Though at first sight a pin-hole may seem to offer great advantages on the score of economy and simplicity, yet there are serious drawbacks. In order to get good definition, the plate must be placed at an inconvenient distance from the hole, and since only a small quantity of light is admitted the exposure must be long. Lord Rayleigh found that to photograph trees he had to expose for an hour and a half, even in sunshine. This length of exposure is, of course, quite prohibitive.

30. Rôle of the Lens.—To increase the quantity of light a larger aperture must be used, and this would destroy the picture, unless some special apparatus could be found to ensure that the illumination at each point of the screen still corresponds solely to that at some point of the object; it is for this purpose that a lens is used. The most important part of our subject is, therefore, that dealing with lenses, and these, being complicated and liable to many defects, require very careful study.

[1] *Photographie sans Objectif.* Gauthier-Villars, Paris.

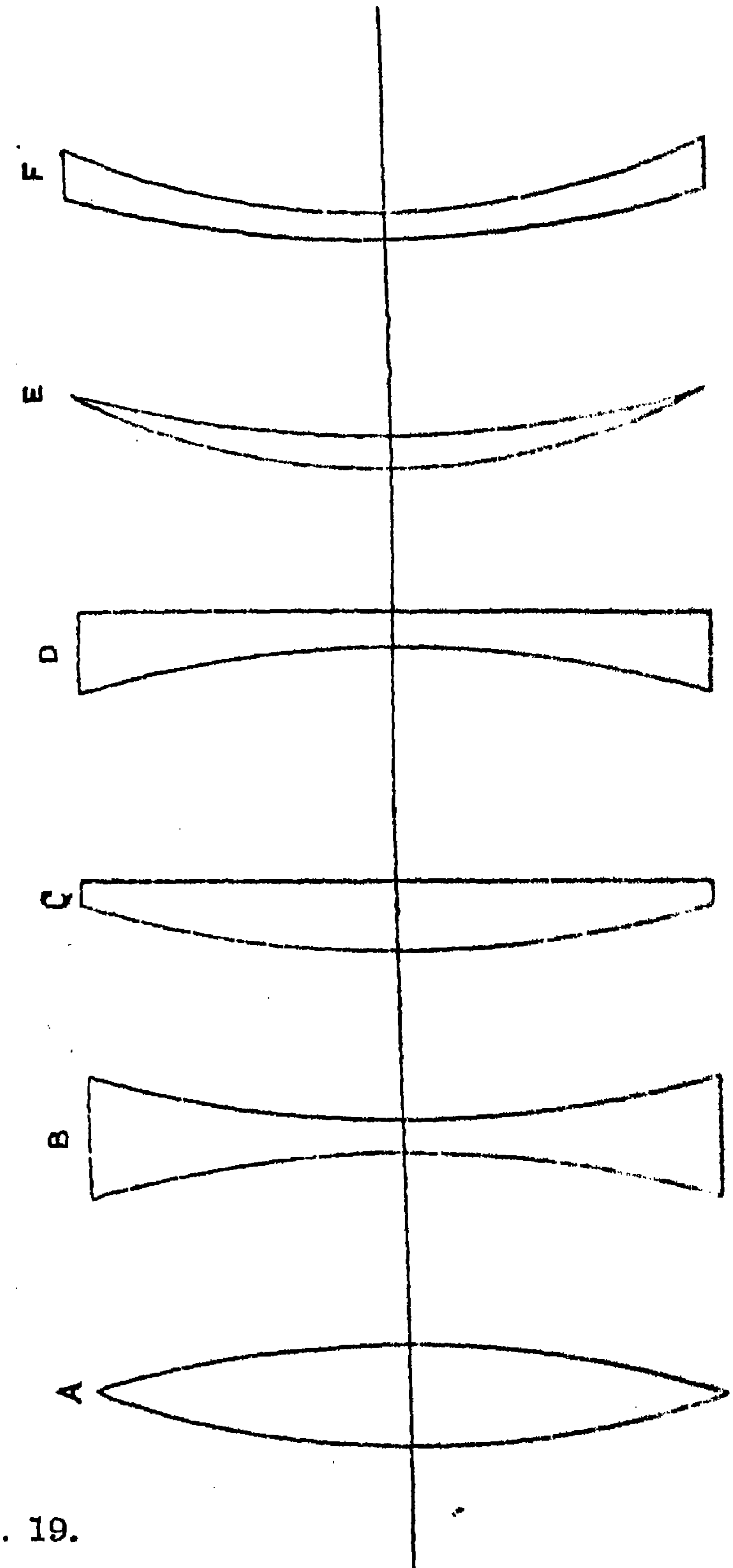


FIG. 19.

A lens may be defined to be a piece or assemblage of pieces of glass bounded by surfaces which are portions

of spheres; it has been proposed to make the surfaces portions of ellipsoids of revolution on account of some advantage which these forms seem to offer. But it is doubtful, considering all the corrections necessary, whether anything would be gained by deviating from the spherical form, and even if there were any advantage the labour of the necessary calculations and the mechanical difficulty of grinding would be prohibitive. Astronomical lens grinders finally correct lenses after they are ground, testing the surface point by point, and rubbing where necessary, and this may amount in some cases to giving the surfaces an ellipsoidal form.

Lenses are made of various types and kinds of glass, and are usually named according to their shape (Fig. 19). A is a double convex lens, B double concave, C plano-convex, D plano-concave, and so on; the lens E, bounded by two spherical surfaces with their concavities in the same direction and thicker in the centre than at the edge, is called *meniscus*.

For purposes of theory, lenses are divided into two classes, thick and thin lenses, the thickness in the latter being small compared with their radii of curvature. These are very rarely realized in practice, but their consideration is easier than that of thick lenses, and leads to many useful results.

We shall therefore begin with thin lenses, and then proceed to extend the theory to thick lenses, to which it will afford an introduction.

We must begin with the refraction of rays from a luminous point, passing from air into glass bounded by a spherical surface, and in this chapter we shall confine ourselves to a small pencil of rays all near the line joining the luminous point to the centre of the surface. In the next chapter the case of larger pencils and oblique small pencils will be treated; the former will be enough for our immediate purpose, and will yield many useful results.

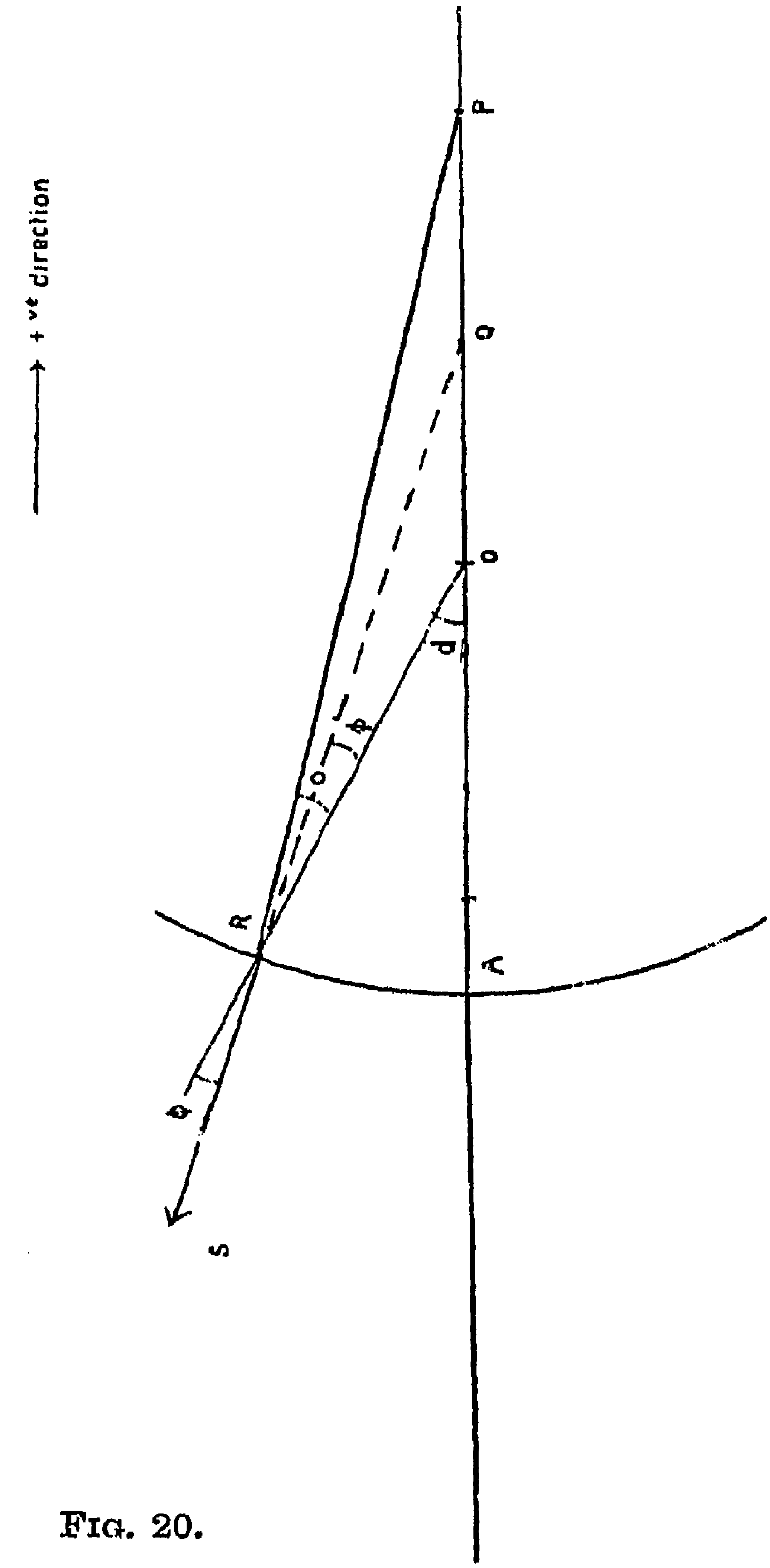


FIG. 20.

31. Refraction at a Spherical Surface.—Let R A (Fig. 20) be the section by the plane of the paper of the

spherical bounding surface, and P the luminous point. To begin with, we must make careful conventions about the directions which are to be reckoned positive and negative; by doing this we can avoid a multiplicity of formulæ, which leads only to confusion.

The following convention will always be adhered to in this treatise :

(*a*) In the case of spherical surfaces or thin lenses all distances are to be measured from the point in which the axis cuts the surface. In the case of a thick lens the lengths will be measured from two points called nodal points, as explained further on.

(*b*) Lengths are reckoned positive when they are measured from the starting-point in a direction opposite to that of the incident light.

The axis of a single spherical surface is that line which joins the centre of the sphere to the middle point of the surface; and the axis of a lens is the line joining the centres of the bounding spherical surfaces.

In all figures the light is taken as coming from the right hand towards the left; the positive direction is, therefore, from left to right.

Thus in Fig. 20 all the lines A O, A Q, A P are in the positive direction.

Let P, R, S be the path of a ray of light near the axis, and let S R produced backwards meet the axis A P in Q; let O be the centre of the surface.

Let A O $= r$, A P $= u$, A Q $= w$, and let μ be the refractive index of the glass.

Let the angle P R O, the angle of incidence, be θ, and the angle Q R O, which is equal to the angle of refraction, be ϕ, and let the angle R O A be a.

Then
$$\frac{\mathrm{O\,P}}{\mathrm{R\,P}} = \frac{\sin \mathrm{O\,R\,P}}{\sin \mathrm{R\,O\,P}} = \frac{\sin \theta}{\sin (180^\circ - a)} = \frac{\sin \theta}{\sin a}$$
$$\frac{\mathrm{O\,Q}}{\mathrm{R\,Q}} = \frac{\sin \mathrm{O\,R\,Q}}{\sin \mathrm{R\,O\,Q}} = \frac{\sin \phi}{\sin (180^\circ - a)} = \frac{\sin \phi}{\sin a}$$

$$\therefore \frac{OP}{RP} \times \frac{RQ}{OQ} = \frac{\sin\theta}{\sin\phi} = \mu, \therefore OP \times RQ = \mu RP \times OQ$$

(Note that r is measured from A to O, not *vice versâ*.) Now $OP = AP - AO = u - r$, $OQ = AQ - AO = w - r$. When the pencil of rays is small and *confined near the axis*, so that the *angle R P A is small for all rays of the pencil*, then we have *approximately*

$$RP = AP = u, \qquad RQ = AQ = w$$

and the relation found above becomes

$$w(u - r) = \mu u (w - r)$$

or

$$\mu u r - w r = (\mu - 1) u w$$

or dividing by $u w r$ we get—

$$\frac{\mu}{w} - \frac{1}{u} = \frac{\mu - 1}{r}$$

This is the relation connecting the distances from A of the points P, and Q the point in which one of the rays cuts the axis after refraction. Since this relation does not involve the inclination of the ray to the axis, it is true for *all* rays *near* the axis, and these will therefore, after refraction, all pass through Q, which may be called the image of P.

Hence, if only a small portion of the refracting surfaces be used, all rays, which before refraction pass through a point (or converge towards one), will, after refraction, either pass through or converge towards another point.

In our figure the rays, after refraction, will, if produced backwards, pass through a point, and the image is called virtual, the effect being that to an observer in the glass the light would appear to come from Q.

Some numerical examples will serve to drive home the convention of signs, and show the meaning of the relation obtained.

Example I.—Let the surface be as on Fig. 20, and let $r = AO = 6$ inches, $u = AP = 18$ inches, $\mu = 1{\cdot}5$ required the position of Q.

We have

$$\frac{\mu}{w} = \frac{1}{u} + \frac{\mu - 1}{r} = \frac{1}{18} + \frac{\cdot 5}{6} = \frac{1}{18} + \frac{1}{12} = \frac{5}{36}$$

$$\therefore \frac{1 \cdot 5}{w} = \frac{5}{36} \text{ or } w = \frac{36 \times 1 \cdot 5}{5} = 10 \cdot 8 \text{ inches.}$$

$\therefore$ A Q $= w = 10 \cdot 8$ inches, or Q is $10 \cdot 8$ inches from the surface on the positive side; that is, on the same side as the object, and is virtual.

Example II.—Take the same surface, but turned in the opposite direction; the radius measured from the surface is now in the negative direction, hence (Fig. 21)

$$r = -\ 6 \text{ inches, } u = 18 \text{ inches, } \mu = 1 \cdot 5$$

$$\therefore \frac{\mu}{w} = \frac{1}{u} + \frac{\mu - 1}{r} = \frac{1}{18} - \frac{1}{12} = -\ \frac{1}{36}$$

$$\text{Or } \frac{v}{1 \cdot 5} = -\ 36, \therefore v \doteq -\ 54 \text{ inches.}$$

Or the image is in this case on the negative side, that opposite to the object, at a distance of fifty-four inches from the surface, and it will be real.

We have proved the relation

$$\frac{\mu}{w} - \frac{1}{u} = \frac{\mu - 1}{r}$$

between the distances of object and image from the surface; if the luminous point from which the light comes is very distant we may put

$$u = \infty \text{ or } \frac{1}{u} = 0 \text{ and then}$$

$$\frac{1}{w} = \frac{\mu - 1}{\mu\, r} \text{ or } w = \frac{\mu\, r}{\mu - 1}$$

and this value of w we may conveniently call the "focal length of the surface" for entering rays, and denote it by the letter f.

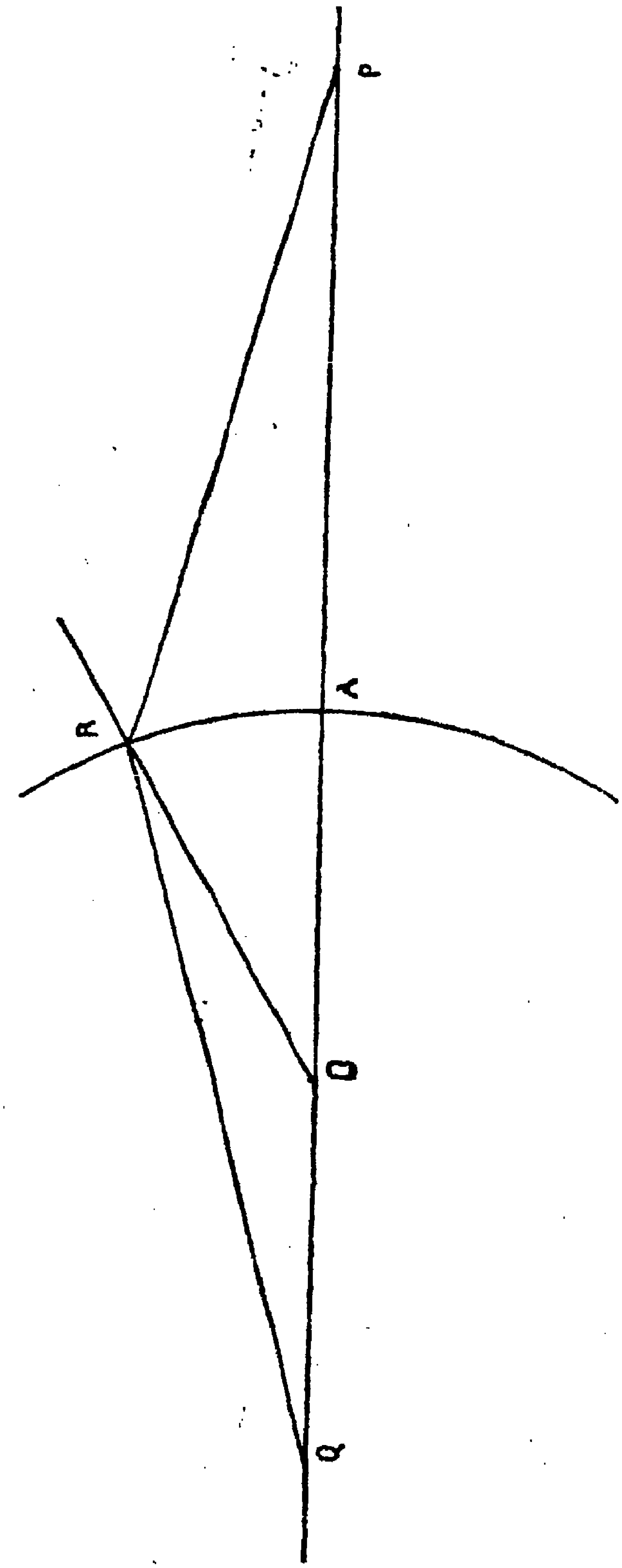


FIG. 21.

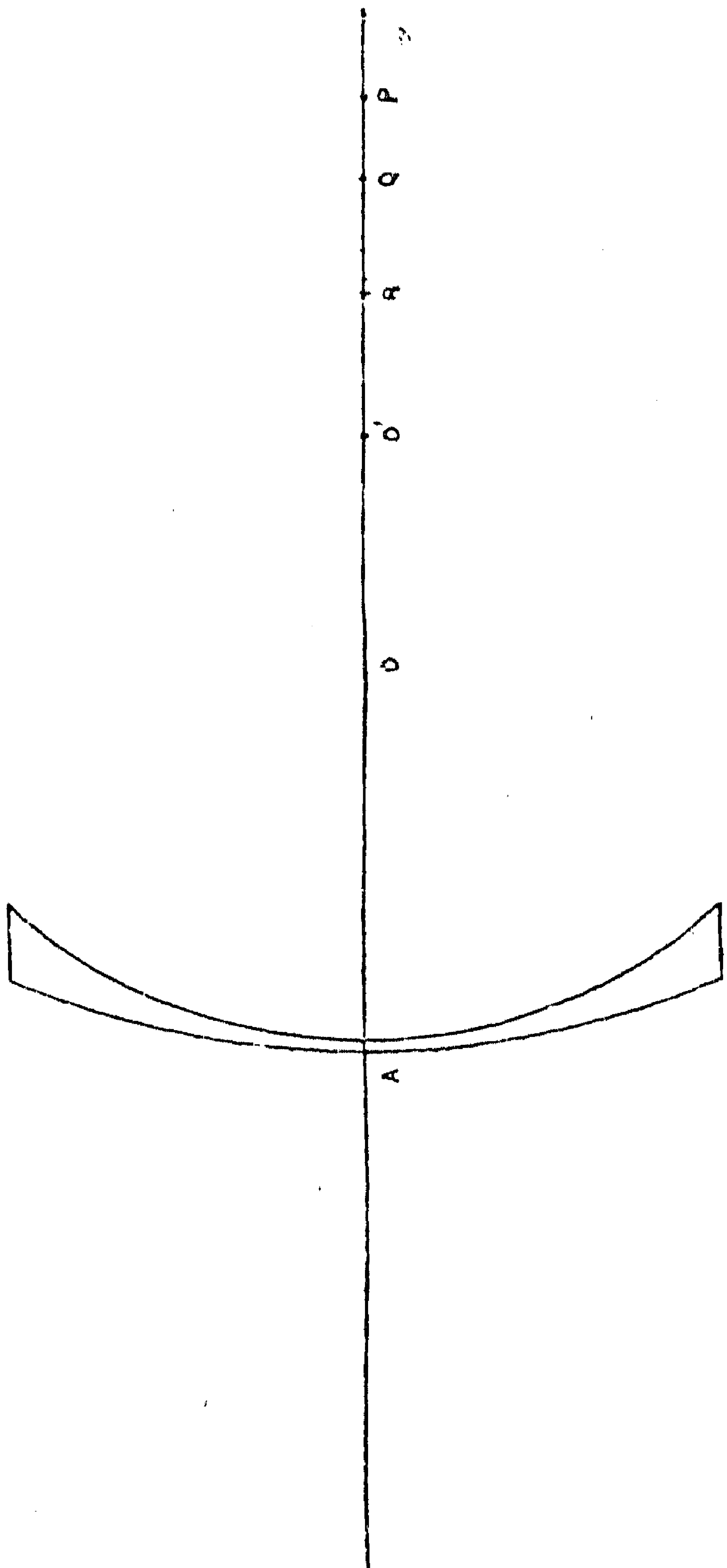


FIG. 22.

Thus $f = \dfrac{\mu r}{\mu - 1}$ and it is proportional to r.

32. Thin Lens.—To get a thin lens put two spherical surfaces together, the distance between them being small compared with their radii. Consider a lens of the form in Fig. 22; this is not a usual shape, but it is convenient for calculation, since its radii are both in the positive direction. Relations for other cases can be found by giving the proper signs to all the lengths.

Let O and O′ be the centres of the front and back surfaces respectively, and let A O $= r$, A O′ $= s$. Let P be the object, Q its image by refraction at the first surface, and R the image of Q by refraction at the second surface, so that R is the image of P produced by the two refractions.

Let A P $= u$, A Q $= w$, A R $= v$, then from the last article

$$\frac{\mu}{w} - \frac{1}{u} = \frac{\mu - 1}{r} = \frac{\mu}{f}$$

But the distances A R and A Q are connected together in the same way as A P and A Q, for if R be regarded as the object Q will evidently be its image by refraction at the second surface. This is, in fact, a special case of the general principle that if a ray of light be reversed, it will exactly retrace its path, and so object and image are always interchangeable terms. The truth of this statement will be made evident by examining the laws of reflection and refraction, by all of which, if a ray be exactly reversed, it will retrace its path. Hence, for R and Q we have

$$\frac{\mu}{w} - \frac{1}{v} = \frac{\mu - 1}{s}$$

For convenience we may call $\dfrac{-\mu s}{\mu - 1}$ the focal length

of the second surface for refraction out, and denote it by f', hence we get

$$\frac{\mu}{w} - \frac{1}{v} = \frac{\mu - 1}{s} = -\frac{\mu}{f'}$$

Subtract the latter equation from the former, and we get

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \mu\left(\frac{1}{f} + \frac{1}{f'}\right)$$

This is the general formula connecting the distances of the object and image from a thin lens. P and R, the positions of object and image, are called *conjugate foci*. It is clear that there can be any number of such pairs of points, for we could take P anywhere along A O O′, and then find the position of the image of Q from the general relation.

33. Principal Focus and Focal Length.—Particular cases arise when either object or image is very distant; if the object is distant, u is very large and $\frac{1}{u}$ very small, and we can neglect it, hence

$$\frac{1}{v} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \mu\left(\frac{1}{f} + \frac{1}{f'}\right)$$

This shows that if the object is so distant that the incident rays are practically parallel, they converge after refraction to a point on the axis at a distance from the lens given by the relation above; this point is called the principal focus of the lens, and its distance from the lens is called the focal length of the lens.

If the focal length be called F we have

$$\frac{1}{F} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \mu\left(\frac{1}{f} + \frac{1}{f'}\right)$$

and the fundamental relation becomes

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \frac{1}{F}$$

If the image is distant, v is large and $\frac{1}{v}$ small, hence

$$-\frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \frac{1}{F}, \therefore u = -F$$

or the rays which, before refraction, converge to a point distant F from the lens on the negative side are parallel after refraction. Both the points so found are sometimes called principal foci, but there will be less danger of confusion if we restrict this name to the first point, calling the other the second principal focus. We now proceed to give examples of the application of the formula found above to lenses of various shapes; in these the two surfaces will be taken, always having the same radii, five and seven inches, the differences being in their arrangement; μ, the refractive index, will be taken as 1·5.

(*a*) The form of the lens is that in Fig. 19 F. $r = 5$ inches, $s = 7$ inches, $\mu = 1{\cdot}5$.

$$\therefore \frac{1}{F} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = {\cdot}5 \times \left(\frac{1}{5} - \frac{1}{7}\right) = \frac{1}{35}$$

$\therefore F = 35$ inches, or the principal focus is thirty-five inches from the lens on the positive side. Let there be an object sixty inches in front of the lens, then $u = 60$ and

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{F} = \frac{1}{60} + \frac{1}{35} = \frac{1}{22{\cdot}1}$$

$\therefore v = 22{\cdot}1$, showing that the image is in front of the lens, and therefore vertical.

(*b*) Double concave lens, as in Fig. 19 B. Here $r = 5$ inches, $s = -7$ inches, $\mu = 1{\cdot}5$.

$$\therefore \frac{1}{F} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = {\cdot}5 \times \left(\frac{1}{5} + \frac{1}{7}\right) = \frac{6}{35}$$

$$\therefore F = \frac{35}{6} = 5{\cdot}83 \text{ inches.}$$

Take the object sixty inches in front as before,

$$\therefore \frac{1}{v} = \frac{1}{u} + \frac{6}{35} = \frac{1}{60} + \frac{6}{35} = \frac{79}{420} \therefore v = 5 \cdot 32 \text{ in.}$$

and the image is again in front and virtual.

(*c*) Meniscus as in Fig. 19 E. Here $r = 7$ inches, $s = 5$ inches, $\mu = 1 \cdot 5$.

$$\therefore \frac{1}{\text{F}} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = -\cdot 5 \times \left(\frac{1}{7} - \frac{1}{5}\right) = -\frac{1}{35}$$

$\therefore$ F $= -35$ inches, showing that the focal length is the same as in example (*a*), but the principal focus is on the opposite side of the lens. Take the object sixty inches in front as before,

$$\therefore \frac{1}{v} = \frac{1}{60} - \frac{1}{35} = -\frac{5}{420} = -\frac{1}{84} \therefore v = -84 \text{ in.,}$$

showing that the image is behind the lens on the side opposite to that of the object and is real.

(*d*) Double convex lens as in Fig. 19 A. Here $r = -7$ inches, $s = 5$ inches, $\mu = 1 \cdot 5$,

$$\therefore \frac{1}{\text{F}} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = -\cdot 5 \times \left(\frac{1}{7} + \frac{1}{5}\right) = -\frac{6}{35}$$

$$\therefore \text{F} = -\frac{35}{6} = -5 \cdot 83 \text{ inches.}$$

Or the focal length is the same as that in Example 2, but negative. Take the object sixty inches in front of the lens as before, then

$$\frac{1}{v} = \frac{1}{60} - \frac{6}{35} = -\frac{65}{420} \therefore v = 6 \cdot 5 \text{ inches nearly,}$$

or the image is on the side opposite from that of the object and real.

These four examples show how the calculations are made; if any difficulty is found in understanding them the reader is recommended to have recourse to pencil and paper, and to *find out for himself* where the figures

come from; a short time thus spent will make many things further on much easier to follow, for it will be impossible to give all the calculations at full length.

Two points in these examples should be noted: First, that whenever the lens was thinner in the middle than at the edge the focal length was positive, and when thicker at the centre than at the edge negative; secondly, that when the focal length is positive the image is virtual, and when negative real (the object is supposed real in both cases).

These remarks will be found to hold good generally, and it is useful to bear them in mind.

34. Principal and Secondary Axes.—The line joining the centres of the spherical bounding surfaces of a lens is called its principal axis, and the centre of the lens is the point in which the principal axis meets it; any other line drawn through the centre of the lens is called a secondary axis.

We have hitherto found only the relation between the distances of conjugate foci on the principal axis; we shall now show that a similar relation holds good for conjugate foci on a secondary axis inclined at a small angle to the principal axis. The following proof is adapted from Wallon:[1]

Let P be a point near to but not on the principal axis; draw the axis P O and produce it (Fig. 23).

We know that an image of P is produced by the lens, for we could find its image by refraction into the glass at the first surface, which will be on the line joining P to the centre of that surface, and we can then find the image of this by refraction out at the second surface. Since all the rays from P pass, after refraction, through one point, we can find the position of its image, if we can trace the path of two different rays, for they must, after refraction, intersect at the image.

The first ray to be traced is P O, in the immediate

[1] *L'Objectif Photographique.*

neighbourhood of O; here the two sides of the lens are practically parallel planes, and they are also near

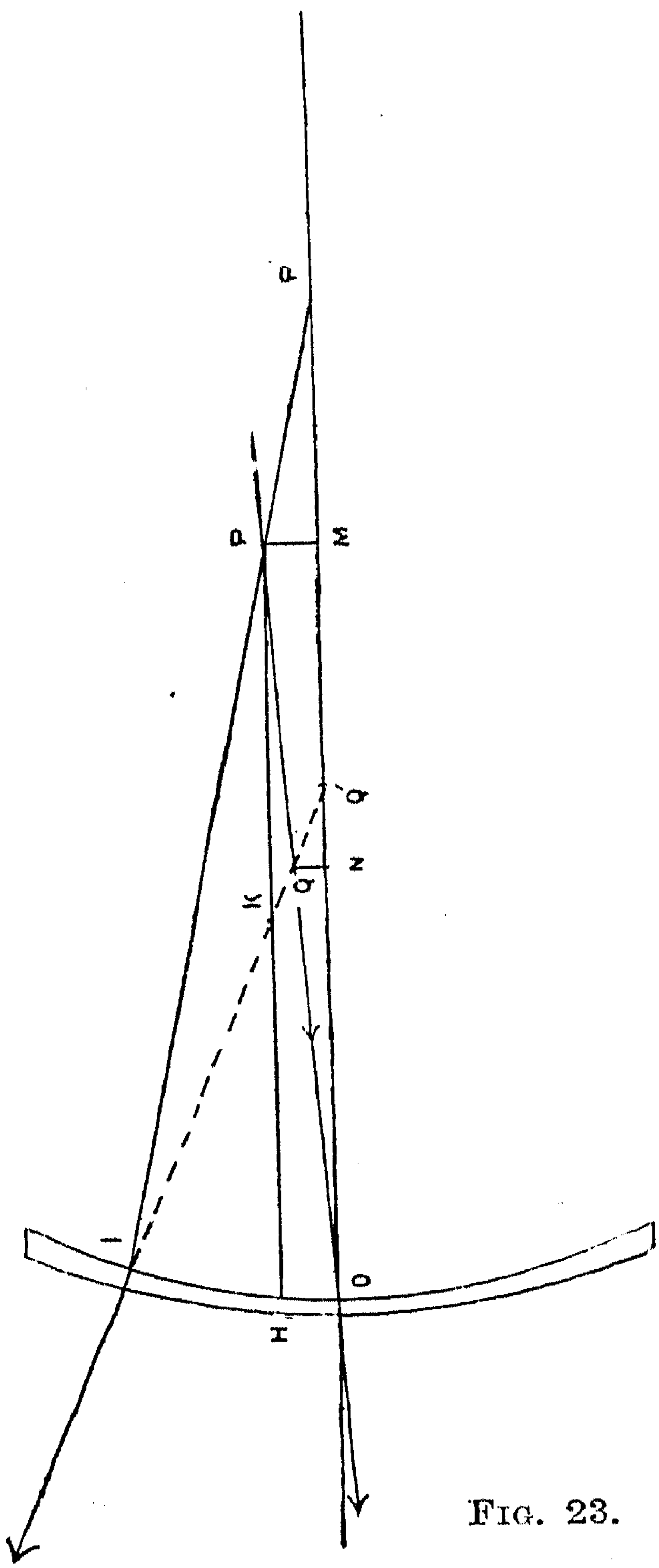


FIG. 23.

together, and the lens at this point acts as a very thin parallel-sided plate of glass, which produces neither

deviation nor sensible lateral shift of a ray. Hence, all rays passing through O will proceed, after refraction, in the same straight line as before, so that P O will pass undeviated, and the image of P must be in P O produced, if necessary.

Trace another ray P I inclined at a small angle to the axis meeting the lens in I, and let this, when produced backwards, cut the principal axis in P′. Then we may regard the ray P′ I as coming from P′. Let Q′ be the focus conjugate to P′. P′ I will clearly, after refraction, pass through Q′, and the refracted ray will be I Q′.

If I Q′ intersect P O in Q, this point is the image of P; draw P M, Q N perpendicular to P′ O Q′. We are now going to show that M and N are approximately conjugate foci.

Draw a line through P parallel to the principal axis to meet the lens in H, and I Q′ in K; since I is near the axis, I O may be taken as perpendicular to P′ Q′ O. Since P H is parallel to P′ Q′ O, we get

$$\frac{PK}{PH} = \frac{P'Q'}{P'O}$$

and by similar triangles, P K Q, O Q′ Q, we get

$$\frac{OQ'}{PK} = \frac{OQ}{PQ}$$

Multiplying these ratios together, we have, since P K cancels out

$$\frac{OQ'}{PH} = \frac{P'Q'}{P'O} \times \frac{OQ}{PQ}$$

or transposing

$$\frac{PQ}{PH \times OQ} = \frac{P'Q'}{OQ' \times P'O}$$

Now the angle P O P′ being small, we may take P Q = M N, O Q = O N, P H = O M.

$$\therefore \frac{MN}{OM \times ON} = \frac{P'Q'}{OQ' \times P'O}$$

but $MN = MO - ON$, and $P'Q' = P'O - OQ'$

$$\therefore \quad \frac{MO - ON}{OM \times ON} = \frac{P'O - OQ'}{OQ' \times OP'}$$

or

$$\frac{1}{ON} - \frac{1}{OM} = \frac{1}{OQ'} - \frac{1}{OP'} = \frac{1}{f} \text{ (see § 32)}$$

where f is the focal length.

Hence M and N are conjugate foci.

This is proved for one particular kind of lens, but it will hold good for any lens if we give the various lines their proper signs.

We therefore have the following method of finding the image of a point P near the axis: Draw P M perpendicular to the axis and find N the focus conjugate to M; join P O and produce it if necessary; at N erect a perpendicular to the axis, meeting the secondary axis, through P, in Q, thus Q is the image of P.

35. Conjugate and Principal Focal Planes.—Now suppose the object to be an extended one covering a small plane perpendicular to the axis, let the axis cut this plane in P, and let the image of P be at Q. To find the image of the extended object we must find the images of all points on the plane, and their assemblage will be the image required; from the last article we see that the images of all points in this plane lie in a plane perpendicular to the axis, which is cut by the axis in Q.

We see then that a plane object near the axis gives rise to a plane image; the case of a large object will be considered when we come to the more complete theory of a lens under the head of spherical aberration. Such planes as the above are called conjugate focal planes; when the object is very distant the rays of light coming from its various points form parallel pencils, and as long as these have only a small inclination to the axis they will all give rise to images on a plane through the principal focus F perpendicular to

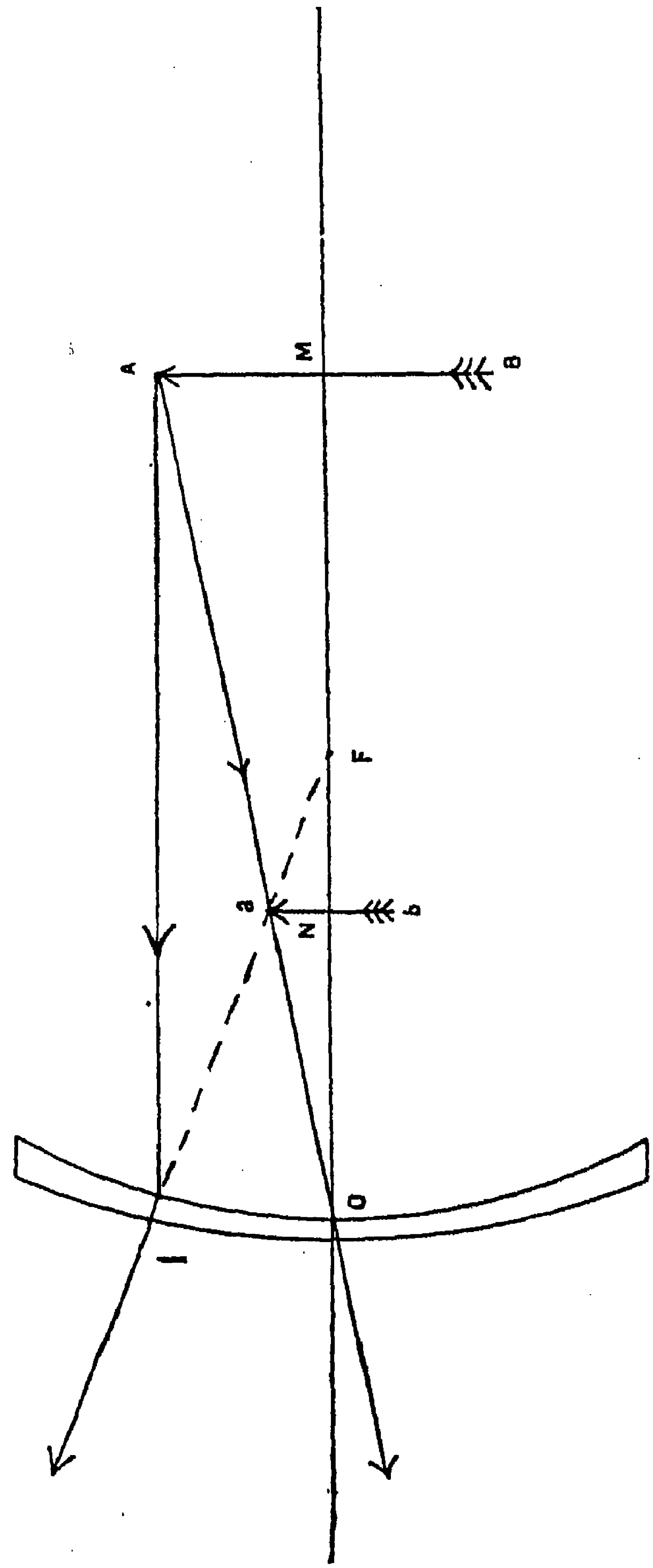


FIG. 24.

the axis, which is called the principal focal plane. There will of course be a second principal focal plane

corresponding to the second principal focus, such that its image will be at a great distance from the lens.

36. Geometrical Construction for the Image.—In § 34 we found the image of P by tracing the paths of two rays; we can in this way find the position of the image of a plane object, for if we can find a point on the image we know the plane on which it lies.

The two rays whose path is traced will, in this case, be, firstly, the ray through the centre of the lens which passes unaltered; secondly, a ray parallel to the axis which, after refraction, must pass through the principal focus.

There are two cases to be considered—(a) When the focal length is positive; (b) when it is negative.

(a) *Positive focal length.*—In this case the principal focus is on the same side of the lens as the object; let it be F (Fig. 24); let the object be an arrow A B which has the advantage of having its ends dissimilar, so that it is evident at a glance which way up it is drawn. To trace the first ray join A O, for the second draw A I parallel to the axis to meet the lens in I; this ray must, after refraction, when produced backwards, meet the axis in F; hence join F I cutting A O in a. Then a is the image of A, and we can construct in a similar manner the image of b, and $a\ b$ will be the image of A B; it is erect but virtual.

(b) *Negative focal length.*—In this case F will be on the other side of the lens; the construction will be similar to that of the last case, and (Fig. 25) the image will be inverted but real.

We thus see the reason of the remark at the end of § 33, that if the focal length of a lens is positive, the image is virtual, but if it is negative the image is real.

The former kind of lens is called *divergent*, for it is

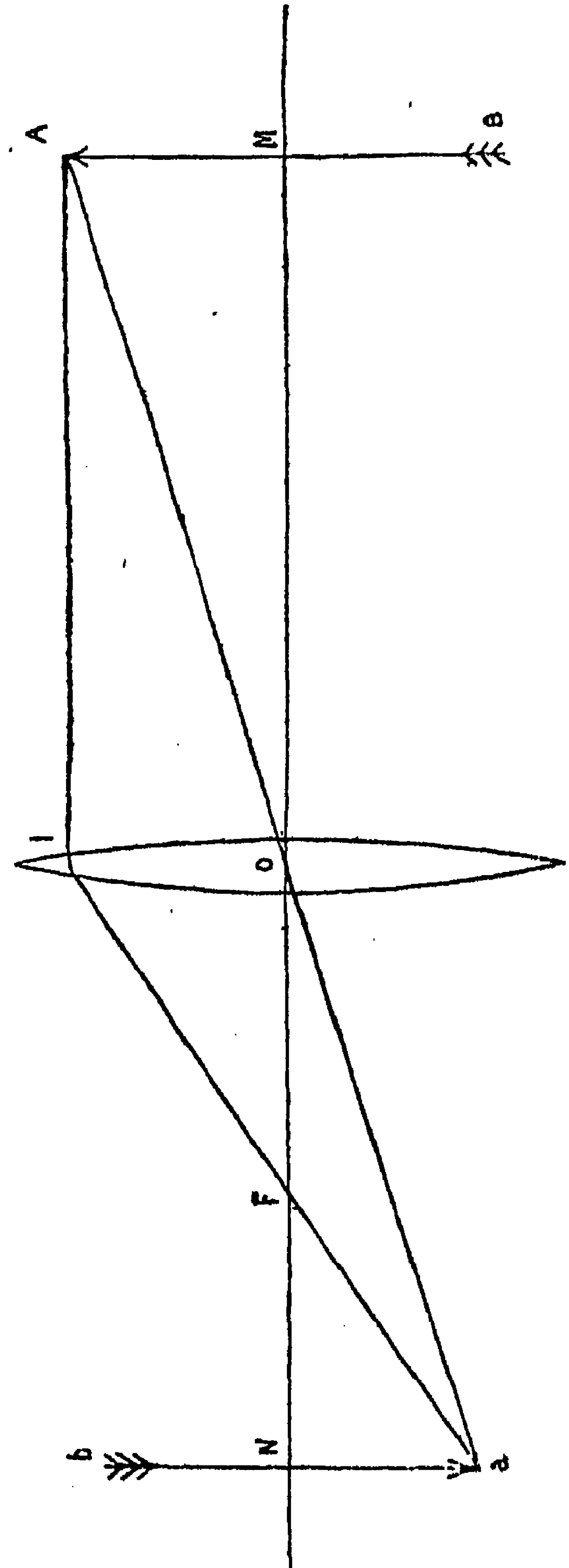


FIG. 25.

easy to see that a pencil of rays is always spread out by refraction through it, while the second kind of lens produces the opposite effect, and is *convergent.*

37. Magnification.—In some kinds of work, such as copying or enlarging, it is necessary to know the relative sizes of object and image; magnification is defined to be the ratio of the size of the image to that of the object whether the image be larger or smaller than the object. The case now considered is that when the object is near; the case when the object is distant is considered further on (§ 56).

In the two figures of the last article, let A B and $a\ b$ cut the principal axis in M and N respectively; then

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{a\ b}{\text{A B}} = \frac{\text{O N}}{\text{O M}} = \frac{v}{u} \text{ by similar triangles,}$$

$$\text{but } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{which gives } \frac{v}{u} = \frac{f}{u+f}$$

$$\therefore \frac{\text{Size of image}}{\text{Size of object}} = \frac{f}{u+f}$$

Example.—An object is placed three feet in front of a converging lens of 6-in. focal length; here $u = 36$ inches, $f = -6$ inches.

$$\therefore \frac{\text{Size of image}}{\text{Size of object}} = \frac{-6}{36-6} = -\frac{6}{30} = -\frac{1}{5}$$

Hence the size of the image is one-fifth of that of the object; the negative sign means that the image is inverted, as we know it should be. The application to enlargements will be given later (§ 140). It should be carefully noted that the ratio above is that of the linear and not areal dimensions of the image and object; the areal dimensions will be proportional to the squares of the linear; thus in the two examples any area in the image will be one twenty-fifth of the corresponding area of the object.

38. Calculation of the Distance of the Image.—We have found the relation connecting the distances of the object and image from a lens, so that if we know the focal length f of the lens and u the distance of the

object we can find v the distance of the image from the lens.

In these calculations we have to deal with reciprocals,[1] and, except with round numbers, the work is apt to be tedious, but it is much simplified by the use of a table of reciprocals, which reduces the work to addition and subtraction only. Such tables are given in many sets of mathematical tables, such as Bottomley's four figure tables and many engineering handbooks.

The reciprocals are usually calculated to four significant figures, and will give an accuracy of one in a thousand, which is all that is generally required.

Example.—Find the position of the image formed by a convergent lens of focal length 5·813 inches of an object placed 30·56 inches in front.

$$\text{Here } f = -5{\cdot}813,\ u = 30{\cdot}56.$$

$$\therefore \frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{30{\cdot}56} - \frac{1}{5{\cdot}813} = {\cdot}03273 - {\cdot}1720$$

$$= -{\cdot}1392 = -\frac{1}{7{\cdot}183}$$

$$\therefore v = -7{\cdot}183 \text{ inches}$$

or the image is on the opposite side from the object at a distance of 7·183 inches from the lens.

39. Graphical Calculations.—Lens calculations can be performed graphically with a fair amount of accuracy by means of a geometrical construction. Let the parallel straight lines A B and C D (Fig. 26) be drawn to meet B D at any angle (it will be convenient as a rule to make them perpendicular to B D), and at any points B and D; join A D and C B intersecting in P, and draw P N parallel to A B or C D to meet B D in N; then will

$$\frac{1}{\mathrm{PN}} = \frac{1}{\mathrm{AB}} + \frac{1}{\mathrm{CD}}$$

[1] The reciprocal of a number is unity divided by that number thus the reciprocals of 2 and 4 are $\frac{1}{2}$ and $\frac{1}{4}$, or ·5 and ·25.

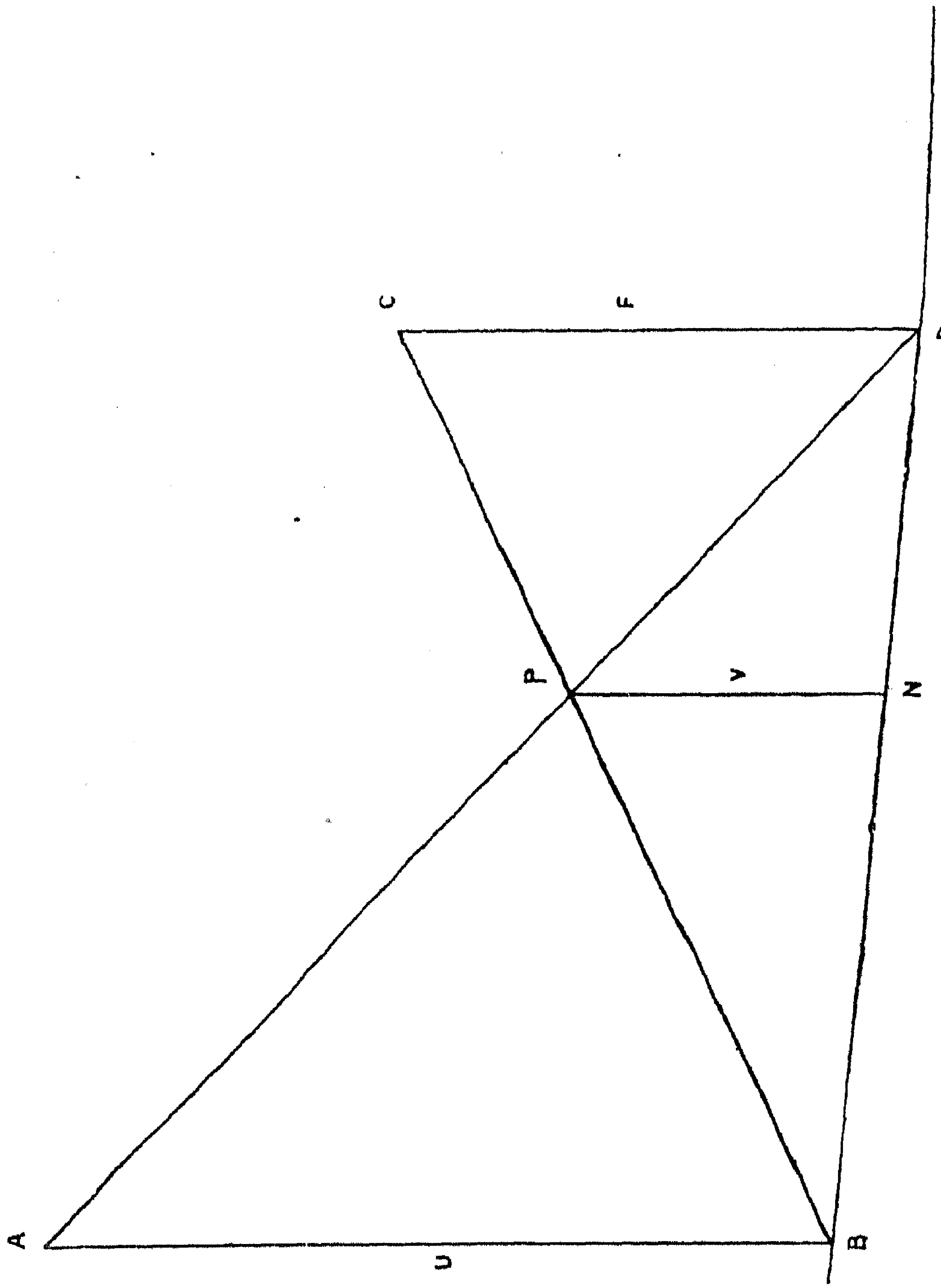


FIG. 26.

By similar triangles P N D, A B D we have—

$$\frac{DN}{BD} = \frac{PN}{AB}, \text{ similarly } \frac{BN}{BD} = \frac{PN}{CD}$$

$\therefore$ By addition—

$$\frac{PN}{AB} + \frac{PN}{CD} = \frac{DN + BN}{BD} = \frac{BD}{BD} = 1$$

$$\therefore \frac{1}{AB} + \frac{1}{CD} = \frac{1}{PN}$$

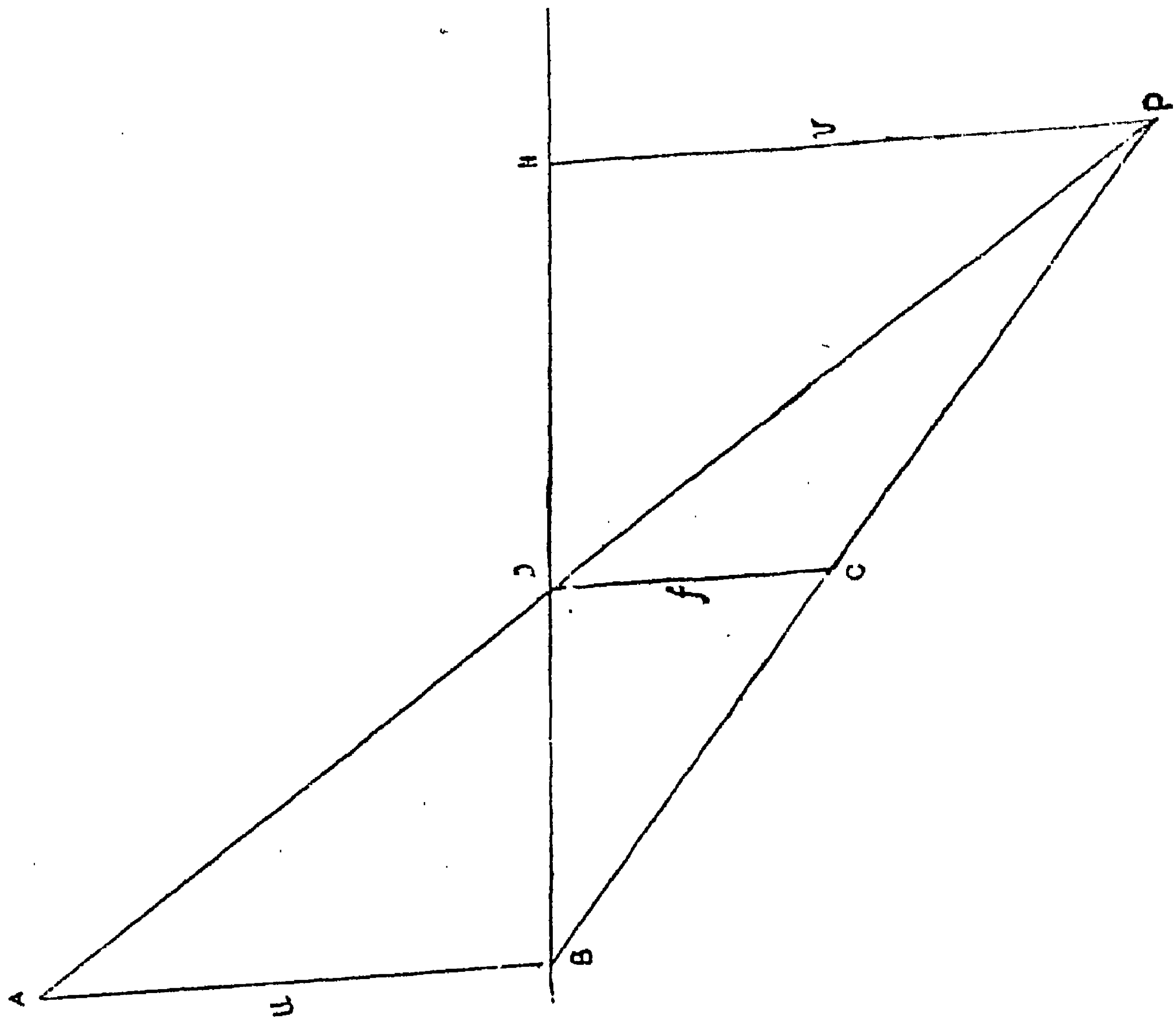


FIG. 27.

To apply this, suppose we have to calculate $1/v = 1/u + 1/f$; on any convenient scale make $AB = u$, $CD = f$, and construct as above, then evidently PN will represent v.

If we wish to calculate $1/v = 1/u - 1/f$ we have only to draw CD downwards instead of upwards, and construct as before (Fig. 27); in this case PN lies below

the line B D, and is therefore to be reckoned negative, showing, as we already know, that when the focal length is negative, the image is on the opposite side of the lens from the object and is real.

Another advantage of the construction is that it shows at a glance the relative sizes of object and image, for

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{v}{u} = \frac{\mathrm{P\,N}}{\mathrm{A\,B}}$$

In Fig. 26 the image is erect, which is shown by P N being above B D; in Fig. 27 it is inverted. The construction can be made use of for any calculations involving the sum or difference of reciprocals. There is a geometrical property of the figure which will be found very useful further on when we come to the combination of lenses not in contact.

By similar triangles B P N, B C D

$$\frac{\mathrm{B\,N}}{\mathrm{B\,D}} = \frac{\mathrm{P\,N}}{\mathrm{C\,D}}, \text{ similarly } \frac{\mathrm{D\,N}}{\mathrm{B\,D}} = \frac{\mathrm{P\,N}}{\mathrm{A\,B}}$$

Combining these

$$\frac{\mathrm{B\,N}}{\mathrm{B\,D}} \times \frac{\mathrm{B\,D}}{\mathrm{D\,N}} = \frac{\mathrm{P\,N}}{\mathrm{C\,D}} \times \frac{\mathrm{A\,B}}{\mathrm{P\,N}}$$

$$\text{or cancelling } \frac{\mathrm{B\,N}}{\mathrm{D\,N}} = \frac{\mathrm{A\,B}}{\mathrm{C\,D}}$$

Hence N divides B D in the ratio of A B to C D.

40. Combinations of Lenses in Contact.—Combinations of two or more lenses are often used; we must therefore be able to deal with them. We consider only thin lenses in contact; the case of thick lenses in contact, or separated by a sensible interval, will be considered later (§ 52).

Let there be two thin lenses in contact of focal lengths f_1, f_2; let u be the distance of an object in front of the first lens, v_1 the distance of the image formed by this lens, and v_2 the distance of the image of the first image formed by this second lens, all

measured from the surfaces of the lenses; then as before we have

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \qquad \frac{1}{v_2} - \frac{1}{v_1} = \frac{1}{f_2}$$

adding we get
$$\frac{1}{v_2} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

Hence the combination acts like a lens of focal length F, where

$$\frac{1}{\mathrm{F}} = \frac{1}{f_1} + \frac{1}{f_2}$$

for then
$$\frac{1}{v_2} - \frac{1}{v} = \frac{1}{\mathrm{F}}$$

The lens of focal length F is called, the lens equivalent to the two lenses in contact, and as far as the relative positions of object and image are concerned it could replace them.

The calculations of F, when f_1, f_2 are known, can be made either by means of the table of reciprocals or graphically, the proper signs being given to f_1 and f_2.

We can extend the result to any number of thin lenses in contact; take, for example, a third lens, then

$$\frac{1}{v_3} - \frac{1}{v_2} = \frac{1}{f_3}$$

Add this to the last equation obtained, and we get

$$\frac{1}{v_3} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

showing that the focal length F of the equivalent lens now is given by

$$\frac{1}{\mathrm{F}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Proceeding in this way, we shall get the reciprocal of the focal length of the lens equivalent to any number of thin lenses in contact by adding together the reciprocals of their focal lengths.

Example.—Find the focal length of the lens equivalent to a combination of three lenses, two of them being converging and of focal length six inches, and the other diverging and of focal length ten inches.

Here $f_1 = -6$ in., $f_2 = 10$ in., $f_3 = -6$ in.

$$\therefore \frac{1}{\mathrm{F}} = -\frac{1}{6} + \frac{1}{10} - \frac{1}{6} = -\cdot1667 + \cdot1 - \cdot1667$$

$$= -\cdot2334 = -\frac{1}{4\cdot284}$$

$$\therefore \quad \mathrm{F} = -4\cdot284 \text{ inches}$$

which shows that the combination is equivalent to a converging lens of focal length 4·284 inches.

41. Experimental Determination of Focal Length.—It is now clear that the most important thing to know about a lens is its focal length, and this can be found experimentally without knowing either the curvatures of the faces, or the refractive index of the glass. In practice, we should first find the focal length, then the curvatures by the aid of a spherometer, and from these deduce the refractive index.

There are many methods of measuring focal length, which are described in books on optics.[1] We shall here describe a simple method requiring little apparatus, leaving the question of a combination till we come to the subject of lens testing.

(*a*) *Lens of negative focal length.*—Fix the lens upright in a suitable support—an upright piece of board with a hole in it will do—and place in front of it something to act as an object—a pin or a hole in a piece of metal with two cross wires placed in front of a lamp are suitable—and behind it an upright screen of cardboard or paper. Move the lens and screens about till an image of the object is formed on the screen, and adjust till the image is as sharp

[1] Glazebrook and Shaw's *Practical Physics*, edition 4, § 51, etc.

as possible, then measure the distances between the lens, the image, and the object, and from these calculate the focal length. In arranging the lens and screen, it may be found that an image cannot be got. The reason for this will probably be that the object and screen are too near together, for it can be shown that no image is possible unless the distance between them is at least four times the focal length, so it is well to start with the object and screen fairly far apart.

Example.—It is found that with a certain lens the distances of object and image from the lens are twelve inches and four inches respectively.

$$\text{Here } u = 12 \qquad v = -4$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = -\frac{1}{4} - \frac{1}{12} = -\frac{1}{3}$$

$$\therefore f = -3 \text{ inches.}$$

In this and in all cases where accuracy is required several measurements should be made with various distances of object and image, the focal length calculated from each, and the mean of the results taken.

(*b*) *Lens of positive focal length or diverging lens.*—We have seen that we cannot in this case get a real image of a real object; we cannot therefore proceed directly as above, but we may get over the difficulty by placing the diverging lens in contact with a converging one of known focal length, so chosen as to make the focal length of the combination negative, and therefore equivalent to a converging lens. The focal length of the combination is then found as before, and the required focal length calculated.

Example.—A convergent lens of focal length six inches is placed in contact with a divergent lens, and the focal length of the combination is found to be fifteen inches.

Here F = − 15 inches, f_1 = − 6 inches, f_2 = ? The reader is left to work the question out, and to verify that f = 10 inches.

42. Range of Focus.—In outdoor and landscape work the objects are often at a considerable distance, and the image is in consequence very near the principal focus of the lens. From the nature of the formulæ connecting the distances of object and image from the lens, it is easy to show that as the object moves away from the lens the image moves towards the lens, at first rapidly, but less so as the object gets to a considerable distance, and after that motion of the object produces very little further displacement of the image. Hence the images of all objects beyond a certain distance lie fairly near together and close to the principal focal plane.

The following table shows the relative distances of object and image for lenses of focal lengths of six inches, four inches, and three inches, u being the distance of the object measured in feet, and v that of the image in inches :

Distance u of object in feet.	Values of v corresponding to those of u for lenses of focal length.		
	− 6 inches.	− 4 inches.	− 3 inches.
10	− 6·31	− 4·14	− 3·07
20	− 6·15	− 4·07	− 3·04
30	− 6·10	− 4·04	− 3·02
Gt. distance.	− 6·00	− 4·00	− 3·00

From these examples it can be seen that in the case of a 6-in. lens the focussing screen would have to be moved three-tenths of an inch if the object moved from a distance of ten feet to a great distance away ; for a 4-in. lens the movement would be only ·14 inch ; and

for a 3-in. lens only ·07 inch. As will be seen further on, such a small alteration in the position of the focussing screen as the tenth of an inch will produce very little change in the sharpness of the picture. Hence, with a 6-in. lens all objects more than thirty feet away will be approximately in focus at the same time. The same will be the case for lenses of 4-in. and 3-in. focus for objects more than twenty and ten feet distant respectively. If, therefore, we use a lens of fairly short focus for objects not nearer than twenty feet, we can fix the position of the plate, and need not trouble to focus in each particular case.

This is the principle on which hand cameras with a fixed focus are made.

Thick Lenses.

43. Thick Lenses.—The theory of thin lenses is useful as an introduction, and the calculations are for many purposes accurate enough; and thin lenses in contact have been shown to present no difficulty. But for accurate work we cannot neglect the thickness, because for some lenses the relation connecting the distances of object and image is far from accurate, even when the lens is not very thick; and besides this, a combination of lenses separated by a sensible interval can in most cases be replaced by a thick, but not by a thin, lens.

Gauss has worked out the theory of refraction through any number of media separated by spherical surfaces placed with their centres in any positions along a straight line. But we have to consider only a single medium bounded by two surfaces, and shall therefore be able to introduce considerable simplification. We shall afterwards consider a combination of two thick lenses, and from this the calculation can be extended to any number of lenses by taking account of each of them in succession.

44. Optical Centre. Nodal Points.—Consider the

lens in Fig. 28. Let O, O' be the centres of the two bounding surfaces, and A, A' the points in which the axis meets the surfaces. Through O, O' draw two parallel radii to meet the corresponding surfaces in Q and Q', join $Q\,Q'$, and produce it to meet the axis in C.

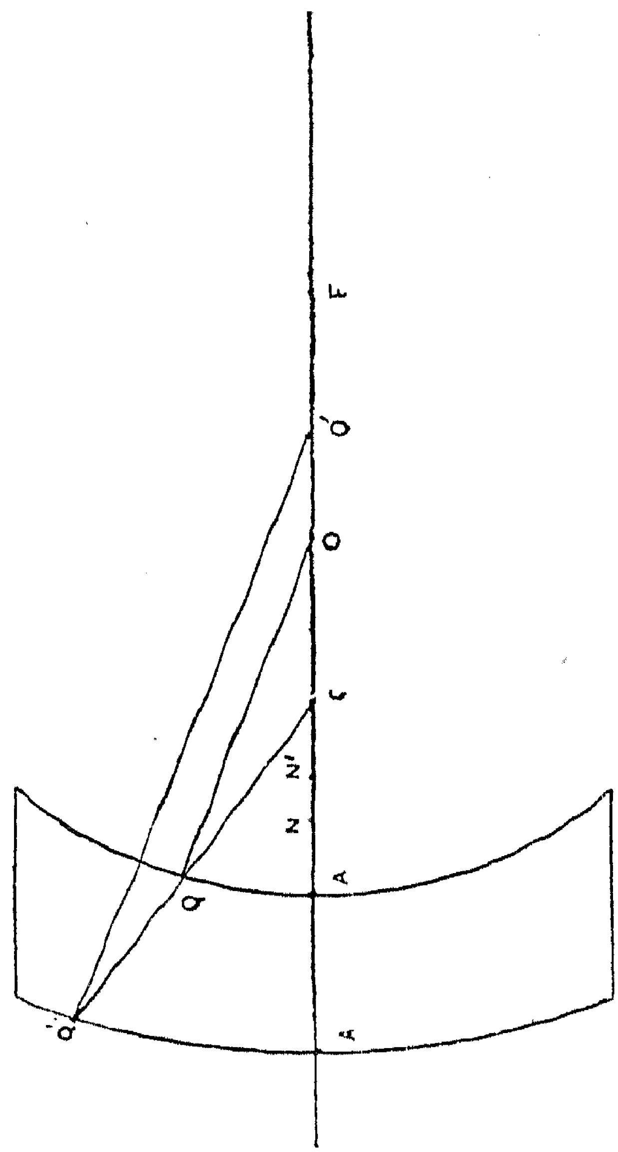


Fig. 28.

Then, by similar triangles, $O\,C\,Q$, $O\,C'\,Q'$, we have

$$\frac{O\,C}{O'\,C} = \frac{O\,Q}{O'\,Q'} = \frac{O\,A}{O'\,A'} = \text{ratio of radii of surfaces.}$$

This shows that C divides the line joining $O\,O'$ externally in the ratio of the radii, and this remains constant whatever the direction of $O\,Q$, $O'\,Q'$, and hence C is a fixed point.

Since O Q, O′ Q′ are parallel, the two tangents to the surfaces at Q Q′, which are perpendicular to the radii, are parallel; therefore, if Q Q′ be the path of a ray in the glass, the lens acts towards it as if it were a parallel-sided plate, and the ray after refraction out from the glass will be parallel to its direction before refraction (§ 11) into it. This will be the case for all rays which when produced pass through C; hence all rays whose directions in the glass (produced if necessary) pass through C traverse the lens without undergoing any final deviation, though they may be shifted parallel to themselves. *The point C is called the optical centre of the lens.*

Let r (A O) and s (A′ O′) be the radii of the surfaces, e the thickness A A′, μ the refractive index of the glass.

Let us make use of the abbreviations we have already employed in §§ 31, 32, *i.e.*

$$f = \frac{\mu r}{\mu - 1},\ f' = \frac{-\mu s}{\mu - 1}$$

f and f' being what we have called the focal lengths of the two surfaces and proportional to their radii.

$$\text{Then} \qquad \frac{\mathrm{C\,O}}{\mathrm{C\,O'}} = \frac{\mathrm{A\,O}}{\mathrm{A'\,O'}} = \frac{r}{s},\ \therefore\ \frac{\mathrm{C\,O}}{\mathrm{O\,O'}} = \frac{r}{s - r}$$

$$\therefore\ \mathrm{C\,O} = \frac{r}{s - r}\,\mathrm{O\,O'} = \frac{r}{s - r}(s - r - e) = r - \frac{e\,r}{s - r}$$

$$\text{Hence} \qquad \mathrm{A\,C} = \mathrm{A\,O} - \mathrm{C\,O} = \frac{e\,r}{s - r} = \frac{e\,f}{f' + f}$$

$$\text{and } \mathrm{A'\,C} = \mathrm{A\,A'} + \mathrm{A\,C} = \frac{e\,s}{s - r} = \frac{e\,f'}{f' + f}$$

which gives the position of C relative to the two surfaces.

Now let N and N′ be the images of C, the optical centre, due to refraction out, at the two surfaces; that is, let N be such a point that rays diverging from it

will, after refraction into the glass at the first surface, all converge towards C, and N′ the point towards which they will all converge after refraction out at the second surface.

Hence all rays which pass through N before refraction will, since they pass through the optic centre, emerge through N′ after refraction parallel to their original direction.

The two points N and N′ are called the nodal points, N being the nodal point of incidence and N′ the nodal point of emergence; planes through N and N′ perpendicular to the axis are called nodal planes.

To find the positions of N and N′ we have § 31.

$$\frac{\mu}{\mathrm{A\,C}} = \frac{1}{\mathrm{A\,N}} - \frac{\mu - 1}{r}$$

for N and C are conjugate foci with respect to the first surface. Hence

$$\frac{1}{\mathrm{A\,N}} = \frac{\mu}{\mathrm{A\,C}} - \frac{\mu - 1}{r} = -\mu\,\frac{f' + f}{ef} - \frac{\mu}{f}$$

$$= -\mu\,\frac{f + f' + e}{e\,f} \quad (\S\ 44)$$

$$\therefore\ \mathrm{A\,N} = \frac{-\,e\,f}{\mu\,(f + f' + e)}$$

It may be shown in a similar manner that

$$\mathrm{A'\,N'} = \frac{e\,f'}{\mu\,(f + f' + e)}$$

Hence $\mathrm{A\,N} : \mathrm{A'\,N'} = -f : f' = -r : s$

or A N, A′ N′ are numerically in the ratio of the radii.

45. Contrast of Thick and Thin Lenses.—In the case of the thin lens we saw that all rays through the centre passed without deviation, while with a thick lens a ray through the nodal point of incidence emerges undeviated through the nodal point of emergence; thus the effect of the thickness of the lens on such a ray is

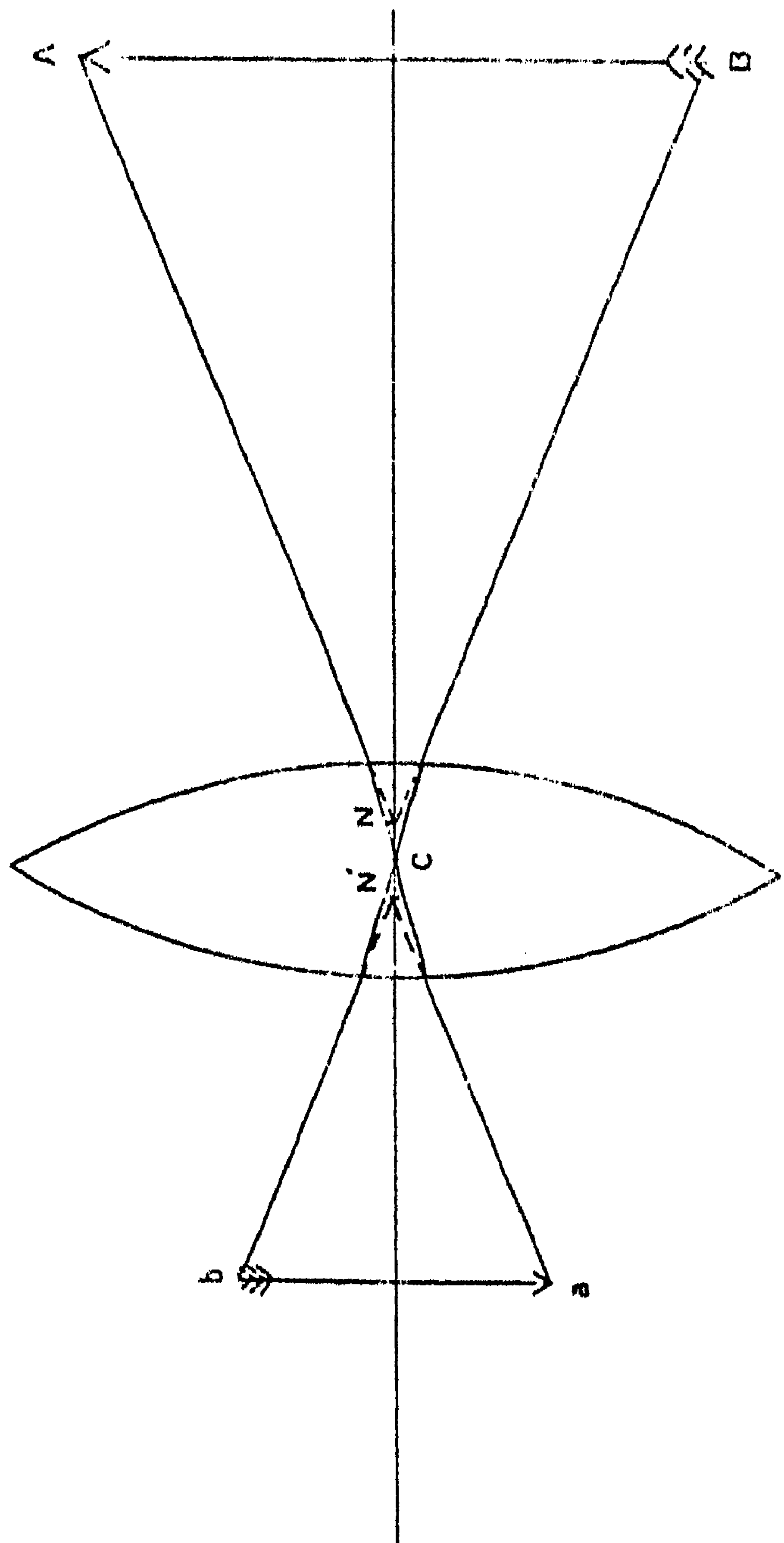


FIG. 29.

not to deviate it, but to give it a lateral shift. There is then some similarity between the centre of a thin lens and the nodal points of a thick lens ; for in Fig. 25, for example, the line joining the corresponding points of image and object passes through O the centre of the lens, while (Fig. 29) we have A N a N′ parallel. We may imagine that Fig. 29 is got from Fig. 25 by cutting the diagram in half by a line through O perpendicular to the axis, and then sliding the two portions apart parallel to each other to a distance N N′, the two points N N′ now taking the place of the single point O. We saw that (Fig. 22)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

where u = A P, v = A Q, and F is the focal length of the lens, so we should expect for a thick lens, if u is the distance of the object from N, and v the distance of the image from N′, we should have a relation of the form.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F'}$$

That this is actually the case will be seen further on, with this difference, that the focal length is not the same as for the thin lens.

46. Size of Image of an Object placed in a Nodal Plane.—We must first show that, if an object be placed in a nodal plane, its image, which will of course be in the other nodal plane, is equal to it in size.

Let P N, Fig. 30, be an object in the nodal plane of incidence ; the image of this, after refraction into the glass, must be in the plane through the optic centre perpendicular to the axis (called the principal plane), and we can find its size if we can trace one ray ; the ray required is P O through the centre of the first surface, which being incident normally passes undeviated.

Let P O meet the principal plane in R, so that C R is the image of P N by refraction at the first surface ; to

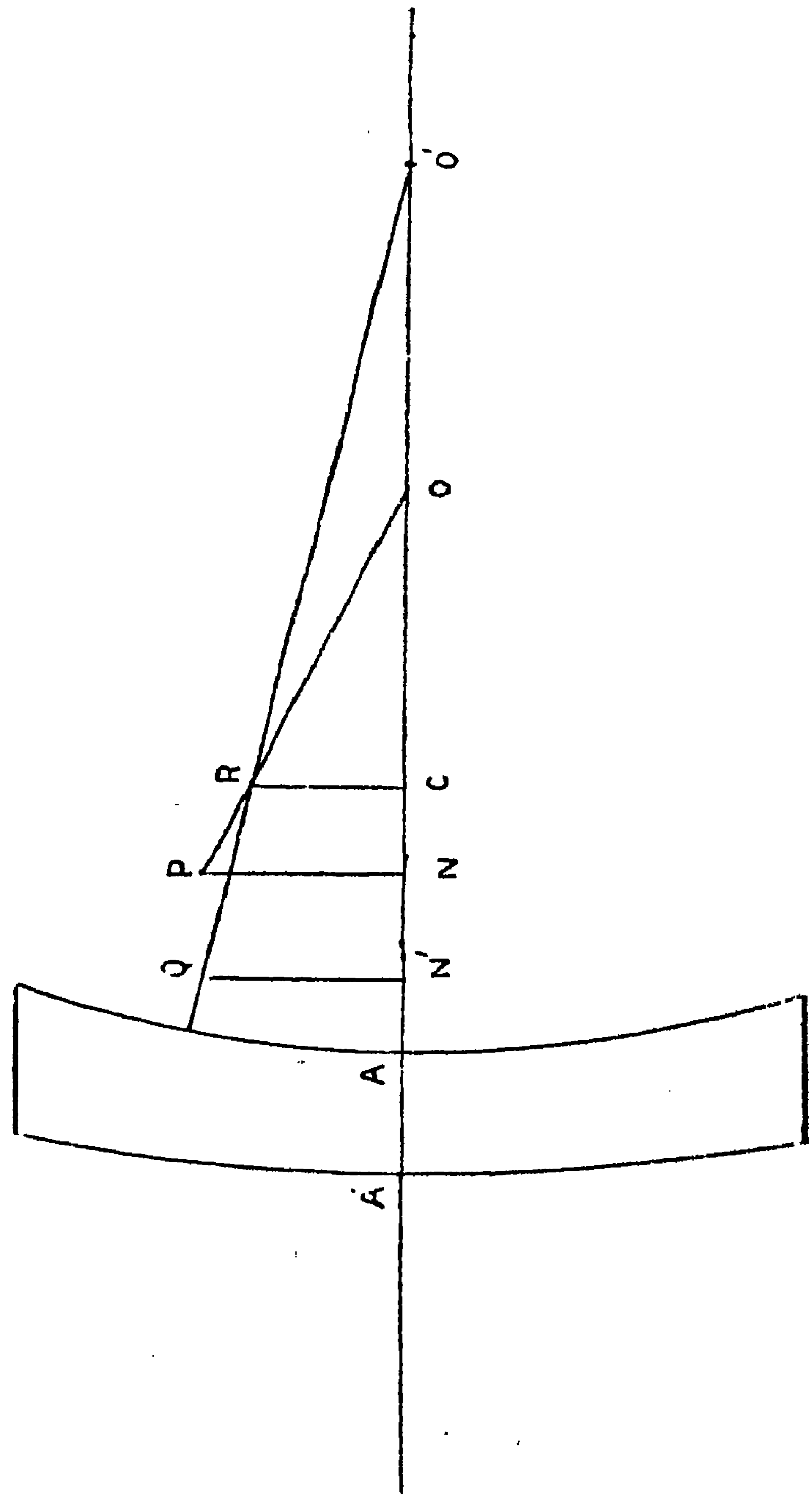


FIG. 30.

get the image of C R by refraction out, join R O′ cutting the nodal plane of emergence in Q, then Q N′ will be the image, for the ray R O′ passes out undeviated. We have to prove that P N, Q N′ are equal. Since the optic centre C divides the distance between the centres of the surfaces in the direct ratio of the radii

$$\frac{\mathrm{C\,O}}{\mathrm{C\,O'}} = \frac{r}{s}$$

$$\text{Also } \frac{\mathrm{P\,N}}{\mathrm{R\,C}} = \frac{\mathrm{N\,O}}{\mathrm{C\,O}} \text{ and } \frac{\mathrm{Q\,N'}}{\mathrm{R\,C}} = \frac{\mathrm{N'\,O'}}{\mathrm{C\,O'}}$$

$$\therefore \frac{\mathrm{P\,N}}{\mathrm{Q\,N'}} = \frac{\mathrm{P\,N}}{\mathrm{R\,C}} \times \frac{\mathrm{R\,C}}{\mathrm{Q\,N'}} = \frac{\mathrm{N\,O}}{\mathrm{C\,O}} \times \frac{\mathrm{C\,O'}}{\mathrm{N'\,O'}} = \frac{\mathrm{N\,O}}{\mathrm{N'\,O'}} \times \frac{\mathrm{C\,O'}}{\mathrm{C\,O}}$$

$$= \frac{\mathrm{A\,O} - \mathrm{A\,N}}{\mathrm{A'\,O'} - \mathrm{A\,N'}} \times \frac{s}{r} = \frac{r - \mathrm{A\,N}}{s - \mathrm{A'\,N'}} \times \frac{s}{r}.$$

but since N A, N′ A′ are proportional to r and s (§ 44), r — A N and s — A′ N′ are also in the same ratio.

$$\therefore \frac{r - \mathrm{A\,N}}{s - \mathrm{A'\,N'}} = \frac{s}{r}, \therefore \frac{\mathrm{P\,N}}{\mathrm{Q\,N'}} = \frac{r}{s} \times \frac{s}{r} = 1$$

$$\text{or } \mathrm{P\,N} = \mathrm{Q\,N'}$$

Hence the object in one nodal plane has an equal and erect image in the other nodal plane ; this means that all rays passing through P in one nodal plane will, after refraction, pass all through Q in the other nodal plane, where P Q is parallel to the axis. From this we can find in what point any ray meets the nodal plane of emergence after refraction, when we know where it meets the nodal plane of incidence before refraction.

47. Construction for the Image.—We can now give a construction for the image analogous to that given for the thin lens (§ 36).

Let F be the principal focus of the lens, N and N′ the nodal points, A B the object (Fig. 31) (the lens itself being omitted to simplify the figure) ; we must trace two rays from A. First take the ray A N

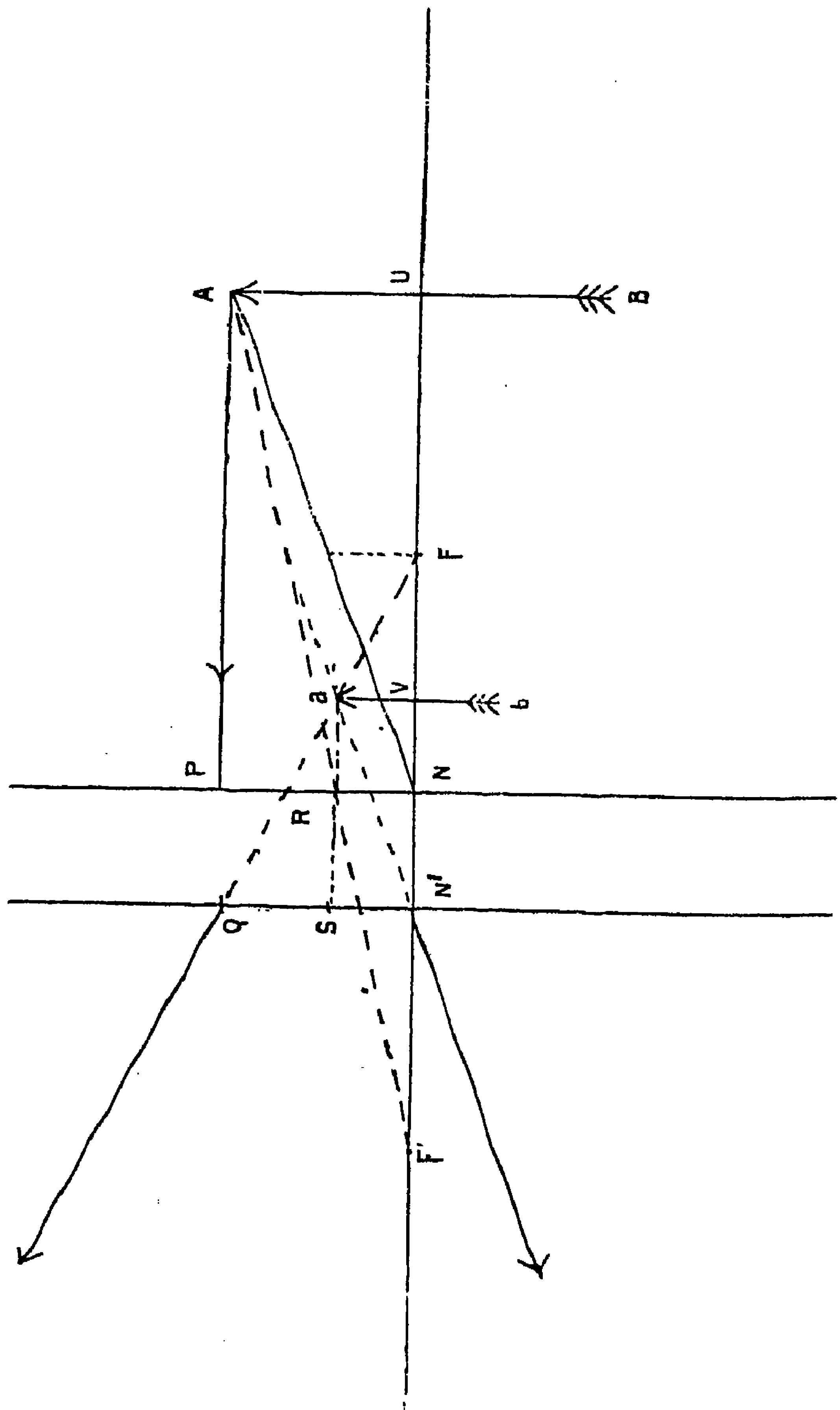


FIG. 31.

through the nodal point of incidence, its direction on emergence $N'a$ is parallel to $A\,N$; secondly, take the ray $A\,P$ parallel to the axis, meeting the nodal plane of incidence in P; by the last section its direction after refraction will pass through Q in the nodal plane of emergence where $Q\,N' = P\,N$, and also it must pass through the principal focus F. Let the two rays meet in a, which is therefore the image of A; similarly we may find the image of any point of the object.

Let $A\,B$ and $a\,b$ cut the axis in U and V, and let

$$N\,U = u, \quad N'\,V = v, \quad N'\,F = F$$

Then by similar triangles $A\,N\,U$, $a\,N'\,V$

$$\frac{N\,U}{N'\,V} = \frac{A\,U}{a\,V} = \frac{Q\,N'}{a\,V} = \frac{N'\,F}{V\,F}$$

$$\text{Or } \frac{u}{v} = \frac{F}{F - v}, \quad \therefore\ u\,(F - v) = v\,F$$

$$\therefore\ u\,F - v\,F = u\,v$$

$$\text{Or } \frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

Hence we have proved that the distances of object and image from the nodal points of a thick lens obey the same law as their distances from the lens obey in the case of the thin lens, the focal length being the distance from the nodal point of emergence to the principal focus.

To avoid any possibility of misunderstanding, we will state the convention of signs as applied to a thick lens.

(*a*) The distance of the object is measured from the nodal point of incidence to the object, and that of the image from the nodal point of emergence to the image.

(*b*) Lengths are reckoned positive when they are measured from the starting-point in a direction opposite to that of the incident light.

In all figures, the light is taken as coming from the

right towards the left; the positive direction is the fore from left to right.

If we wish the rays to emerge parallel the ima will go off to an infinite distance, and $v = \infty$ or $1/v =$ and we get

$$\frac{1}{u} = \frac{1}{F} \text{ or } u = -F.$$

Hence, in this case also there is a second princi focus F′ on the side of the lens opposite to F, and the same distance from N that F is from N′.

47*a*. We must now consider a point which will useful when we come to deal with combinations lenses not in contact.

We can evolve the theory in a different order; if have given the two principal foci F and F′ (Fig. with their properties, and the fact that if an object placed in one focal plane it has an equal image in other focal plane, we can prove that the lines A *a* N′ joining the corresponding points of object image to the nodal points are parallel, and hence ded the relations found in the last article.

Let us trace two rays from A, let A P parallel to axis meet the first nodal plane in P, mark off (equal to P N, then this ray on emergence will proc in the direction F Q; also join A F′ meeting the n plane in R, after refraction it will be parallel to axis as S R. Let the rays F Q, S R meet in *a*, the will be the point of the image corresponding to A; have to prove that A N, *a* N′ are parallel. Since triangles A N U, *a* N′ V are both right-angled, must prove that the sides about the right angles proportionals, and thus the triangles similar, for the angles A N U, *a* N′ V are equal, and thus A *a* N′ will be parallel.

$$\text{Now } \frac{NU}{N'V} = \frac{NU'}{NF'} \times \frac{N'F}{N'V} \text{ for } NF' = N'F \text{ by}$$

properties of the principal foci.

right towards the left; the positive direction is therefore from left to right.

If we wish the rays to emerge parallel the image will go off to an infinite distance, and $v = \infty$ or $1/v = 0$, and we get

$$\frac{1}{u} = \frac{1}{\mathrm{F}} \text{ or } u = -\mathrm{F}.$$

Hence, in this case also there is a second principal focus F′ on the side of the lens opposite to F, and at the same distance from N that F is from N′.

47*a*. We must now consider a point which will be useful when we come to deal with combinations of lenses not in contact.

We can evolve the theory in a different order; if we have given the two principal foci F and F′ (Fig. 31) with their properties, and the fact that if an object be placed in one focal plane it has an equal image in the other focal plane, we can prove that the lines A N, *a* N′ joining the corresponding points of object and image to the nodal points are parallel, and hence deduce the relations found in the last article.

Let us trace two rays from A, let A P parallel to the axis meet the first nodal plane in P, mark off Q N′ equal to P N, then this ray on emergence will proceed in the direction F Q; also join A F′ meeting the nodal plane in R, after refraction it will be parallel to the axis as S R. Let the rays F Q, S R meet in *a*, then *a* will be the point of the image corresponding to A; we have to prove that A N, *a* N′ are parallel. Since the triangles A N U, *a* N′ V are both right-angled, we must prove that the sides about the right angles are proportionals, and thus the triangles similar, for then the angles A N U, *a* N′ V are equal, and thus A N, *a* N′ will be parallel.

$$\text{Now } \frac{\mathrm{N\,U}}{\mathrm{N'\,V}} = \frac{\mathrm{N\,U'}}{\mathrm{N\,F'}} \times \frac{\mathrm{N'\,F}}{\mathrm{N'V}} \text{ for } \mathrm{N\,F'} = \mathrm{N'\,F} \text{ by the}$$

properties of the principal foci.

Also by similar triangles

$$\frac{\mathrm{N\,U}}{\mathrm{N\,F'}} = \frac{\mathrm{A\,R}}{\mathrm{F'\,R}} = \frac{\mathrm{P\,R}}{\mathrm{R\,N}} \text{ and } \frac{\mathrm{N'\,F}}{\mathrm{N'\,V}} = \frac{\mathrm{Q\,F}}{\mathrm{Q}\,a} = \frac{\mathrm{Q\,N'}}{\mathrm{Q\,S}} = \frac{\mathrm{P\,N}}{\mathrm{P\,R}}$$

$$\therefore \frac{\mathrm{N\,U}}{\mathrm{N'\,V}} = \frac{\mathrm{P\,R}}{\mathrm{R\,N}} \times \frac{\mathrm{P\,N}}{\mathrm{P\,R}} = \frac{\mathrm{P\,N}}{\mathrm{R\,N}} = \frac{\mathrm{A\,U}}{a\,\mathrm{V}}$$

Hence the sides of the triangles A N U, a N′ V about the equal angles, are proportionals; the triangles are therefore similar.

Hence the angles A N U, a N′ V are equal.

And therefore A N, a N′ are parallel.

48. To Find the Focal Length.—Let rays parallel to the axis fall on the lens, and let their image by refraction at the first surface be at a distance w from the surface, then (§ 31)

$$\frac{\mu}{w} = \frac{\mu - 1}{r} = \frac{\mu}{f}, \therefore w = f$$

The first image, therefore, will be at a distance $w + e = f + e$ from the second surface, since e is the distance between the surfaces; let v be the distance of the second image from the second surface, then

$$\frac{\mu}{w + e} - \frac{1}{v} = \frac{\mu - 1}{s} = -\frac{\mu}{f'},$$

$$\therefore \frac{1}{v} = \frac{\mu}{w + e} + \frac{\mu}{f'}, = \frac{\mu}{f + e} + \frac{\mu}{f'}, = \mu\,\frac{e + f + f'}{f'\,(f + e)}$$

$$\therefore v = \frac{f'\,(f + e)}{\mu\,(e + f + f')}$$

But (Fig. 28) $v = \mathrm{A'\,F}$

$$\therefore \mathrm{N'\,F} = \mathrm{A'\,F} - \mathrm{A'\,N'} = \frac{f'\,(f + e)}{\mu\,(e + f + f')}$$

$$- \frac{e f'}{\mu\,(e + f + f')} = \frac{f f'}{\mu\,(e + f + f')}$$

Hence if F is the focal length of the lens

$$F = \frac{ff'}{\mu(e + f + f')} \text{ or } \frac{1}{\mu F} = \frac{1}{f} + \frac{1}{f'} + $$

This is usually the most convenient formula t with, but we can express F in terms of the ra thickness by putting in the values of f, f', and

$$\frac{1}{F} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) - \frac{(\mu - 1)^2 e}{\mu r s}$$

Comparing this with § 33 we see that the e the thickness is to introduce an extra term i expression for the focal length depending on th ness e; if we make $e = 0$, or the lens thin, it to the expression for the focal length of a thin

49. Numerical Examples for Thick Lense shall take for calculation lenses similar t treated in § 33, but with a thickness of ·2 in number the cases to correspond.

In each case let x and y denote the distance nodal points of incidence and emergence resp from the corresponding surfaces, and let the dir in § 33 be used.

The values of x, y and F (quoted for referenc

$$x = \frac{-ef}{\mu(e + f + f')}, \quad y = \frac{ef'}{\mu(e + f + f')}$$

$$F = \frac{ff'}{\mu(e + f + f')}$$

$$f = \frac{\mu r}{\mu - 1}, \quad f' = \frac{-\mu s}{\mu - 1}$$

(*a*) $r = 5$ inches, $s = 7$ inches, $\mu = 1·5$ in ·2 inch.

$$f = \frac{\mu r}{\mu - 1} = \frac{1·5 \times 5}{·5} = 15,$$

$$f' = \frac{-\mu s}{\mu - 1} = -\frac{1·5 \times 7}{·5} = -21$$

$$e + f + f' = ·2 + 15 - 21 = -5·8$$

$$F = \frac{ff'}{\mu(e+f+f')} \text{ or } \frac{1}{\mu F} = \frac{1}{f} + \frac{1}{f'} + \frac{e}{ff'}$$

This is usually the most convenient formula to work with, but we can express F in terms of the radii and thickness by putting in the values of f, f', and we get

$$\frac{1}{F} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) - \frac{(\mu - 1)^2 e}{\mu r s}$$

Comparing this with § 33 we see that the effect of the thickness is to introduce an extra term into the expression for the focal length depending on the thickness e; if we make $e = 0$, or the lens thin, it reduces to the expression for the focal length of a thin lens.

49. Numerical Examples for Thick Lenses.—We shall take for calculation lenses similar to those treated in § 33, but with a thickness of ·2 inch, and number the cases to correspond.

In each case let x and y denote the distances of the nodal points of incidence and emergence respectively from the corresponding surfaces, and let the dimensions in § 33 be used.

The values of x, y and F (quoted for reference) are

$$x = \frac{-ef}{\mu(e+f+f')}, \quad y = \frac{ef'}{\mu(e+f+f')}$$

$$F = \frac{ff'}{\mu(e+f+f')}$$

$$f = \frac{\mu r}{\mu - 1}, \quad f' = \frac{-\mu s}{\mu - 1}$$

(*a*) $r = 5$ inches, $s = 7$ inches, $\mu = 1{\cdot}5$ inch, $e = {\cdot}2$ inch.

$$f = \frac{\mu r}{\mu - 1} = \frac{1{\cdot}5 \times 5}{{\cdot}5} = 15,$$

$$f' = \frac{-\mu s}{\mu - 1} = -\frac{1{\cdot}5 \times 7}{{\cdot}5} = -21$$

$$e + f + f' = {\cdot}2 + 15 - 21 = -5{\cdot}8$$

$$\therefore x = \frac{-ef}{\mu(e+f+f')} = \frac{\cdot 2 \times 15}{1\cdot 5 \times 5\cdot 8} = \cdot 34 \text{ inch.}$$

$$y = \frac{ef'}{\mu(e+f+f')} = \frac{-\cdot 2 \times 21}{1\cdot 5 \times 5\cdot 8} = \cdot 48 \text{ inch.}$$

$$\mathrm{F} = \frac{ff'}{\mu(e+f+f')} = \frac{15 \times 21}{1\cdot 5 \times 5\cdot 8} = 36\cdot 2 \text{ inches.}$$

Hence both nodal points are outside the lens and in front of it (Fig. 32 A).

(*b*) Double concave lens.

$r = 5$ inches, $s = -7$ inches, $\mu = 1\cdot 5$ inch, $e = \cdot 2$ inch.

$$\therefore f = \frac{\mu r}{\mu - 1} = 15, f' = \frac{\mu s}{\mu - 1} = 21$$

$$e + f + f' = \cdot 2 + 15 + 21 = 36\cdot 2$$

$$\therefore x = \frac{-ef}{\mu(e+f+f')} = -\frac{\cdot 2 \times 15}{1\cdot 5 \times 36\cdot 2} = -\cdot 055 \text{ inch.}$$

$$y = \frac{ef'}{\mu(e+f+f')} = \frac{\cdot 2 \times 21}{1\cdot 5 \times 36\cdot 2} = \cdot 077 \text{ inch.}$$

$$\mathrm{F} = \frac{ff'}{\mu(e+f+f')} = \frac{15 \times 21}{1\cdot 5 \times 36\cdot 2} = 5\cdot 80 \text{ inches.}$$

Hence the nodal points are both inside the lens, and close to the surfaces.

(*c*) Meniscus (Fig. 32 C).

$r = 7$ inches, $s = 5$ inches, $\mu = 1\cdot 5$ inch, $e = \cdot 2$ inch.

$$f = \frac{\mu r}{\mu - 1} = 21, \quad f' = \frac{-\mu s}{\mu - 1} = -15$$

$$e + f + f' = \cdot 2 + 21 - 15 = 6\cdot 2$$

$$\therefore x = \frac{-ef}{\mu(e+f+f')} = \frac{-\cdot 2 \times 21}{1\cdot 5 \times 6\cdot 2} = -\cdot 451 \text{ inch.}$$

$$y = \frac{ef'}{\mu(e+f+f')} = -\frac{\cdot 2 \times 15}{1\cdot 5 \times 6\cdot 2} = -\cdot 323 \text{ inch.}$$

$$\mathrm{F} = \frac{ff'}{\mu(e+f+f')} = -\frac{15 \times 21}{1\cdot 5 \times 6\cdot 2} = -33\cdot 87 \text{ inches.}$$

Hence the nodal points are both outside the lens and behind it.

(*d*) Double convex lens.

$r = -7$ inches, $s = 5$ inches, $\mu = 1{\cdot}5$, $e = {\cdot}2$ inch.

$$f = \frac{\mu r}{\mu - 1} = -21, \; f' = \frac{-\mu s}{\mu - 1} = -15$$

$$e + f + f' = {\cdot}2 - 21 - 15 = -35{\cdot}8$$

$$\therefore x = \frac{-ef}{\mu(e + f + f')} = -\frac{{\cdot}2 \times 21}{1{\cdot}5 \times 35{\cdot}8} = -{\cdot}078 \text{ inch.}$$

$$y = \frac{ef'}{\mu(e + f + f')} = \frac{{\cdot}2 \times 15}{1{\cdot}5 \times 35{\cdot}8} = {\cdot}056 \text{ inch.}$$

$$\mathrm{F} = \frac{ff'}{\mu(e + f + f')} = \frac{21 \times 15}{1{\cdot}5 \times 35{\cdot}8} = -5{\cdot}85 \text{ inches.}$$

Hence both nodal points lie inside the lens and very near the surfaces, and the focal length is practically the same as that of the corresponding thin lens.

The lenses (*a*) and (*c*) with their nodal points are shown on Fig. 32; the nodal points of (*b*) and (*d*) are too close to the surface to be shown on a figure.

If one surface of a lens be plane, it is not hard to see on inspection that the optical centre lies at the point in which the curved surface is cut by the axis; and one of the nodal points being the image of the optic centre due to refraction at the curved surface will coincide with it.

50. Magnification.—Since the diagram for the thick lens may be got from that for the thin lens by dividing it along the straight line perpendicular to the axis, passing through the centre of the lens, and then sliding the two parts asunder, it is evident that what has been said about magnification for a thin lens will hold good for a thick one, provided the distance (u) is measured from the nodal point of incidence and (v) from the nodal point of emergence.

Or, from Fig. 31,

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{a\text{ V}}{\text{A U}} = \frac{\text{N}'\text{ U}}{\text{N U}} = \frac{v}{u} = \frac{\text{F}}{u + \text{F}} \ (\S\ 37).$$

51. Graphical Calculation.—Since the formulæ for the thick lens are of the same form as those for a thin lens, we can evidently make use of the graphical construction already explained, if we assign the proper meanings to the various lengths.

Note.—It will be useful further on to have the relation connecting the distances of object and image for a thick lens in terms of the distances from the surfaces, instead of the distances from the nodal points.

The positions of the nodal points are dependent on the refractive index, so that they are not the same for different colours; it is important to bear this in mind when considering the chromatic aberration of a thick lens.

Let u and v be the distances of the object and image from the lens measured from the front and back surfaces respectively, r and s the radii of the front and back surfaces, e the thickness of the lens.

Then, with the same convention of signs, the expression required is

$$\frac{1}{v} = \frac{1}{u} + (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) - \frac{e}{\mu}\left(\frac{\mu - 1}{r} + \frac{1}{u}\right)^2$$

This can be deduced without much trouble from the expressions given above.

Combinations of Lenses.

52. We have already (§ 40) considered a combination of two thin lenses in contact; we have now to consider two lenses separated by a sensible interval, and two thick lenses in contact. It will be seen that these two cases can be dealt with at the same time.

We shall take the case of two thick lenses, then that of two thin lenses can easily be deduced, and in fact the same formulæ will apply, the only difference being that

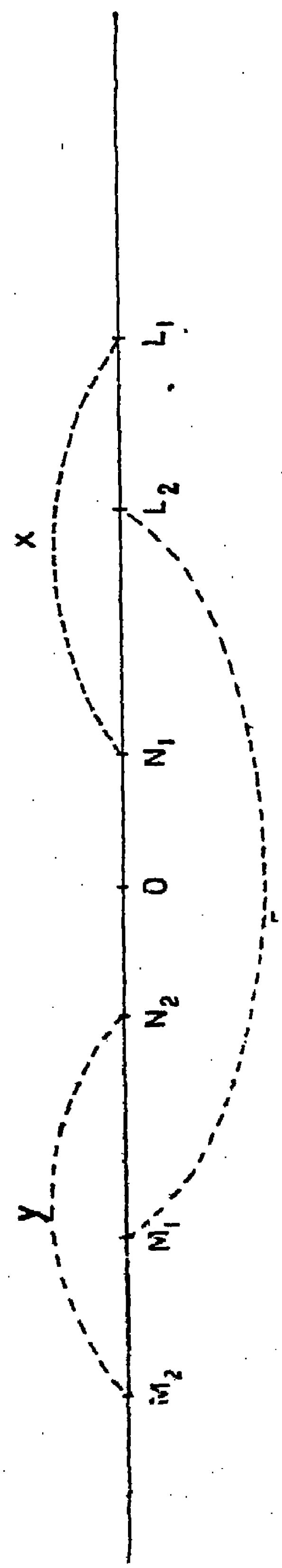


FIG. 33.

in the former case we measure distances from the nodal points, and in the latter from the surface of the lens.

The complete treatment of the subject involves a considerable amount of algebra, so we shall first state the results arrived at, and then give a geometrical verification by the aid of the graphical construction already explained.

53. Statement of Results of Combination.—Let there be two thick lenses of focal lengths f_1, f_2, separated by a sensible interval; we shall show that the combination is equivalent (both as regards the relative positions of object and image, and also as regards magnification) to a certain thick lens.

Let (Fig. 33) L_1 L_2 be the nodal points of the front lens focal length f_1, M_1 M_2 the nodal points of the back lens focal length f_2, and let the distance between the lenses be given by $M_1 L_2 = e$, measured from M_1 to L_2. Now let N_1 N_2 be the nodal points of the equivalent thick lens, and let F be its focal length, and let

$$L_1 N_1 = x, \quad M_2 N_2 = y, \text{ then will}$$

$$x = \frac{-e f_1}{e + f_1 + f_2}, \quad y = \frac{e f_2}{e + f_1 + f_2},$$

$$F = \frac{f_1 f_2}{e + f_1 + f_2}$$

And if O be the optical centre of the equivalent lens, then will

$$L_2 O = \frac{-e f_1}{f_1 + f_2}, \quad M_1 O = \frac{e f_2}{f_1 + f_2}$$

54. To pass from this case to that of

thin lenses we have only to make the pairs of points L_1 L_2 and M_1 M_2 coincide, they will then be the positions of the thin lenses.

55. Programme of Proof.—To prove the statements above, we have to show two things:

(*a*) That the distances of the object and image by refraction through both lenses from two fixed points are connected by a relation of the same *form* as that which connects the distances of object and image from the nodal points with a single thick lens.

(*b*) That the various quantities involved have the values stated above.

We have then, first of all, to show that the combination has two points resembling the nodal points of a thick lens, which we shall identify by the following characteristic property of nodal points (§ 46):

If an object be placed in one nodal plane, its geometrical image, by refraction through both lenses, lies on the other nodal plane, and is equal in size to the object.

We have next to show that the combination has two principal foci.

When these two points are established, we know (§ 47*a*) that any incident ray passing through one nodal point of the combination emerges through the other nodal point, and is parallel to its original direction. The resemblance between the combination and a thick lens will then be complete.

55*a*. Proof.—In the first place, the two lenses will form a definite image of any object, for if we take the effects of the two lenses in succession, the first lens will form an image of the object, and the second lens will then form an image of the first image.

We see also from this that there must be two points, corresponding to the principal foci of a single lens, to which rays which are parallel to the axis before refraction converge after refraction, or from which the rays proceed which are parallel after refraction.

We shall take the case of two diverging lenses, that the focal lengths may be positive; any other case can be got by giving the focal lengths their proper signs.

Let us now determine whereabouts the nodal points of the combination—if they exist—must lie. Turning to Fig. 24, we see that the image of a distant object, formed by a concave lens, is smaller than the object, and this will always be the case as long as the object is real.

For if u, v be the distances of the object and image from the lens of focal length f, we have as usual

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

and f is positive, so v must be less than u.

Also by § 50

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{v}{u}$$

hence the image is less than the object.

Again, by refraction at the second lens the size of the image will be still further reduced. Hence, if we are to have the final image equal to the object, the object cannot be in front of the first lens, but must be virtual and behind it; similarly, the equal image cannot be behind the second lens.

We conclude, therefore, that with two concave lenses the nodal points of the combination must lie between those of the component lenses, as shown in Fig. 33.

We must now turn to graphical methods to prove the existence of the nodal points N_1, N_2, and to determine their position.

To the straight line B D (Fig. 34) erect perpendiculars A B ($= f_1$), C D ($= f_2$), produce these to E and F, so that A E $=$ C F $= e$; join A D, B F intersecting in L_1', and C B, D E intersecting in M_2', and let B C, A D intersect in O′.

Drop perpendiculars L_1' F_1, M_2' F_2, O′ R to B D, and let L_1' F_1 intersect B C in N_1', M_2' F_2, intersect A D

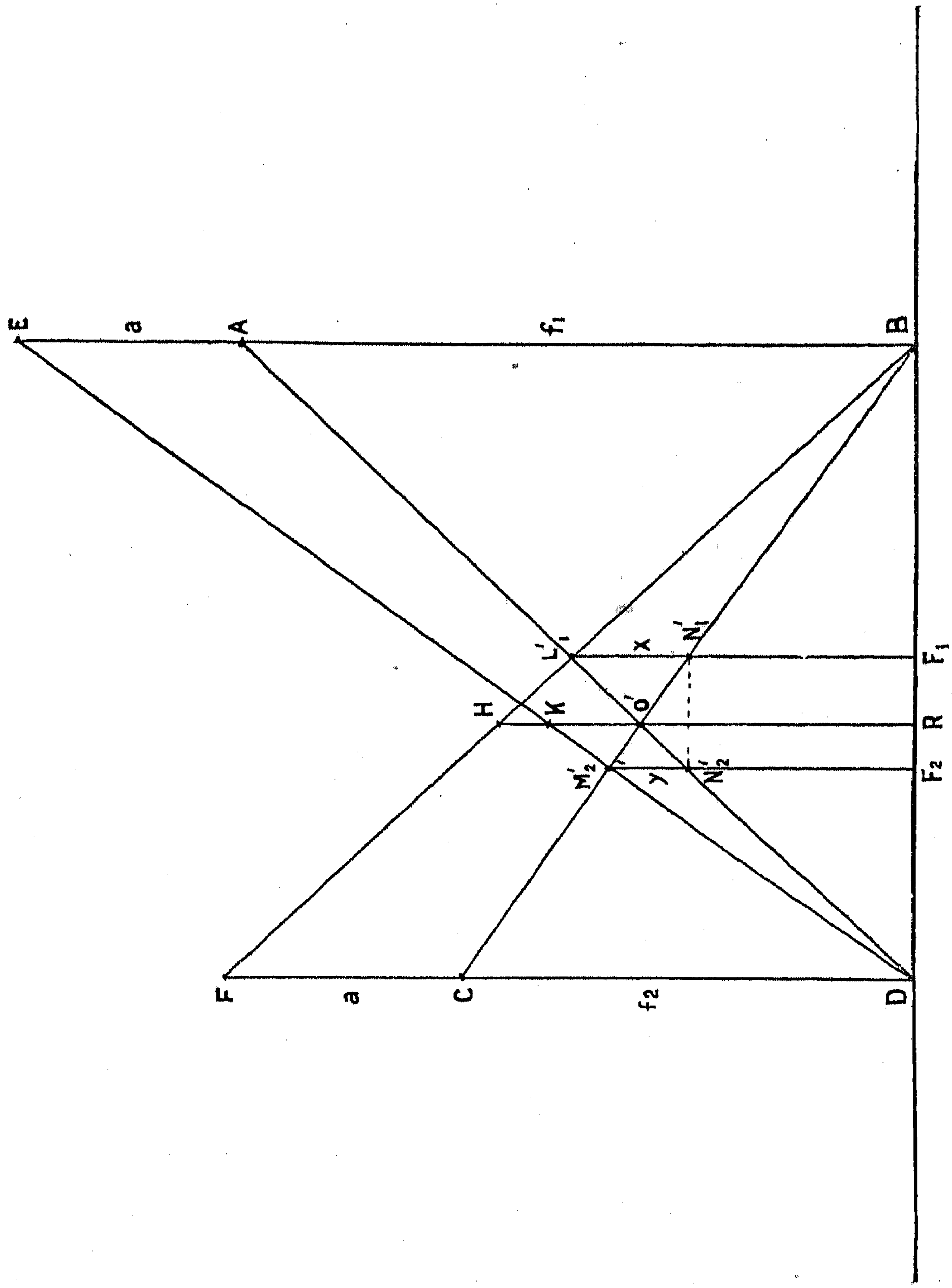


FIG. 34.

in N_2' and O' R produced, intersect B F, D E in H and K respectively.

First find the positions of the principal foci of the

combination; if parallel rays strike the first lens they will pass after refraction through its principal focus distant f_1, or A B from L_2 the nodal point of emergence. This point will be distant $e+f_1$ or E B from the nodal point of incidence of the second lens; the image of this point by refraction through the second lens of focal length f_2 is, by the graphical construction, at a distance $M_2' F_2$ from the nodal point of emergence.

Again, by symmetry it can be seen from the figure that the rays emanating from a point distant $L_1' F_1$ from the second lens will after refraction by both lenses be parallel.

It is evident also that $M_2' F_2$ must represent a length measured in the positive direction and $L_1' F_1$ is measured in the negative direction, for the two principal foci are always on opposite sides of the nodal points.

Next consider the lengths $L_1' N_1'$ and $O' H$; L_1' is at the intersection of A O and B H, so that

$$\frac{1}{L_1' N_1'} = \frac{1}{O' H} + \frac{1}{A B} = \frac{1}{O' H} + \frac{1}{f_1}$$

$$\text{or } -\frac{1}{O' H} + \frac{1}{L_1' N_1'} = \frac{1}{f_1}$$

Hence if $L_1' N_1'$ represents a length in the negative direction, an object at that distance from the first lens will have an image due to that lens at a distance $-O' H$ from it.

We must find the distance of this image from the nodal point of emergence of the second lens, by parallels.

$$\frac{O' H}{C F} = \frac{B O'}{B C} = \frac{B R}{D B} = \frac{f_1}{f_1 + f_2}, \therefore O' H = \frac{e f_1}{f_1 + f_2}$$

$$\frac{O' K}{A E} = \frac{D O'}{D A} = \frac{D R}{D B} = \frac{f_2}{f_1 + f_2}, \therefore O' K = \frac{e f_2}{f_1 + f_1}$$

$$\therefore O' H + O' K = e \text{ or } O' K = e - O' H$$

Hence $O' K$ is the distance of the image from the nodal point of incidence of the second lens. Again by con-

struction M_2 is the intersection of C O′ and D K, so that

$$\frac{1}{M_2' N_2'} = \frac{1}{O' K} + \frac{1}{C D} = \frac{1}{O' K} + \frac{1}{f_2}$$

$$\text{or } \frac{1}{M_2' N_2'} - \frac{1}{O K} = \frac{1}{f_2}$$

Hence the second image found by refraction at the second lens is at a distance $M_2' N_2'$ from the nodal point of emergence of that lens. If, then, there be a virtual object at distance $L_1' N_1'$ behind the nodal point of incidence of the first lens, it will have an image by refraction through both lenses at a distance $M_2' N_2'$ in front of the nodal point of emergence of the second lens; also

$$\frac{\text{Size of object}}{\text{Size of first image}} = \frac{L_1' N_1'}{O' H},$$

$$\frac{\text{Size of first image}}{\text{Size of second image}} = \frac{O' K}{M_2' N_2'}$$

$$\therefore \frac{\text{Size of object}}{\text{Size of final image}} = \frac{L_1' N_1'}{O' H} \times \frac{O' K}{M_2' N_2'} \quad (\alpha)$$

$$\text{Now } \frac{L_1' N_1'}{C F} = \frac{B N_1'}{B C} = \frac{B F_1}{B D} = \frac{f_2}{(e + f_1 + f_2)} \quad (\S\ 39)$$

$$\frac{M_2' N_2'}{A E} = \frac{D N_2'}{D A} = \frac{D F_2'}{D B} = \frac{f_2}{(e + f_2 + f_1)}$$

$$\therefore L_1' N_1' = \frac{e f_1}{e + f_1 + f_2}, \quad M_2' N_2' = \frac{e f_2}{e + f_1 + f_2}$$

$$\therefore \frac{L_1' N_1'}{M_2' N_2'} = \frac{f_1}{f_2} \text{ also } \frac{O'K}{O'H} = \frac{e f_2}{f_1 + f_2} \times \frac{f_1 + f_2}{e f_1} = \frac{f_2}{f_1}$$

Hence, since the product of these latter ratios is unity, we see from (α) that

Size of image = size of object.

Thus, the points distant $- L_1' N_1'$ from the nodal point of incidence of the front lens, and $M_2' N_2'$ from

the nodal point of emergence of the second lens, fulfil all the conditions for being the nodal points of the combination; we therefore conclude that such points exist, and that the combination can be replaced, as far as the positions of object and image, and size of object and image, are concerned by a single thick lens.

The two figures (33 and 34) have been lettered to correspond, hence

$$x = -L_1 N_1 = -L_1' N_1' = \frac{-ef_1}{e + f_1 + f_2}$$

$$y = M_2 N_2 = M_2' N_2' = \frac{ef_2}{e + f_1 + f_2}$$

The focal length of the combination is the distance of the principal focus from N_2, the nodal point of emergence, or of the second principal focus from N_1, the nodal point of incidence.

These lengths are evidently given in the diagram by $N_1' F_1$ and $N_2' F_2$, and we can show that these are equal, as they should be, for

$$\frac{N_1 F_1'}{L_1 N_1'} = \frac{C D}{C F} = \frac{f_2}{e}, \therefore N_1' F_1 = \frac{f_2}{e} \times \frac{ef_1}{e + f_1 + f_2}$$

$$\therefore N_1' F_1 = \frac{f_1 f_2}{e + f_1 + f_2}$$

and this being symmetrical with respect to f_1 and f_2, we shall evidently get the same value for $N_2 F_2'$

It is evident since $O'K$ and $O'H$ give the position of the image produced by the first lens of an object placed at N_1, that these lengths give the position of the optical centre of the combination, and hence

$$O N_1 = O' K = \frac{-ef_1}{f_1 + f_2}, \quad O N_2 = O' H = \frac{ef_2}{f_1 + f_2}$$

This completes the proof of all the statements in § 53.

56. Combinations in General.—We have shown how to find the lens equivalent to two lenses; if

there are three or more lenses, these can be taken in pairs, and replaced by equivalent thick lenses, and these can, in turn, be taken in pairs and replaced by other equivalent lenses, till we finally arrive at a single thick lens which is equivalent to the whole system. Thus we see that *any* combination of lenses can be replaced by a single thick lens. This lens may not, in all cases, be such as can be conveniently realized in practice, but this is not often required; its use is to simplify calculations.

It must be clearly understood, when we say that a single lens is equivalent to a combination, we mean only that it will act in the same way as the combination as regards the relative positions and sizes of object and image; but there are other properties of combinations important in photography, such as corrections for spherical and chromatic aberrations treated of in the next chapter, which cannot be reproduced by a single lens.

57. Reversibility of Optical Instruments. From what we know about the nodal points of lenses and principal foci, it is clear that if a lens be reversed so that the positions of the nodal points are interchanged, no change in the position of the image of any object will be produced; and as the combination is equivalent to a single thick lens it can be reversed in like manner.

This is evident at first sight when combinations are symmetrical, but not in other cases.

58. Numerical Example.—Dallmeyer's wide-angle landscape lens.

This objective consists of three lenses in contact. The component lenses are shown in Fig. 35; the two outer lenses are of crown glass and convergent, and the middle lens is of flint glass and divergent, the concave surfaces being all turned towards the incident light.

Let A, B, C, D, as in the figure, be the points in which the surfaces are cut by the axis of the system; also let

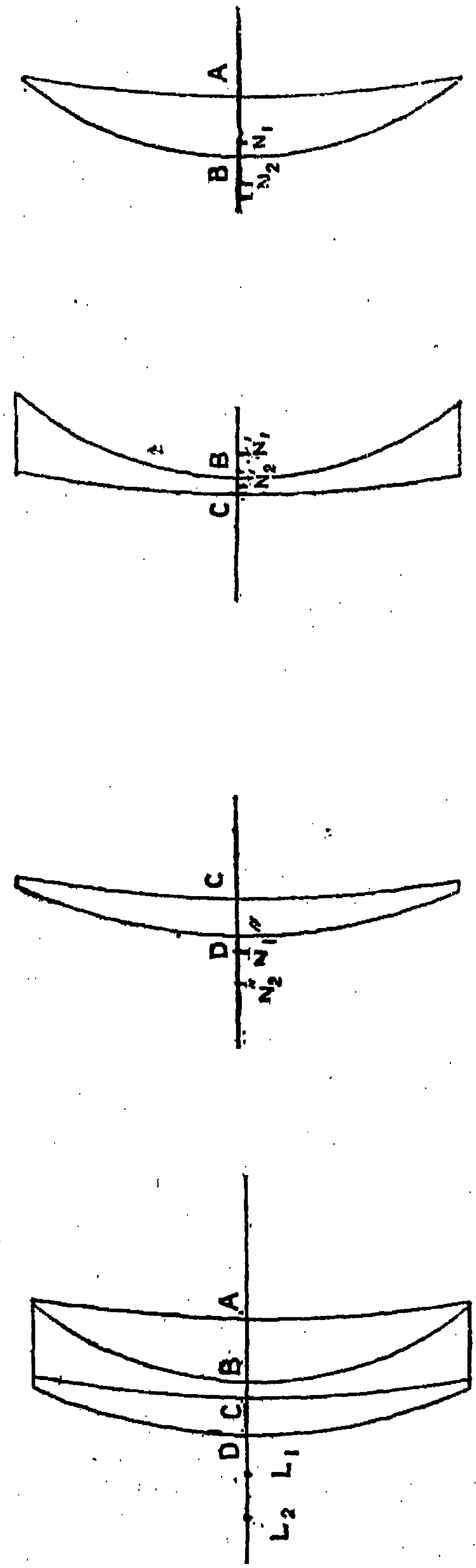


FIG. 35.

N_1 N_2 be the nodal points of the first lens:
N_1' N_2' ,, ,, ,, second lens.

$N_1'' \, N_2''$ be the nodal points of the third lens.
$M_1 \, M_2$,, ,, ,, combination of 1st and 2nd lenses.
$L_1 \, L_2$,, ,, ,, whole combination.

These points as determined below are shown on a magnified scale in Fig. 36.

To give the calculations in full would occupy too much space, so the summary only is given.

Let f, f', r, s, e, etc., have the meanings assigned to them in § 44, then:

First lens: $r = 4{\cdot}29$ inches, $s = 1{\cdot}20$ inch, refractive index $= \mu_1 = 1{\cdot}5146$, $e = {\cdot}230$ inch; from this we get

$$A\,N_1 = x = -\,{\cdot}206 \text{ inch.} \quad (\S\ 44)$$
$$B\,N_2 = y = -\,{\cdot}057 \text{ inch.}$$
$$F_1 = \text{focal length} = -\,3{\cdot}159 \text{ inches.}$$

Second lens: $r = 1{\cdot}20$ inch, $s = 3{\cdot}75$ inches, refractive index $= \mu_2 = 1{\cdot}574$, $e = {\cdot}050$ inch; from this we get

$$B\,N_1' = x' = {\cdot}015 \text{ inch.}$$
$$C\,N_2' = y' = {\cdot}047 \text{ inch.}$$
$$F_2 = \text{focal length} = 3{\cdot}095 \text{ inches.}$$

Third lens: $r = 3{\cdot}75$ inches, $s = 1{\cdot}80$ inch, refractive index $= \mu_3 = 1{\cdot}517$, $e = {\cdot}151$ inch; from this we get

$$C\,N_1'' = x'' = -\,{\cdot}186 \text{ inch.}$$
$$D\,N_2'' = y'' = -\,{\cdot}089 \text{ inch.}$$
$$F_3 = \text{focal length} = -\,6{\cdot}51 \text{ inches.}$$

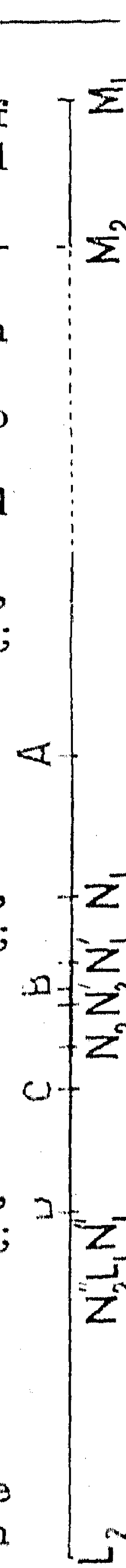


Fig. 36.

Now combine the first and second lenses, § 53, we have e = distance between N_1 and N_1', which is in the negative direction (see figure) $= -\,(B\,N_2 + B\,N_1') = -\,({\cdot}057 + {\cdot}015) = -\,{\cdot}072$ inch.

$$F_1 = -\,3{\cdot}158 \text{ inches}, \; F_2 = 3{\cdot}095 \text{ inches.}$$

Hence

$$M_1 N_1 = x = \frac{-e F_1}{e + F_1 + F_2} = -\frac{\cdot 072 \times 3\cdot 158}{\cdot 135} = 1\cdot 68 \text{ inch.}$$

$$M_2 N_2' = y = \frac{e F_2}{e + F_1 + F_2} = \frac{\cdot 072 \times 3\cdot 095}{\cdot 135} = 1\cdot 65 \text{ in.}$$

$$\phi = \text{focal length} = \frac{F_1 F_2}{e + F_1 + F_2} = \frac{3\cdot 158 \times 3\cdot 095}{\cdot 135} = 71\cdot 76 \text{ inch.}$$

$$\therefore M_1 A = 1\cdot 68 - \cdot 206 = 1\cdot 47 \text{ inch.}$$

$$M_2 C = 1\cdot 65 + \cdot 05 = 1\cdot 70 \text{ inch.}$$

Now add on the third lens, we have

$$e = \text{distance between } N_1'' \text{ and } M_2.$$
$$= \cdot 186 + 1\cdot 70 = 1\cdot 886 \text{ inch.}$$
$$\phi = 71\cdot 76 \text{ inches, } F_3 = -6\cdot 51 \text{ inches.}$$

$$\therefore L_1 M_1 = \frac{-e \phi}{\phi + F_3 + e} = -\frac{1\cdot 886 \times 71\cdot 76}{67\cdot 14} = -2\cdot 013 \text{ inches.}$$

$$L_2 N_2'' = \frac{e F_3}{\phi + F_3 + e} = -\frac{1\cdot 886 \times 6\cdot 51}{67\cdot 14} = -\cdot 183 \text{ in.}$$

$$F = \text{focal length} = \frac{\phi F_3}{\phi + F_3 + e} = -\frac{71\cdot 76 \times 6\cdot 51}{67\cdot 14} = -6\cdot 96 \text{ inches.}$$

$$\therefore A L_1 = -(L_1 M_1 - A M_1) = -(2\cdot 013 - 1\cdot 478) = -\cdot 535 \text{ inch.}$$

$$D L_2 = -(L_2 N_2'' - N_2'' D) = -(\cdot 183 + \cdot 089) = -\cdot 272 \text{ inch.}$$

Thus we have the focal length of the combination, and the positions of its nodal points.

This example is enough to show how the calculations are worked; the case in which the lenses are not in contact will only differ from this in altering the values of e, the distances between the nodal points, and presents no difficulty.

59. Definitions of Certain Terms.—In the present section, definitions will be given of several angles connected with a lens.

Let us consider the illumination of a plate at points at varying distances from the axis; at a point on the axis the aperture appears circular. As the point considered recedes from the axis the pencils which illuminate it become oblique, and the aperture is foreshortened, in consequence of which the illumination decreases regularly, and its rate of decrease can be calculated; but after a certain distance the light begins to be cut off by the mounting, and if we go far enough is altogether intercepted.

If we consider the whole circumference, we can then imagine two cones having their vertices at the nodal point of emergence.

On the surface of the first cone, called the *cone of illumination*, lie the axes of the extreme pencils, any portion of which escapes the mounting, it therefore contains *all* the light which reaches the plate; on the surface of the second cone lie the axes of the extreme pencils which are not at all cut off by the mounting, this may be called the *cone outside which the aperture begins to be eclipsed.*

Inside the latter cone the illumination decreases regularly and not very rapidly as we leave the axis, but outside it the decrease is irregular and generally rapid; it is therefore necessary to place the plate inside the inner cone to obtain anything like equal exposure all over.

In judging of a lens it is therefore important to know the angles of these cones (§ 115).

The angle of *sharpness* is the angle between the

axes of the extreme pencils which can be focussed on the plate to the sharpness required; it is a matter of every-day experience that this angle varies with the stop used, for the usual way to make the definition sharp at the edge of a picture is to reduce the stop.

If this angle is known for a lens with a particular stop, we can determine how large a plate the lens will cover with that stop.

Let (Fig. 37) N, N′ be the nodal points of incidence and emergence, x y the axis of the lens, and let A N, N′ D and B N, N′ C be the axes of the extreme pencils which give sharp enough images with the stop of aperture E F; for landscape work the plate must be placed either at, or very near to, the principal focal plane of the lens, since the objects are distant.

Let F be the principal focus, and let N′ C and N′ D meet the focal plane in C and D; then C D will be the greatest dimension of the largest plate that can be used.

The angle A N B (or C N′ D which is equal to it) is the angle of sharpness; denote this by 2 θ degrees, and let F be the focal length of the lens, then

$$\frac{\mathrm{C\,F}}{\mathrm{N'\,F}} = tan\ \mathrm{C\,N'\,F} = tan\ \theta, \therefore \mathrm{C\,F} = \mathrm{N'\,F}\ tan\ \theta.$$

$$\therefore \mathrm{C\,D} = 2\,\mathrm{C\,F} = 2\,\mathrm{N'\,F}\ tan\ \theta = 2\,\mathrm{F}\ tan\ \theta.$$

C D can therefore be found by the aid of a table of tangents when θ and F are known.

If we do not wish to use the lens for landscape work, but for copying, the plate will no longer be at distance F from the lens; we must in this case find the distance from N′ (call it v) at which the plate is placed, and we evidently get

$$\mathrm{C\,D} = 2\ v\ tan\ \theta.$$

It is, however, possible that θ may not be the same in this latter case; whether it is so or not can be found only by experiment.

We must next consider what the length C D is; it is

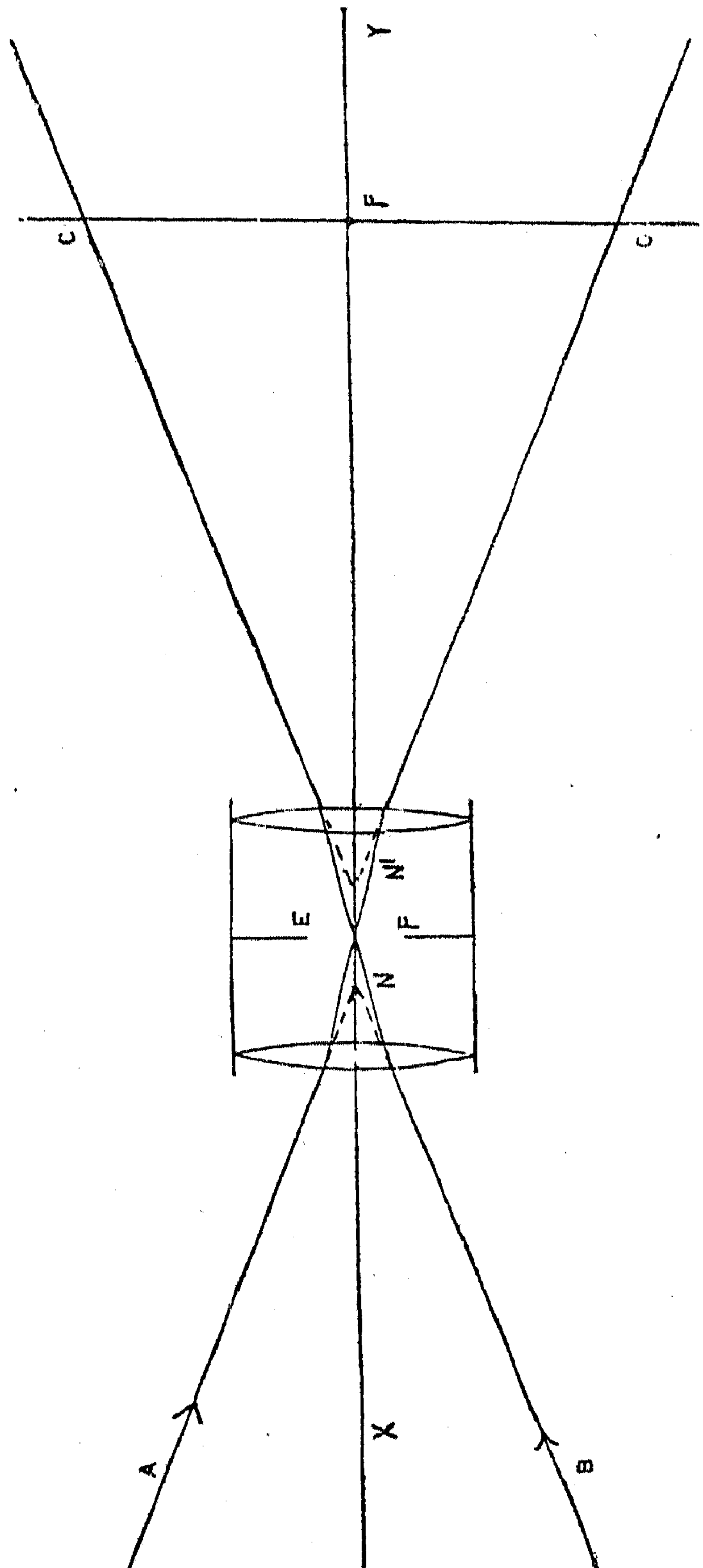


FIG. 37.

the diameter of a circle within which everything is as sharp as required.

If any particular sized plate will go inside that circle the lens will cover it sharply.

Thus (Fig. 38) the plate A C B D will just fit inside the circle of which C D is the diameter, and C D is therefore the diagonal of the plate.

If we know the dimensions of the plate we can find its diagonal, for

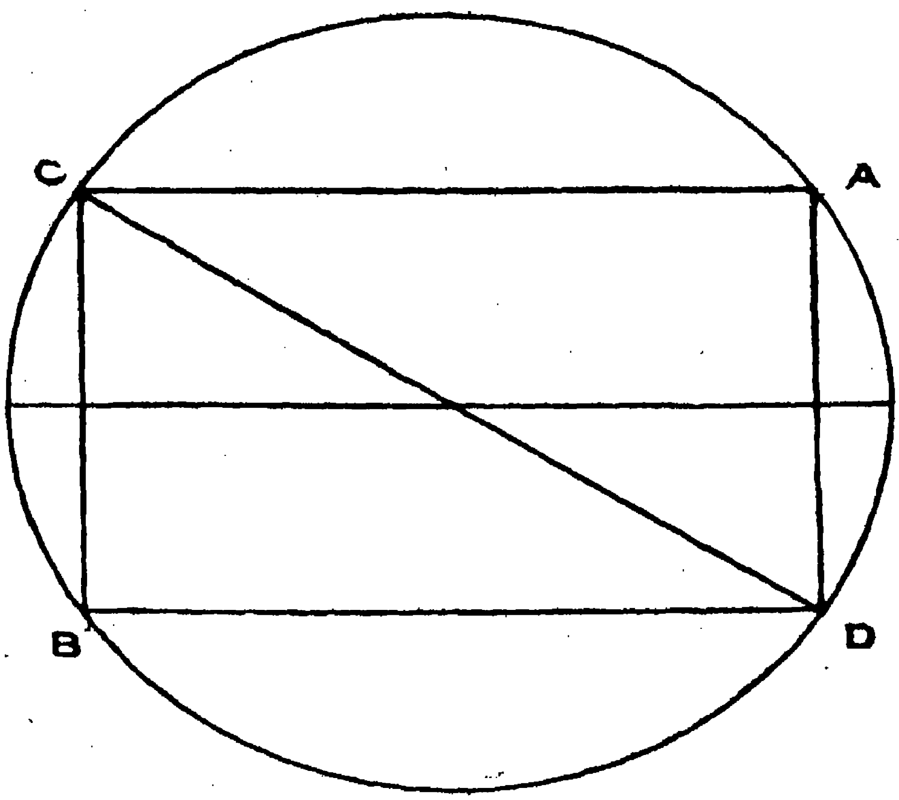


FIG. 38.

Square on diagonal = sum of squares on the sides.

Example.—Will a lens of six inches focal length, whose angle of sharpness is fifty degrees with its largest stop, cover a plate $3\frac{1}{4} \times 4\frac{1}{4}$ inches?

Here diagonal $= \sqrt{(3{\cdot}25^2 + 4{\cdot}25^2)} =$
$\sqrt{28{\cdot}62} = 5{\cdot}35$ inches.

$$\text{And } 2\,\theta = 50°, \therefore \theta = 25°$$

$\therefore$ C D $= 2$ F $\tan\theta = 2 \times 6 \times \tan 25° =$
$12 \times {\cdot}4663 = 5{\cdot}595$ inches.

Hence C D, being greater than the diagonal, the lens will cover the plate sharply.

The *angle of view* of the lens is the angle between the axes of the two extreme pencils of rays which strike the plate. This is the same as the angle between the lines joining the nodal point of incidence to the most widely separated objects that will be included in the picture.

There is here a danger of confusion, for we may take the most widely separated pencils to be those which strike the plate at the extremities of a diagonal, or else those which strike it at the extremities of either a horizontal or vertical line. We shall take the angle between the pencils which meet the plate at the extremities of a horizontal line, but the reader should be on his guard, as makers in their catalogues often use the diagonal. The angle will of course depend on which side of the plate is horizontal. For purposes of comparison it will be well to keep to the case when the longer side is horizontal.

To find the angle of view, let C D (Fig. 37) represent the longest side of the plate, which, in landscape work, is at the principal focus.

Let $2\,\phi$ be the angle of view, then

$$\mathrm{C\,N'\,D} = 2\,\phi \text{ and } \tan\phi = \frac{\mathrm{C\,F}}{\mathrm{N'\,F}} = \frac{\mathrm{C\,F}}{\mathrm{F}} = \frac{\mathrm{C\,D}}{2\cdot\mathrm{F}}$$

If, therefore, C F and F are known we can calculate $\tan\phi$, and hence find ϕ from the tables.

Example.—Find the angle of view of a lens of six inches focal length used with a plate $3\frac{1}{4} \times 4\frac{1}{4}$ inches.

Here F = 6 C D = 4·25 C F = 2·125.

$$\therefore \tan\phi = \frac{2\cdot125}{6} = \cdot3541$$

$$\therefore \phi = 19^\circ\ 30' \text{ and } 2\,\phi = 39^\circ$$

Hence the angle of view is 39°.

59*a*. Size of Image. — When a lens is used for such work as reducing and enlarging, in which it is comparatively close to the object to be photographed,

the relative sizes of object and image can be varied by varying their distances from the lens; and this can be done in many cases out of doors, the camera being moved till the picture on the ground glass is of the right size.

But in landscape work, specially where the scene is fairly distant, this cannot be done, for often the picture can be obtained from one point of view only; neither can we adjust the size of the picture by moving the focussing screen, for that would throw the picture out of focus. The only adjustment available is that of changing the lens and using one of a different focal length. By doing this we can vary the distance of the plate from the lens and yet keep the picture in focus.

The shorter the focus of the lens the nearer is the plate to it, and hence the larger the angle of view; also the larger the region whose picture is included on the plate and the smaller is any particular object. If the lens is of long focus, and the angle of view small, the smaller will be the region pictured, and the larger any particular object.

Hence to increase the size of the image of a particular object we increase the focal length of the lens used; this proceeding is limited only by the possible extension of the camera.

When very distant objects are to be photographed a lens of very long focus must be used. The difficulty of the extension of the camera has been met by the use of lenses of a special design called "telephotographic"; one of the best known of these is that of Dallmeyer, and as it furnishes an excellent example of the principles of lens combination we shall describe it in the next section.

60. Dallmeyer's Telephotographic Lens. — This objective consists of a converging lens or combination in front, and a diverging lens or combination behind; for simplicity these will here be represented by single lenses.

It is not hard to see that this arrangement can be made to act as a lens of great focal length and small angle of view. For (Fig. 39) let A be the converging lens, parallel rays falling on this would ordinarily converge to the point P; but they are made to fall on the diverging lens B, which causes them to converge to a point F much further back, where F is the image of P produced by the second lens. The rays which reach F are therefore inclined as if they came from some lens N very much further away than either A or B.

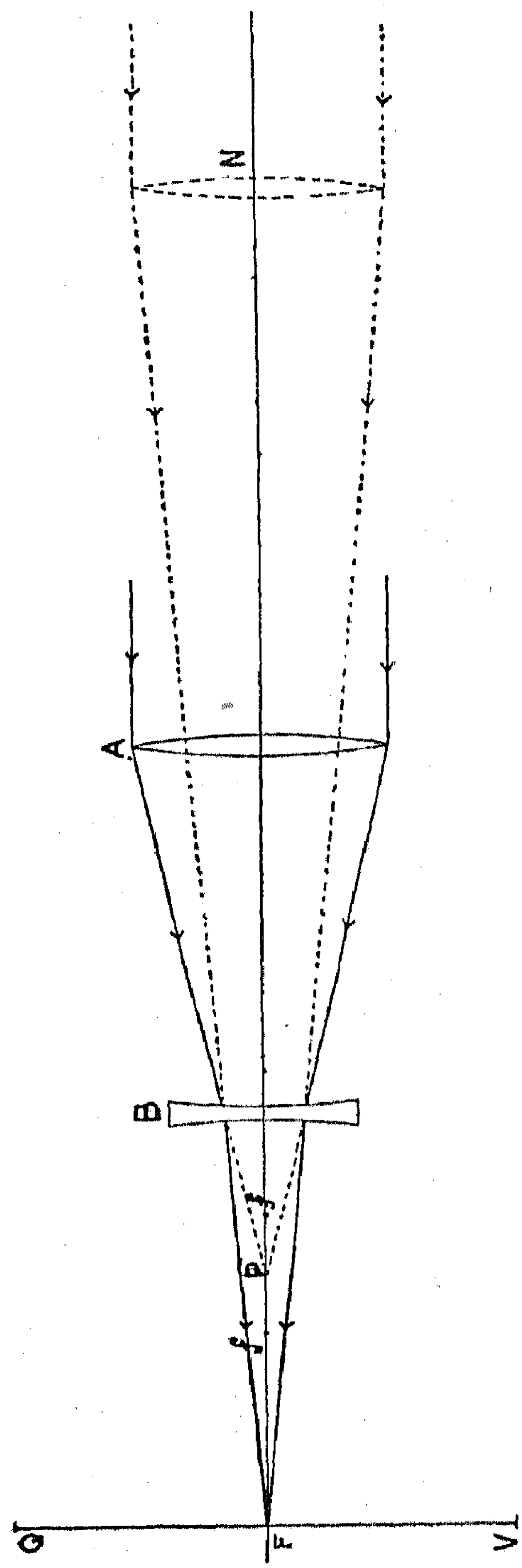


FIG. 39.

Thus the combination is equivalent to a lens of very much greater focal length than that of either of the component lenses, and thus gives us the means of obtaining by the aid of lenses comparatively near the plate the effect of a lens placed at a much greater distance away.

To calculate the exact effect of any particular combination we can use the formulæ we have already obtained, *i.e.* (§ 53)—

$$x = \frac{-ef_1}{e + f_1 + f_2},\ y = \frac{ef_2}{e + f_1 + f_2},$$

$$F = \frac{f_1 f_2}{e + f_1 + f_2}$$

where x, y, e, etc., have the meanings already assigned.

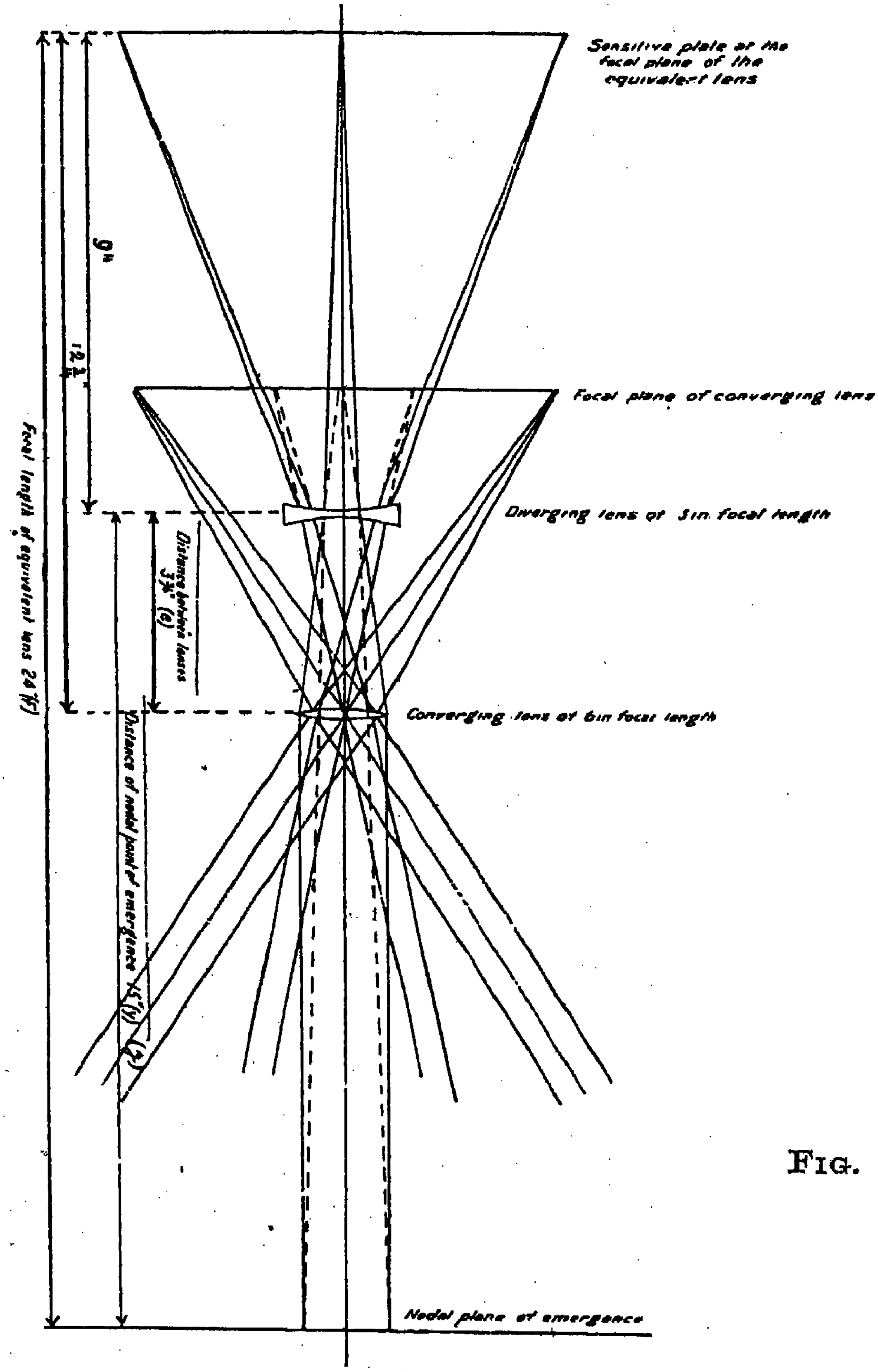


FIG. 40.

Example (Fig. 40).—Let $f_1 = -6$ inches, $f_2 = 3$ inches, then

$$x = \frac{6\,e}{e-3},\quad y = \frac{3\,e}{e-3},\quad \mathrm{F} = \frac{-18}{e-3}$$

and we can make e to have any value that may be convenient.

The equivalent lens must be convergent, and hence F negative or $e - 3$ positive, $\therefore e > 3$.

If $e = 3$ then $x = -\infty$, $y = +\infty$, $F = -\infty$, or the combination produces no effect.

Take, for example, $e = 3\frac{3}{4}$ inches, then we get $x =$ 30 inches, $y =$ 15 inches, $F = -$ 24 inches. The nodal point of emergence of the equivalent lens is therefore fifteen inches in front of the nodal point of emergence of the diverging lens, and its focal length is twenty-four inches. The lengths in this case are shown on Fig. 40, adapted from that in Dallmeyer's pamphlet, and will repay careful study. The position of one only of the nodal points is shown, the other being beyond the limits of the figure.

Two sets of oblique pencils of rays are shown; the first pair are near enough to the axis to fall on the second lens and be focussed on the plate, but the second pair do not strike the lens at all, but are absorbed by the mount of the diverging lens.

By varying the distance e we can vary the focal length of the combination, and could make it as great as we like were it not that a practical limit is placed by the possible extension of the camera.

If we suppose the diverging lens rigidly fixed to the camera, while the adjustment of e is made by moving the front lens, we can, when we know the possible extension, find the greatest focal length obtainable.

Example.—With the lens of the last example, if the camera can be extended till the ground glass is twelve inches distant from the diverging lens, find the distance e between the lenses and the corresponding focal length.

The distance between the plate and the back lens is equal to the difference between the focal length of the combination and the distance of the back lens from the nodal point of emergence of the combination $F - y$.

$\therefore$ Distance between plate and back lens

$$= -(F - y) = \frac{-f_1 f_2}{e + f_1 + f_2} - \frac{e f_2}{e + f_1 + f_2} = -\frac{f_2 (e + f_1)}{e + f_1 + f_2}$$

In the present case this becomes

$$12 = -\frac{3(e - 6)}{e - 3}$$

This is a simple equation for e, and when solved gives us $e = 3\frac{3}{5}$ inches. From this we can find that $F = -30$ inches. Hence thirty inches is the greatest focal length that can be obtained with the given camera.

61. Angle of View of Telephotographic Lens.—The angle of view can of course be found as before when the focal length used is known.

Example.—Find the angle of view of the lens adjusted as in the first example of the last section when used with a plate $4\frac{1}{4} \times 3\frac{1}{4}$ inches.

Here $F = -24$, $CD = 4\frac{1}{4}$ (§ 59), $CF = 2{\cdot}125$

$$\therefore \tan\theta = -\frac{CF}{F} = \frac{2{\cdot}125}{24} = {\cdot}0886$$

$$\therefore \theta = 5° \; 4' \qquad \therefore 2\theta = 10° \; 8'$$

$$\therefore \text{Angle of view} = 10° \; 8'$$

62. Magnification.—Dallmeyer reckons as the magnification, the ratio of the size of image of a distant object produced by the compound lens to that of the image produced by the converging lens alone.

The advantage of this proceeding is that we use the image produced by the converging lens alone as the standard with which we compare the size of the image produced by the combination.

To find the ratio, we notice first that the distant object in question will subtend angles at the nodal points of incidence of the converging and of the equivalent lens which are practically equal, and hence the angles subtended by the images at the nodal points of emergence of their respective lenses will be equal.

Let L and M, Fig. 41, be the nodal points of emergence of the equivalent and converging lenses respectively, and let A F B, C F D be the corresponding images, F being the point where the axis of the lens meets the image; then

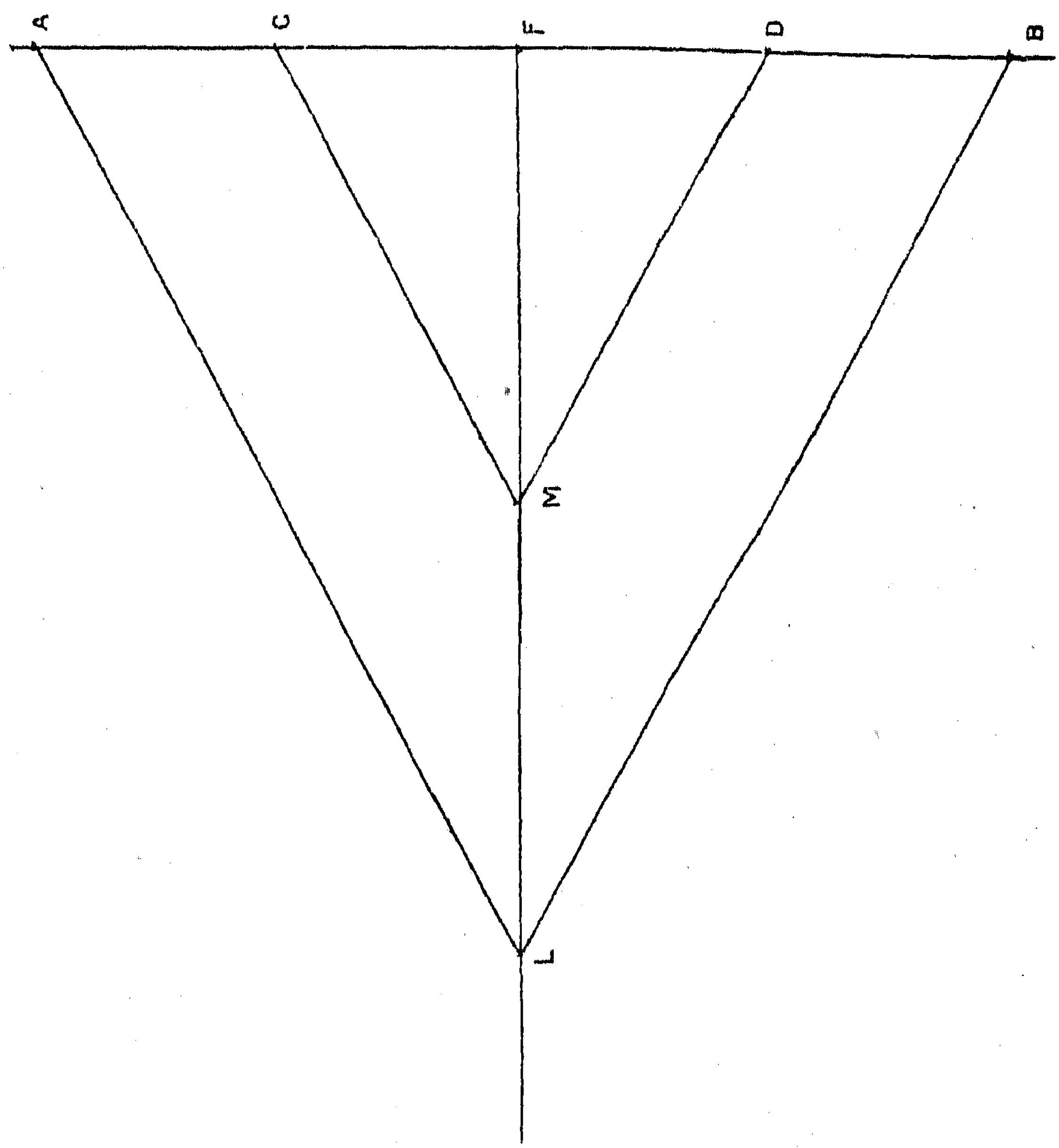


FIG. 41.

$$\frac{\text{Size of image by combination}}{\text{Size of image by converging lens}} = \frac{A\,B}{C\,D} = \frac{A\,F}{C\,F} = \frac{L\,F}{M\,F}$$

$$= \frac{\text{focal length of combination.}}{\text{focal length of converging lens.}}$$

For since the angles A L B, C M D are equal, triangles A L B, C M D will be similar, and so also will be the

triangles A L F, C M F. We see, therefore, that the magnification is expressed by the ratio of the focal lengths.

In the case considered above this ratio is $24/6 = 4$, and the size of the image produced by the combination is four times that produced by the converging lens alone.

It should be remarked that no attempt has been made to make the notation of these sections correspond with that used by Dallmeyer, nor have either general expressions or rules been given. Any question that may arise can easily be treated when the general principles of the combination are known; rules tend only to produce confusion.

63. Perspective.—Photographs of buildings when taken with a wide-angle lens often present a strained or distorted appearance, which is due to the fact that the distances between the various points in the photograph do not subtend at the eye the same angles as the distances between the corresponding points in the object photographed, or, in other words, the perspective of the photograph is not correct. This defect is particularly noticeable when the photograph is taken with a lens of very short focal length; if, however, an enlargement of such a picture is made its appearance is usually very much better than that of the original picture.

To explain this, let A B, C D (Fig. 42) be distant objects, and let $a\,b$, $c\,d$ be the corresponding images; if N_1 N_2 be the nodal points of incidence and emergence respectively, the lines joining these points to the corresponding points of object and image are parallel—for stance, A N_1, a N_2 are parallel. Thus the angles N_1 B, $a\,N_2\,b$ are equal, and so also are the angles N_1 D, $c\,N_2\,d$, or the angles subtended by the images at the nodal point of emergence are equal to those subtended by the corresponding points of the object at the nodal points of incidence.

If then the eye be placed opposite the middle of the

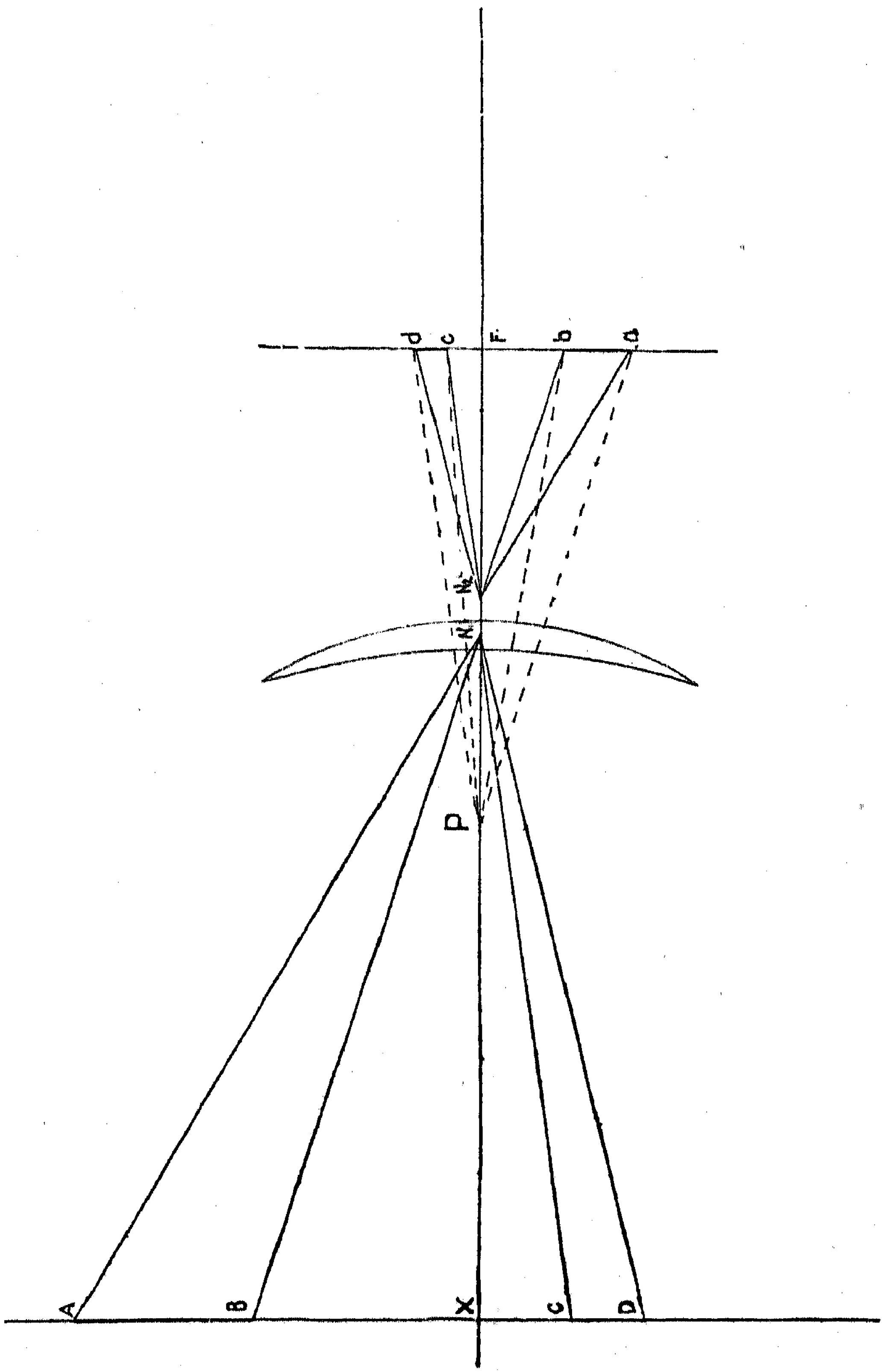


FIG. 42.

picture and at a distance from it equal to N_2 F, the various parts will subtend at the eye angles equal to those subtended by the corresponding parts of the

object, and the perspective of the picture will be correct. But in many cases the distance N_2 F is less than the distance of distinct vision for normal eyes, and to see the picture at all it must be held at a distance greater than N_2 F from the eye, such, for instance, as P F.

The angles which a P b, c P d now subtend at the eye are less than the angles a N_2 b, c N_2 d (which we have seen are the correct angles), and distortion will therefore result; the picture will appear crowded into a smaller space than it ought to occupy. And it can be shown that the angles between lines at the edges of the picture will appear diminished if they are acute and increased if they are obtuse; on both these accounts the picture will not appear to have the proper perspective.

Now, suppose that an enlargement is made—the effect of this is to increase all the lengths in the picture proportionally—thus if the picture be enlarged three times, it will be exactly similar to a picture of the same object taken with a lens of focal length three times that of the lens originally used. The distance N_2 F from the picture of the point where the eye should be placed to get the proper perspective is three times as great for the enlargement as for the original picture. If it is greater than the least distance of distinct vision the picture can be seen by an eye placed at that distance. This explains why an enlargement is often more pleasing than the original picture.

The least distance of distinct vision for a normal eye is somewhere about ten inches; it follows therefore that a picture to have the proper perspective must be taken with a lens whose focal length is greater than ten inches. A telephotographic lens, the focal length of which can be as much as five feet, will obviously produce pictures with much better perspective than an ordinary lens.

It follows, therefore, that for each picture there is a definite position from which it should be viewed if the

proper effect is to be obtained, *i.e.* a point on a line at right angles to the picture opposite to its centre and at a distance from it equal to the focal length of the lens with which it was (or in the case of an enlargement with which it could have been) taken.

64. The Use of the Swing Back.—A defect in the picture which must not be confounded with that of the last article is caused by the plate being placed in a wrong position. When arranging the camera it is often necessary to give it a tilt (to get on the plate all that is required), which tilts the plate also; if the photograph is taken with the plate in that position a certain kind of distortion is produced. If the picture is a landscape with no near or large buildings in it the distortion is not noticeable, but if it contains parallel straight lines such as occur in buildings or diagrams, these straight lines will in the picture run together at one end or another according to the way in which the camera is tilted, giving to buildings the appearance of tumbling down.

To understand the cause of this running together let the object be a rectangle A B C D (Fig. 43), and let N_1 N_2 be the nodal points of incidence and emergence respectively; join N_1 to A B C D, and let $N_2\,a$, $N_2\,b$, $N_2\,c$, $N_2\,d$ be lines through the other nodal point parallel to the lines through the first. Suppose the plate to be parallel to A B C D and to cut the lines through N_2 in the points $a\ b\ c\ d$ so that $a\ b\ c\ d$ is the image of A B C D.

Then, by similar figures, since A B and C D are equal and parallel, $a\,b$ and $c\,d$ are also equal and parallel; thus the lines $a\,d$, $c\,b$ appear parallel in the picture as they should do.

If, however, the plate be tilted into the position $a'\,b'\,c'\,d'$, where $a'\,b'$ and $c'\,d'$ are parallel to $a\,b$, $c\,d$ respectively, the picture may still be fairly in focus, but $c'\,d'$ is now longer than $c\,d$ and $a'\,b'$ is shorter than $a\,b$, which shows that $c'\,d'$ is now greater than $a'\,b'$.

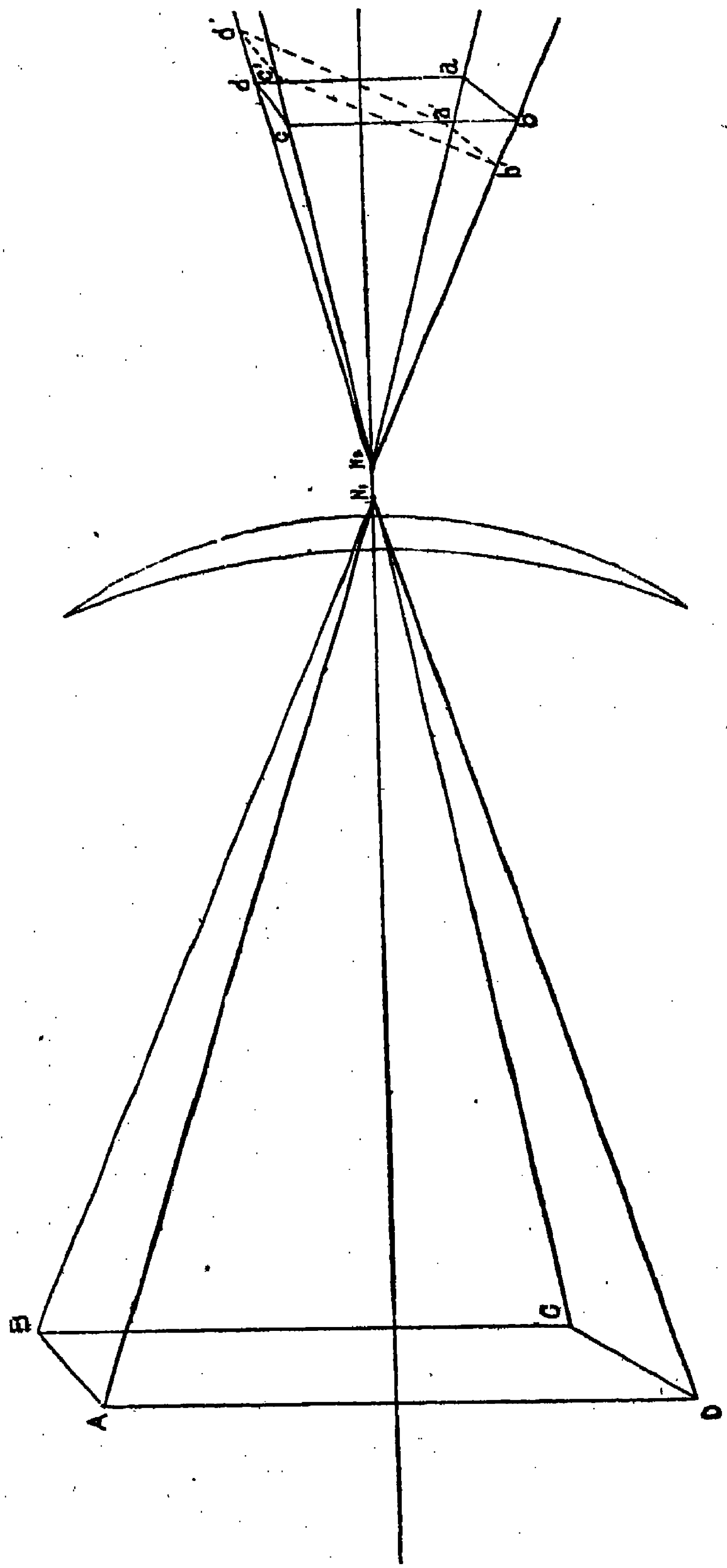


FIG. 43.

The effect of this is that the lines $a' d'$, $c' b'$ in the picture are no longer parallel, and a certain kind of distortion results.

We have here supposed the plate to be tilted about a horizontal axis, but if it be tilted about a vertical axis it is evident that the images of the lines A B, C D will no longer be parallel.

We see thus that if the photograph is to reproduce parallel straight lines correctly, the plate must be placed with its plane parallel to that of the object; for this purpose cameras are now supplied with an arrangement called a swing back, which makes it possible to keep the plane vertical and parallel to the object, however (within certain definite limits) the camera may be placed. The swing about a horizontal axis is that most frequently wanted, but the swing about a vertical axis cannot be dispensed with when buildings are photographed from awkward positions.

In landscape work a little distortion of the kind mentioned is not usually noticeable, and the swing back may in consequence be put to quite a different use from that for

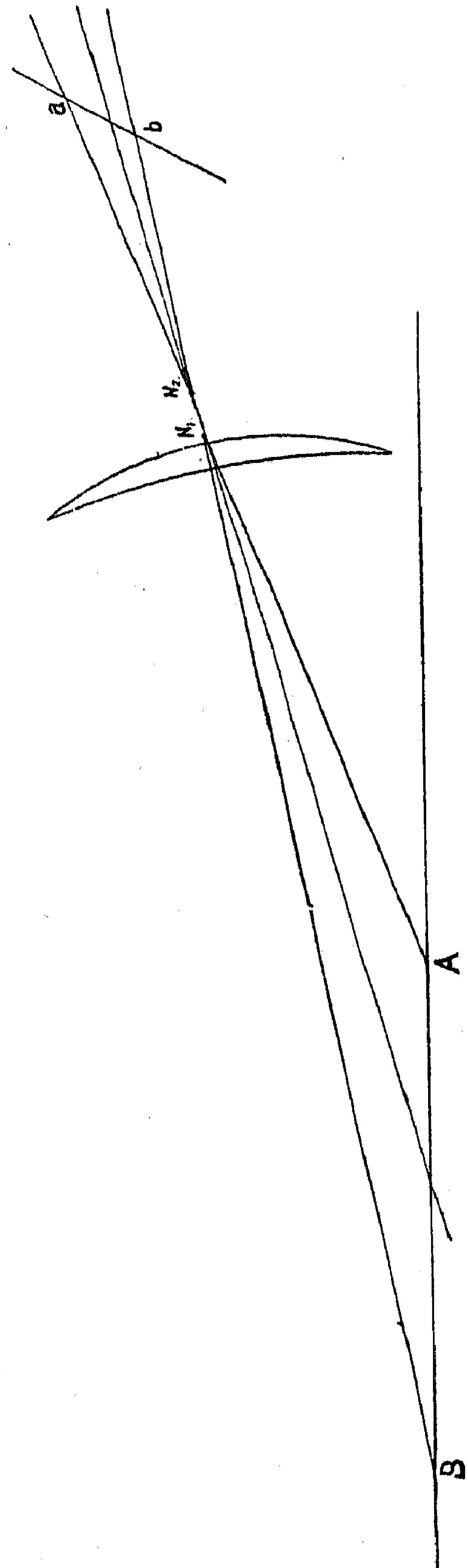


FIG. 44.

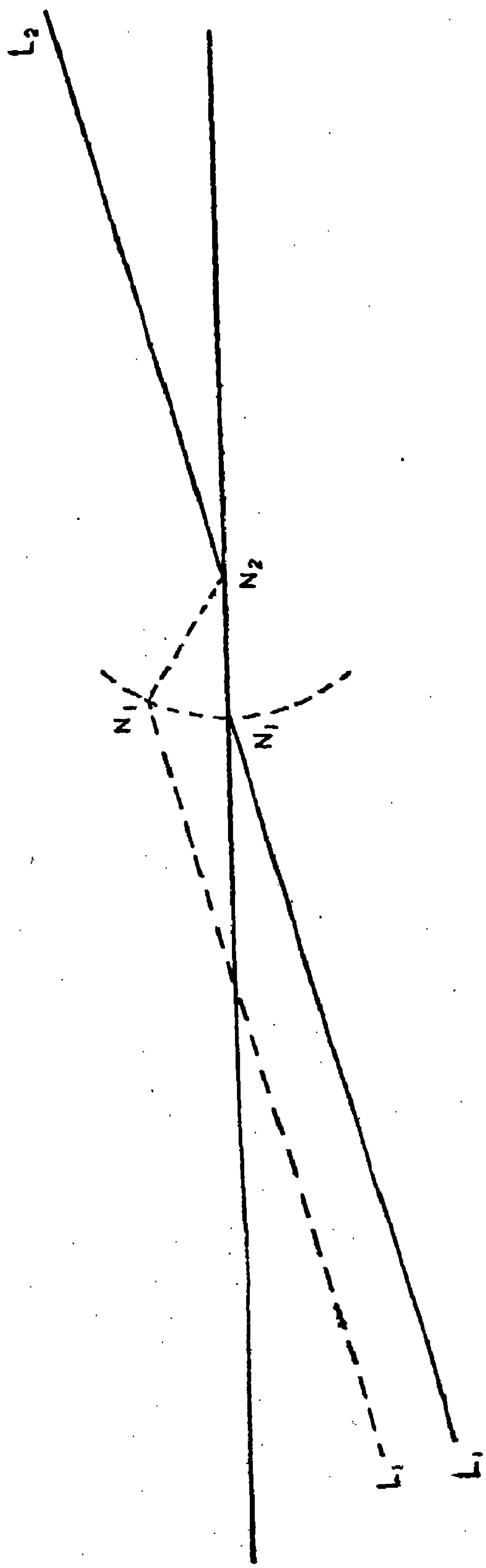


FIG. 45.

which it was designed. If a picture includes objects at very different distances it is impossible to get both near and far objects all into focus at once. Take, for example, a photographer on the top of a sea cliff who wants to photograph a number of ships stretching away from close beneath him to a considerable distance.

Let A and B (Fig. 44) be two of the ships, and let *a* and *b* be the images of these; then if A be nearer than B, *a* will be further from the lens than *b*, or the line *a b* will be inclined to the axis of the lens. The best focussing over the whole picture can therefore be obtained by placing the plate so that both *a* and *b* are on it, which can be done by the aid of the swing back. A similar adjustment can sometimes be made by the swing about a vertical axis.

65. A Property of the Nodal Point of Emergence and Panoramic Photography. — The nodal point of emergence possesses a property which is very useful both in lens testing and panoramic photography. If a lens be pivoted to turn about an axis at right angles to the axis of the lens, passing through the nodal point

of emergence, and the picture of a *distant* object formed by it on a ground glass screen be observed, the position of the picture on the screen will remain stationary while the lens is revolved. To prove this let N_1 and N_2 (Fig. 45 *a*) be the nodal points of incidence and emergence respectively, and let the lens be pivoted about an axis through N_2 perpendicular to the axis of the lens; let $L_1 N_1$ be the line joining a point of the distant object to N_1, and $L_2 N_2$ the line joining the corresponding point of the image to N_2, then we know (§ 44) that $L_1 N_1$ and $L_2 N_2$ are parallel.

Now let the lens be rotated through any angle about N_2 till N_1 comes to N_1', the line $N_1' L$ joining N_1' to the same point of the object will, since the object is distant, be parallel to $N_1 L_1$. The line joining N_2 to the corresponding points of the image will be parallel to $N_1' L$ and hence to $N_2 L_2$, and it has thus not altered its position.

That this is not true when the lens is rotated about any other axis than that stated will be evident from an inspection of Fig. 45*a*.

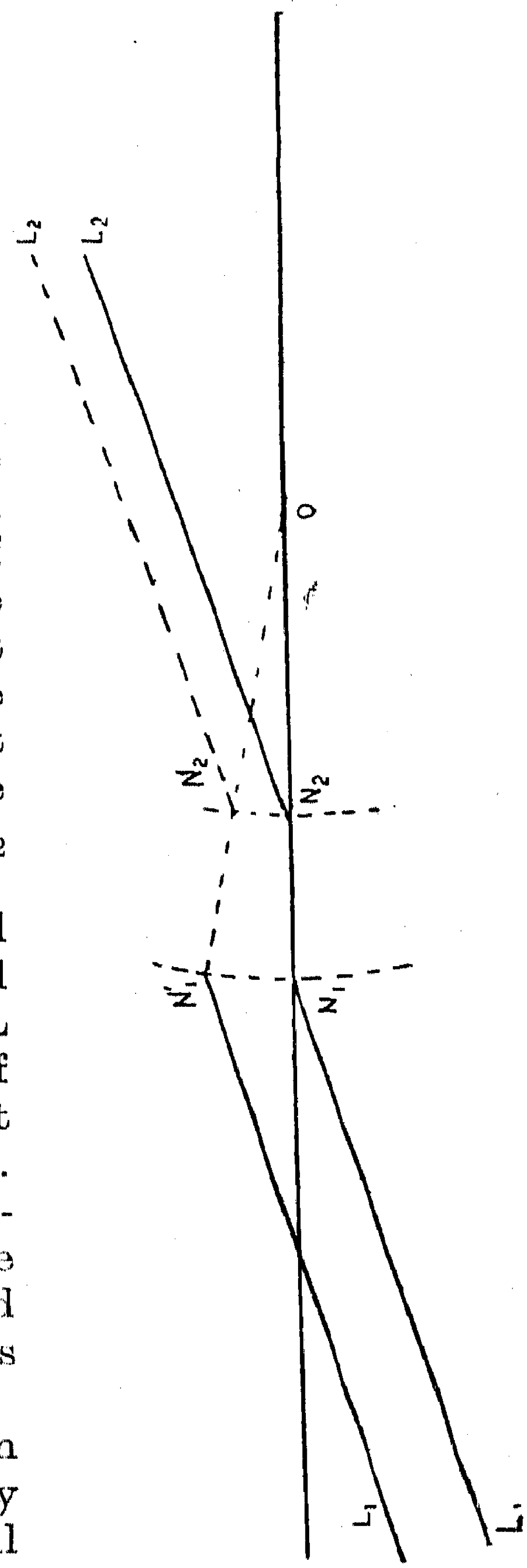


FIG. 45*a*.

Hence the image of the point in question will be in the same position whatever the position of the lens, and this is true of all points of the image. Thus as the

lens is rotated the picture remains stationary, but is extinguished at one side and extended at the other, very much as if a long map on rollers is laid on a table, and one side is rolled up while the other is unrolled without sliding the map along the table.

This principle has been applied both in England and France to the construction of a panoramic camera: the essential parts of such an apparatus are, firstly, a lens pivoted as described, and secondly, a sensitive film

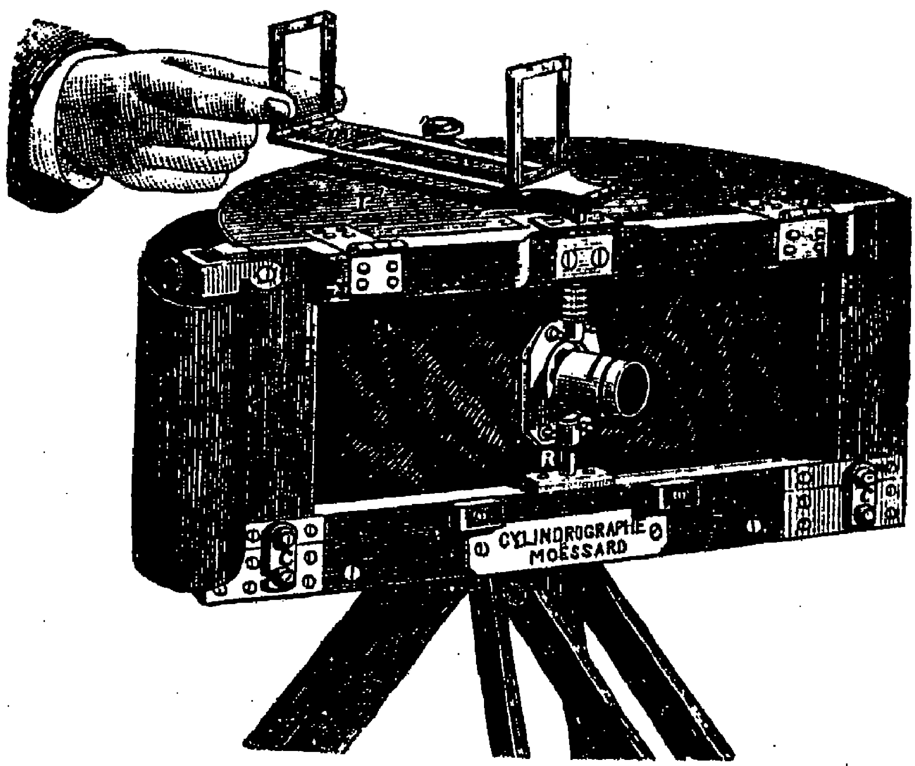


FIG. 46.

arranged in the form of a semi-cylinder with the axis of rotation for axis. To obtain a uniform exposure all along the roll of film, clockwork has been used to rotate the lens uniformly.

A rectilinear lens must be employed, as other lenses have a distortion at the edges (to be described later) which would cause a slight shifting at the edges of the pictures as the lens revolves.

Fig. 46 is a camera, designed by M. A. Möessard, for

panoramic work. A camera of a similar nature has been designed by Col. Stewart, R.E.; the main difference being that the film instead of being in the form of a cylinder is wound on rollers and suitably unrolled as the camera revolves.

CHAPTER III

ABERRATION

65*a*. Introductory.—We have now to see how the theory given in the last chapter must be modified to express the real state of affairs when we come to actual practice. To simplify the work we made several assumptions which are not altogether true; they were as follows:

(*a*) All the pencils of light dealt with were slender, no ray of light being far from any other ray.

(*b*) All pencils of light were incident near the centre of the lens, and thus only a small portion of each surface was used.

(*c*) The axes of the incident pencils were inclined at only a small angle to the axis of the lens.

(*d*) The light was supposed to be monochromatic, so that a single ray gave rise to only one refracted ray.

That these assumptions are not strictly true is a matter of common experience. The aperture of the lens is often by no means small, and the incident pencil of light is in consequence not slender; besides this, even in landscape work, where lenses of a moderate angle of view are mostly used, the axes of the extreme pencils are often inclined at an angle of 30° to the axis of the lens, an angle which cannot be called small; lastly, photographic work is done by the aid of daylight or lamplight, neither of which is even approximately

monochromatic. Why then have we taken so much trouble to investigate an imaginary state of affairs?

It is because this ideal state leads us to a very fair approximation, from which, by the addition of small corrections, we can deduce a more nearly accurate statement of the case; and also because we can, by using two or more lenses, approximate very closely to the ideal state.

The subject is usually divided into two parts: firstly, that of Spherical Aberration, due to the use of large pencils or of pencils oblique to the axis of the lens; secondly, Chromatic Aberration, due to the various coloured rays in the incident light. These two kinds of aberration will be treated in turn, and we shall see how they modify the relations found in the last chapter *without altering their general character.*

The subject of aberration is undoubtedly more difficult than the elementary theory of a lens, not because the fundamental ideas are very difficult, but because the calculations are unavoidably complicated. The practical photographer is not much concerned with aberration, for generally he is content to use the lens which the study and skill of the maker have produced, and to ask no questions provided the results are satisfactory; besides this, the production of a good lens requires so much skill and accuracy of work, that very few amateurs could hope to produce one worth using. The calculations necessary for designing lenses are long and arduous, not so much on account of the intricacy of the principles involved, but on account of the complicated nature of the algebra required to find the magnitudes of the various quantities.

It is doubtful whether these calculations are performed except by lens designers accustomed by practice to such work; or whether it is profitable or even desirable for the general reader to attempt them.

But though the numerical calculations are difficult, a knowledge of general principles is of great use for the

proper understanding of the general character of a lens and in estimating and testing its capabilities.

We shall therefore give in the first place a minute description of the action of a lens, and after this the formulæ usually quoted in optical treatises, but, except in a few cases, we shall not enter on the demonstration of them.

Those readers who wish for further information should consult Coddington's *Optics*, or Wallon's *L'Objectif Photographique*. A great deal of interesting information and specimens of calculations for lens designing are given in a paper by M. Martin [1] on the 'Détermination des courbures des objectifs.'

I.—Spherical Aberration.

65*b*. Let us take first the case of a pencil of light emanating from a point on the axis of a lens and striking the lens symmetrically. Such a pencil can be produced by placing a screen pierced with a small hole in front of a gas flame, the hole being on the axis of the lens. The pencil is here *not* supposed small as in the previous chapter.

If a white screen be placed on the side of the lens away from the light, and be moved backwards and forwards, and kept paralled to the lens, the phenomena presented will be as follows.

When the screen is very close to the lens the appearance on it is a circle of light uniformly illuminated; when the screen moves away from the lens the circle of light contracts, and becomes brighter at the edge than at the centre. As the movement of the screen is continued the circle contracts further and the edge becomes still brighter, and, if looked for carefully, a

[1] *Annales de l'Ecole normale supérieure*, 1877. Gauthier-Villars, Paris.

brighter spot at the centre can also be seen. After this the bright ring shrinks till it becomes a patch.

Up to this the space outside the illuminated circle has been dark; but just about when the bright ring becomes a patch, the space outside it becomes diffusely illuminated.

When the screen is far enough back the bright patch constitutes the image of the luminous point, an image of the hole through which the light is admitted being formed; and further back still the patch becomes indefinite and grows fainter, while the circle of diffuse light round it increases rapidly in size.

It has proved impossible to get really successful photographs of these phenomena, owing mainly to diffuse light which cannot be got rid of, but the phenomena themselves can easily be reproduced by any one possessing a lens of fair size, such as a magnifying glass or one of the condensing lenses of an enlarging lantern; since the object is to get as much aberration as possible an uncorrected lens should be used.

The incident light should be rendered monochromatic by interposing a coloured glass between the luminous point and the lens.

The various appearances described are all of them sections, by planes perpendicular to the axis of the lens, of the assemblage of the rays of light after refraction by the lens.

Let us proceed to form some idea of the nature of the assemblage which gives rise to these sections. The arrangement of rays which will be found to answer all requirements is given in Fig. 47, which represents a section by a plane passing through the axis of the lens.

[In the figure the incident rays are, for a subsequent purpose, taken to be parallel to the axis of the lens, but the general character of the phenomenon is the same as when the rays proceed from a point on the axis.]

It should be noticed that the rays from the centre of the lens are shown as converging to F, which is therefore the focus conjugate to the luminous point; while

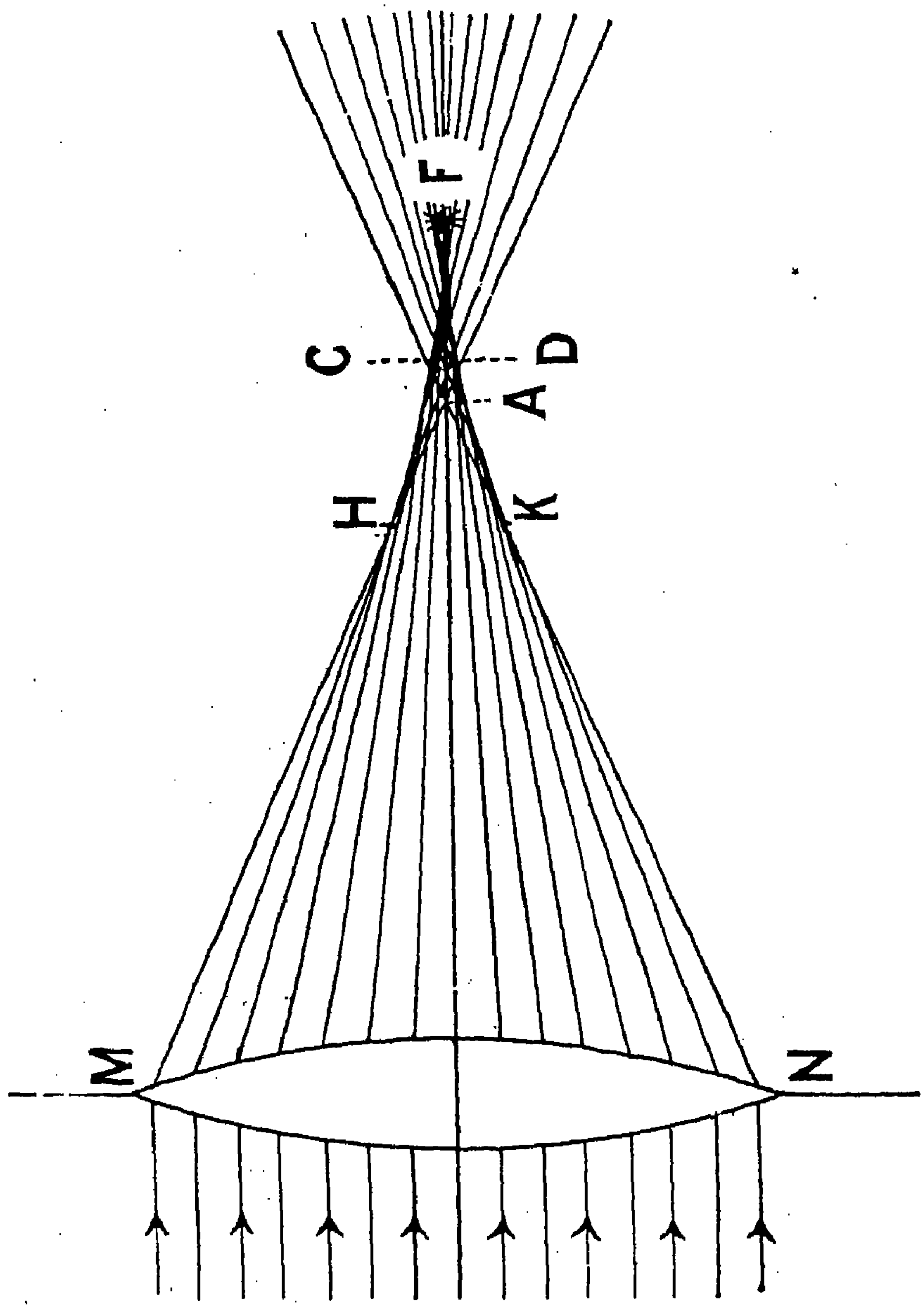


Fig. 47.

those refracted through the edges converge to A, a point nearer to the lens than F. Rays through other portions of the lens cut the axis in points lying between A and F.

At a certain distance from the lens (beyond H and K in the figure) the rays intersect, and there is between H, K and F a curve formed by the intersections of consecutive rays. Since through each point of this curve more than one ray passes, it follows that the illumination there is brighter than at points inside it; or in other words, the curve is one of maximum illumination. This curve is called a *caustic* curve, and the surface of which it is the section a caustic surface. The points on the axis lying between A and F have also several rays passing through each of them, and the line A F is therefore one of maximum illumination and may be reckoned as part of the caustic.

That this agrees with observation will be evident if the section by a screen, parallel to the lens at different distances from it, be considered. At C D the section by the screen is evidently of a minimum size; beyond C D the rays which converged to A spread out beyond the caustic surface, producing a bright circle of light surrounded by diffused light.

Careful examination shows that rays from a considerable portion of the lens converge to F, making it the most brightly illuminated point, so that the image formed at F is much brighter than that formed at any other point between A and F, and will stand out clearly in spite of the diffuse light surrounding it.

Beyond F there is no point where the rays intersect, and consequently no point of maximum illumination, though for some distance the patch of light on the screen will be brightest at the centre fading away *gradually* to the edge.

This description may be still further verified by first placing in front of the lens and close to it a screen with a small hole cut in it which allows rays to strike only the central portions of the lens, when the focus will be found to be at F; and afterwards removing the screen and placing a circular disc in front of the lens which cuts off all the rays except those near the edges, when

the focus will be found at a point nearer to the lens than F.

Definition.—If the incident rays be parallel to the axis as in the figure, then the length A F between the points in which the central and marginal rays come to a focus is called the *Longitudinal Aberration* of the lens. Also if F B be drawn, so that it is the radius of the section of the cone of rays by a plane at F perpendicular to the axis, then F B is called the *Lateral Aberration* of the lens. (See note on p. 130.)

65c. Calculation of Aberration.—We have in the last chapter considered the case of rays passing symmetrically through the centre of the lens; we have now to take the case of rays passing through or near the edge.

We shall consider the lens to be thin, as the result will be quite close enough for most purposes. Let u be the distance of the object on the axis from the lens, and v the distance from the lens at which the marginal rays come to a focus, the positive and negative directions being taken as before. Let r and s be the radii of the front and back surfaces respectively, f the focal length as already found, μ the refractive index, and z the radius of the aperture of the lens.

Then if z is so small compared with f, r and s that we may neglect powers of z/f, z/r, z/s beyond the second; in place of the old value for v

$$\text{i. e.} \quad v = \frac{uf}{u+f}$$

we now get [1]

$$v = \frac{uf}{u+f} - \frac{z^2}{2}\left(\frac{uf}{v+f}\right)^2 \frac{1}{f}\left\{A - \frac{B}{u} + \frac{C}{u^2}\right\}$$

where $\mu\,A = (2 - 2\,\mu^2 + \mu^3)\,\dfrac{1}{r^2} +$

$$(\mu + 2\,\mu^2 - 2\,\mu^3)\,\frac{1}{rs} + \mu^3\,\frac{1}{s^2}$$

[1] See M. Martin's paper quoted above, or Coddington's *Optics*.

$$\mu B = (4 + 3\mu - 3\mu^2)\frac{1}{r} + (\mu + 3\mu^2)\frac{1}{s}$$

$$\mu C = 2 + 3\mu.$$

It should be noticed in this relation that the first power of z does not occur, hence if the aperture is such that we can neglect squares of z/f, z/r, z/s, the elementary relation will be near enough to accuracy for practical purposes even though we cannot neglect first powers of these quantities.

To facilitate calculation a table is given below, showing the values of the coefficients in A, B, and C for different values of μ; the range of μ is taken from 1·50 to 1·70, which includes all that is usually needed.

μ	$\frac{2}{\mu} - 2\mu + \mu^2$	$1 + 2\mu - 2\mu^2$	$\frac{4}{\mu} + 3 - 3\mu$	$1 + 3\mu$	$\frac{2}{\mu} + 3$
1·50	·5834	— ·5000	1·167	5·500	4·333
1·51	·5847	— ·5400	1·119	5·530	4·325
1·52	·5862	— ·5810	1·072	5·560	4·316
1 53	·5881	— ·6220	1·024	5·590	4·307
1·54	·5904	— ·6630	·9776	5·620	4·299
1 55	·5924	— ·7050	·9308	5·650	4·290
1·56	·5956	— ·7470	·8840	5·680	4·282
1·57	·5987	— ·7900	·8376	5·710	4·274
1·58	·6022	— ·8330	·7916	5·740	4·266
1·59	·6059	— ·8760	·7456	5·770	4·258
1·60	·6100	— ·9200	·7000	5·800	4·250
1·61	·6143	— ·9640	·6544	5·830	4·242
1·62	·6190	— 1·009	·6092	5·860	4·235
1·63	·6239	— 1·054	·5640	5·890	4·227
1·64	·6292	— 1·099	·5192	5·920	4·219
1·65	·6347	— 1·145	·4744	5·950	4·212
1·66	·6404	— 1·191	·4296	5·980	4·205
1·67	·6465	— 1·238	·3852	6·010	4·198
1·68	·6528	— 1·285	·3408	6·040	4·190
1·69	·6595	— 1·332	·2908	6·070	4·183
1·70	·6664	— 1·380	·2528	6·100	4·176

If the values of the coefficients corresponding to values of μ lying between those given are required, they can be found by interpolation in the ordinary way.

In photography the object is often at a considerable distance, in which case we can get a close approximation if we make u infinite or $1/u$ zero; the expression $\frac{u f}{u+f}$, which may be written $\frac{f}{1+f/u}$, reduces to f, and we get

$$v = f - \frac{z^2 f A}{2}$$

or the longitudinal aberration A F (Fig. 47), usually called a, is given by

$$a = \frac{z^2 f A}{2}$$

This is the formula usually quoted.

To find the lateral aberration denoted by b, we have (Fig. 43) by similar triangles M L A, A F B,[1]

$$\frac{FB}{FA} = \frac{ML}{MA} \quad \text{or } b = \frac{ML}{LA} a$$

Now M L $= z$, and we may take L A as approximately equal to L F or f.

$$\therefore b = \frac{z}{f} a = \frac{z}{f} \cdot \frac{z^2 f A}{2} = \frac{z^3 A}{2}$$

Note.—The complete expression connecting the distances of object and image for a thick lens may be interesting to some.

If e be the thickness, u and v the distances of the object and image from the front and back surfaces respectively, the relation required is

$$\frac{1}{v} = \frac{1}{u} + (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) - \frac{e}{\mu}\left(\frac{\mu - 1}{r} + \frac{1}{u}\right)^2$$
$$+ \frac{\mu - 1}{2}\left(\frac{1}{r} - \frac{1}{s}\right)\left(A - \frac{B}{u} + \frac{C}{u^2}\right) z^2$$

where A, B, C have the meanings given above.

[1] B is not shown in the figure; to get it draw from F a perpendicular to the axis of the lens to cut the extreme ray on either side in B.

The only extra term is that depending on e, terms in $e\,z^2$ are omitted, as e is usually a small quantity.

The other quantity required to fix completely the position of the emergent ray is the angle which it makes with the axis of the lens.

If ϵ be the angle made by the incident ray, and η that made by the emergent ray with the axis of the lens, these angles are connected by the relation

$$\frac{\tan\,\eta}{\tan\,\varepsilon} = \frac{u}{v}\left[1 + \frac{e}{\mu}\left(\frac{\mu-1}{r} + \frac{1}{u}\right) - \frac{\mu-1}{2\,\mu}\left\{\frac{1}{r}\left(\frac{1}{r} - \frac{1}{u}\right) - \frac{1}{s}\left(\frac{1}{s} - \frac{1}{v}\right)\right\} z^2\right]$$

If more than one lens is employed we can calculate successively the angles which the rays emergent from each lens make with the axis.

66. Numerical Examples of Aberration.—Consider the convergent lenses treated in § 33 :—

(a) Meniscus as in Fig. 19, E.

$r = 7$ inches, $s = 5$ inches, $\mu = 1{\cdot}5$ inch, and take $z = {\cdot}5$ inch.

$$\therefore\ \mathrm{A} = \left(\frac{2}{\mu} - 2\mu + \mu^2\right)\frac{1}{r^2} + (1 + 2\mu - 2\mu^2)\frac{1}{r\,s} + \mu^2\frac{1}{s^2}$$

$$= \frac{{\cdot}5834}{49} - \frac{{\cdot}5000}{35} + \frac{2{\cdot}25}{25} = {\cdot}0119 - {\cdot}0143 + {\cdot}0900$$

$$= {\cdot}0876.$$

From § 33 we get f the focal length $= 35$ inches.

$$\therefore\ a = \frac{z^2 f\,\mathrm{A}}{2} = \frac{{\cdot}25 \times 35 \times {\cdot}0876}{2} = {\cdot}383 \text{ inch.}$$

If the lens be reversed we get

$r = -\,5$ inches, $s = -\,7$ inches.

$$\therefore\ \mathrm{A} = \frac{{\cdot}5834}{25} - \frac{{\cdot}5000}{35} + \frac{2{\cdot}25}{49} = {\cdot}0232 - {\cdot}0143 + {\cdot}0459 = {\cdot}0458$$

$$\therefore\ a = \frac{z^2 f\,\mathrm{A}}{2} = \frac{{\cdot}25 \times 35 \times {\cdot}0548}{2} = {\cdot}201 \text{ inch.}$$

(*b*) Double convex lens as in Fig. 19, A.

$r = -7$ inches, $s = 5$ inches, $\mu = 1{\cdot}5$, $f = -5{\cdot}83$ inches.

$$\text{Hence } A = \frac{{\cdot}5834}{49} + \frac{{\cdot}5000}{35} + \frac{2{\cdot}25}{25} = {\cdot}0119 + {\cdot}0143 + {\cdot}0900 = {\cdot}1162$$

$$\therefore\ a = \frac{z^2 f A}{2} = \frac{{\cdot}25 \times 5{\cdot}83 \times {\cdot}1162}{2} = {\cdot}085 \text{ inch.}$$

If the lens be reversed we get

$r = -5$ inches, $s = 7$ inches.

$$\therefore\ A = \frac{{\cdot}5834}{25} + \frac{{\cdot}5000}{35} + \frac{2{\cdot}25}{49} = {\cdot}0232 + {\cdot}0143 + {\cdot}0459 = {\cdot}0834$$

$$\therefore\ a = \frac{z^2 f A}{2} = \frac{{\cdot}25 \times 5{\cdot}83 \times {\cdot}0834}{2} = {\cdot}061 \text{ inch.}$$

In these cases, the lenses being totally uncorrected, the aberration is larger than could be tolerated in practice, but the results serve to illustrate one or two important points.

In the first part of example (*a*) when the meniscus lens is placed with its concave face towards the incident pencil of parallel rays, the aberration is nearly double what it is when the lens is reversed. Examination of diagrams will show that the angles of incidence and refraction at both surfaces are less in the latter than in the former case.

This is an example of the general principle that the aberration of a lens depends on the nature of the face which receives the incident light, but that the smaller the angle of incidence and the smaller the angle of refraction the less is the aberration. To secure this condition with a single lens or cemented combination, for incident rays parallel to the axis, the face whose radius of curvature is greatest, whether convex or concave, must receive the incident light, and this is usually the case with simple objectives.

For compound objectives the problem is not quite so simple, as the rays, after passing through the first combination, are not parallel for incidence as the second combination.

It will be found in the majority of cases, that in each combination the radii of curvature of the outside surfaces are greater than those of the faces cemented together, and as a rule the flattest faces of the combinations face each other.

67. Aberration for two Lenses.—When there are two lenses in contact, let F be the focal length of the combination, f and f' those of the component lenses, and let the quantities for the second lens corresponding to those for the first lens be denoted by dashed letters; then the value of v for the marginal rays is given by

$$v = \frac{u\,\mathrm{F}}{u+\mathrm{F}} - \frac{z^2}{2}\left(\frac{u\,\mathrm{F}}{v+\mathrm{F}}\right)^2\left[\frac{1}{f}\left(\mathrm{A} - \frac{\mathrm{B}}{u} + \frac{\mathrm{C}}{u^2}\right) + \frac{1}{f'}\left\{\mathrm{A}' - \mathrm{B}'\left(\frac{1}{f} + \frac{1}{u}\right) + \mathrm{C}'\left(\frac{1}{f} + \frac{1}{u}\right)^2\right\}\right]$$

where A, B, C, etc. have the values assigned to them in § 65. If the object is distant, and hence very large, we get for the longitudinal aberration—

$$a = \frac{z^2\,\mathrm{F}^2}{2}\left\{\frac{\mathrm{A}}{f} + \frac{\mathrm{A}'}{f'} - \frac{\mathrm{B}'}{ff'} + \frac{\mathrm{C}'}{f^2 f'}\right\}$$

For the treatment of three or more lenses in contact, reference should be made to M. Martin's paper.

68. Trigonometrical Method.—The method already given is useful for designing lenses, but when the aberration of a given lens is required a more direct method may be adopted.

The method is that of tracing the course of a ray, originally parallel to the axis of the lens at any required distance till it cuts the axis; this will give the position of the point A (Fig. 47), the position of F can be found in the usual way, and thus A F, the aberration, will be known.

Use the following notation in the calculations: μ_1,

μ_2, etc., are the refractive inches of the lenses; R_1, R_2, R_3, etc., are the radii of the successive surfaces; a is the angle of incidence, and α the corresponding angle of refraction at the first surface, b and β the corresponding angles for the second surface, and so on. The elongations of the successive portions of the ray, after the refractions which it undergoes, cut the axis in points called A, B, C, etc.; the distances of these points from the first, second, third, etc. surfaces respectively are called A, B, C, etc., and the angles which they make with the axis (A), (B), (C) etc., and e_1 e_2, etc. are the distances between the successive surfaces measured along the axis.

The different portions of the refracted ray make with the axis and the radii of the surfaces a series of rectilineal triangles which can be solved in succession.

The formulæ required should in each case be written down from a consideration of the particular figure.

In the case of a double convex front lens in contact with a double concave lens, the two being cemented, the following will be the formulæ required; they should be verified from a figure.

The incident ray is parallel to the axis and at a distance z from it:—

For the first refraction—

$$\text{I}\left\{\begin{array}{l} \sin a = \dfrac{z}{R_1}, \ \sin \alpha = \dfrac{\sin a}{\mu_1} \\ (A) = a - \alpha, \ A = \dfrac{R_1 \sin a}{\sin (A)} + R_2 \end{array}\right.$$

For the refraction from the first to the second lens—

$$\text{II}\left\{\begin{array}{l} \sin b = \dfrac{A + R_2 - e_1}{R_2} \sin (A), \sin \beta = \dfrac{\mu_1}{\mu_2} \sin b, (\S 11) \\ (B) = (A) + \beta - b, \ B = \dfrac{R_2 \sin \beta}{\sin (B)} - R_2 \end{array}\right.$$

For the third refraction, out at the last surface—

$$\text{III} \begin{cases} \sin c = \dfrac{R_3 - B + e_2}{R_3}, \sin \gamma = \mu_2 \sin c \\ (C) = (B) + c - \gamma, C = \dfrac{R_3 \sin \gamma}{\sin (C)} + R_3 \end{cases}$$

As a numerical example, consider the lens in § 33. $r = 7$ inches, $s = 5$ inches, $\mu = 1{\cdot}5$, and let $z = {\cdot}5$ inch.

For the first surface the formulæ (I) above are applicable, the results of the calculation are—

$$\left.\begin{aligned} a &= 4^\circ\ \ 5'\ 46'' \\ \alpha &= 2^\circ\ 43'\ 46'' \\ (A) &= 1^\circ\ 22'\ \ 0'' \end{aligned}\right\} \begin{aligned} A &= 13{\cdot}976 + 7 \\ &= 20{\cdot}976 \text{ inches} \end{aligned}$$

For the second surface the formulæ are—

$$\sin b = \frac{A - R_2}{R_2} \sin (A), \sin \beta = \mu \sin b,$$

$$(B) = \beta - A - b, B - \frac{R_2 \sin \beta}{\sin (B)} - R_2$$

The results are—

$$\left.\begin{aligned} b &= 4^\circ\ 22'\ 14'' \\ \beta &= 6^\circ\ 33'\ 50'' \\ (B) &= 0^\circ\ 49'\ 36'' \end{aligned}\right\} \begin{aligned} B &= 39{\cdot}615 - 5 \\ &= 34{\cdot}615 \end{aligned}$$

Hence a ray which before incidence is parallel to the axis and half an inch from it, after refraction cuts the axis at a distance of 34·615 inches from the lens; now the focal length of the lens (§ 33) is 35 inches, hence we have for the aberration

$$a = 35{\cdot}000 - 34{\cdot}615 = {\cdot}385 \text{ inch.}$$

69. Minimum Aberration.—We have seen (§ 65) that the lateral aberration of a lens is given by

$$a = \frac{z^2 f}{2} A$$

where

$$\mu A = \frac{2 - 2\mu^2 + \mu^3}{r^2} + \frac{\mu + 2\mu^2 - 2\mu^3}{r s} + \frac{\mu^3}{s^3}$$

If the aberration is to vanish (for terms as far as z^2)

we must have $A = 0$. This gives us a quadratic equation to determine the ratio of $r : s$ when the refractive index μ is given; it can be shown that if the roots of this equation are to be real, we must have μ less than one quarter; no substance is known which has such a refractive index, and hence a single lens free from aberration cannot be made.

We can, however, choose the ratio $r : s$ when the value of μ is given to make the aberration a minimum, and if we wish the lens to be of given focal length, we can completely determine r and s.

The two equations for finding r and s are (Wallon, p. 275)—

$$\frac{r}{s} = -\frac{4 + \mu - 2\mu^2}{\mu + 2\mu^2}, \quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

where f is the focal length required.

Such a lens is called a crossed lens.

Example.—The following is taken from Wallon. Let $\mu = 1{\cdot}5$; then

$$\frac{r}{s} = -\frac{4 + \mu - 2\mu^2}{\mu + 2\mu^2} = -\frac{1}{6}$$

The negative sign shows that the radii of curvature of the surface must be in opposite directions, hence the lens must either be double convex or double concave.

We have also $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = {\cdot}5\left(\frac{1}{r} - \frac{1}{s}\right)$

whence we get—

$$r = -\frac{7}{12}f, \qquad s = \frac{7}{2}f$$

If the lens is convergent f is negative, which makes r negative and s positive, or the lens is double convex, as we should expect.

If these values of r and s are used to calculate the aberration we get—

$$a = \frac{15}{14}\ \frac{y^2}{f}$$

If the lens be turned round so that

$$r = -\frac{7}{2}f,\ s = \frac{7}{12}f$$

we shall get for the aberration

$$a' = \frac{45}{14}\ \frac{y}{f},\ \text{which is } 3\ a.$$

or the aberration in the latter case is three times as large as in the former.

The form of the lens of least aberration changes with the refractive index of the substance employed; looking at the expression for r/s we see that the ratio will be negative only so long as

$$4 + \mu - 2\,\mu^2 > o$$

or

$$2\,\mu^2 - \mu - 4 < o$$

which is the case only so long as

$$\mu < 1{\cdot}686$$

If $\mu = 1{\cdot}686$, then $2\,\mu^2 - \mu - 4 = o$

or the ratio $r/s = 0$, and as r cannot be zero $\frac{1}{s}$ must be zero, or the back face of the lens is in this case plane.

70. Oblique Pencils.—Now take the case of a pencil of rays striking the lens obliquely. If the region behind the lens be explored by means of a screen (as in § 64) the appearances on it will be like those in Fig. 47*a*, which is reproduced from photographs of the actual phenomena. The arrangements in this case are similar to those in § 64, with the exception that the lens is turned, round a vertical axis through its centre through a suitable angle. A cursory examination of the figures shows that the rays, after refraction,

do not pass nearly through one point, but are in a seemingly inextricable jumble.

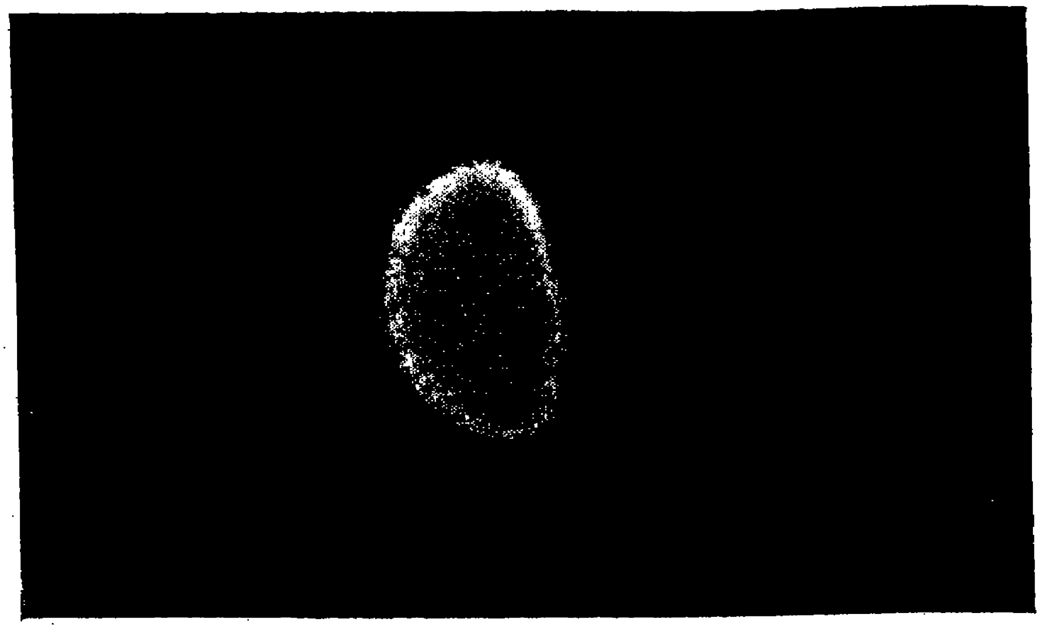


FIG. 47*a* (1).

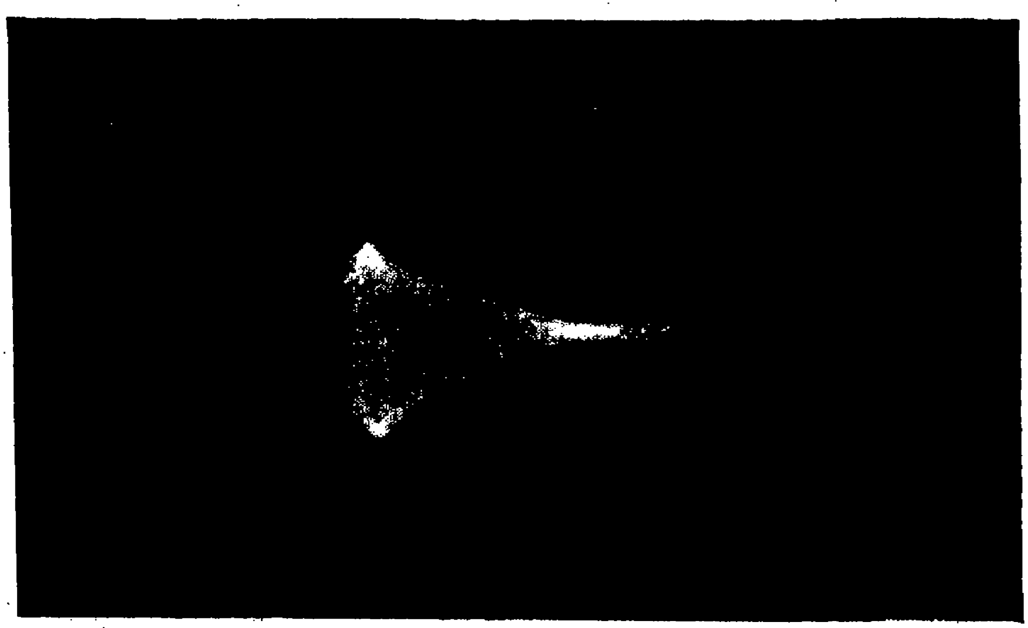


FIG. 47*a* (2).

If, however, a diaphragm is placed in front of the lens, so that the central portion only is used, the figures corresponding to the former become now those of Fig.

47*b*; here, if the screen be placed near the lens, the appearance is an elliptical-shaped figure; on moving the

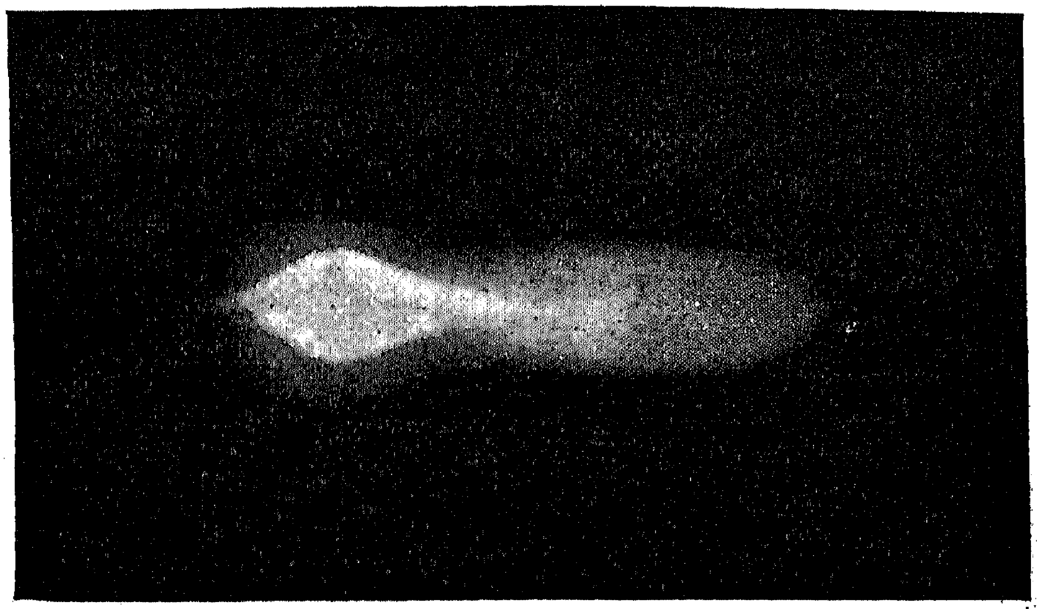


FIG. 47*a* (3).

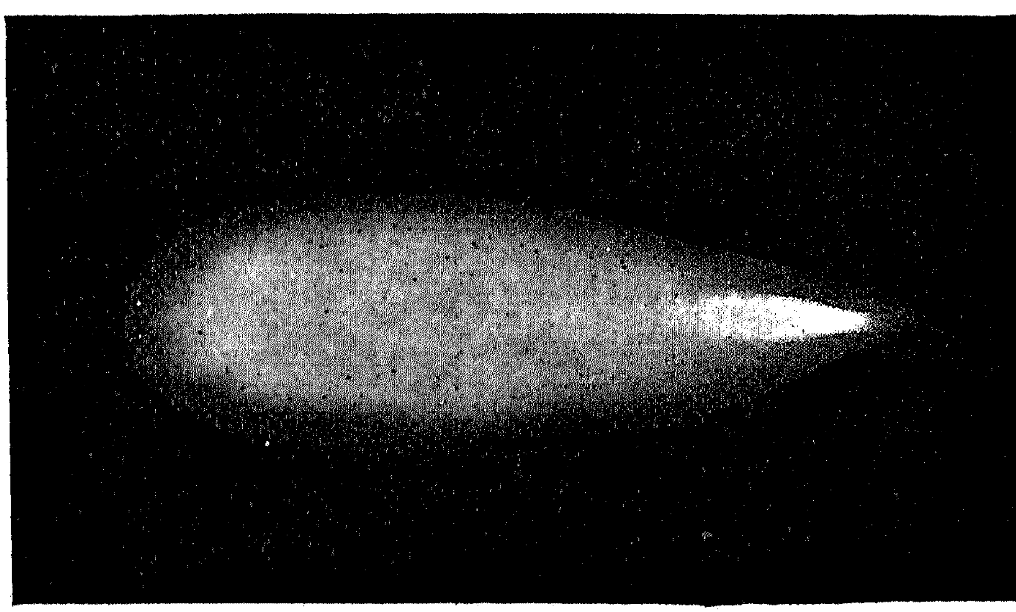


FIG. 47*a* (4).

screen away from the lens the ellipse shrinks very nearly into a straight line, then it broadens out and approximates to a circle. On further movement of the

screen this circle lengthens out into a straight line at right angles to the former, and this again broadens out into an oval.

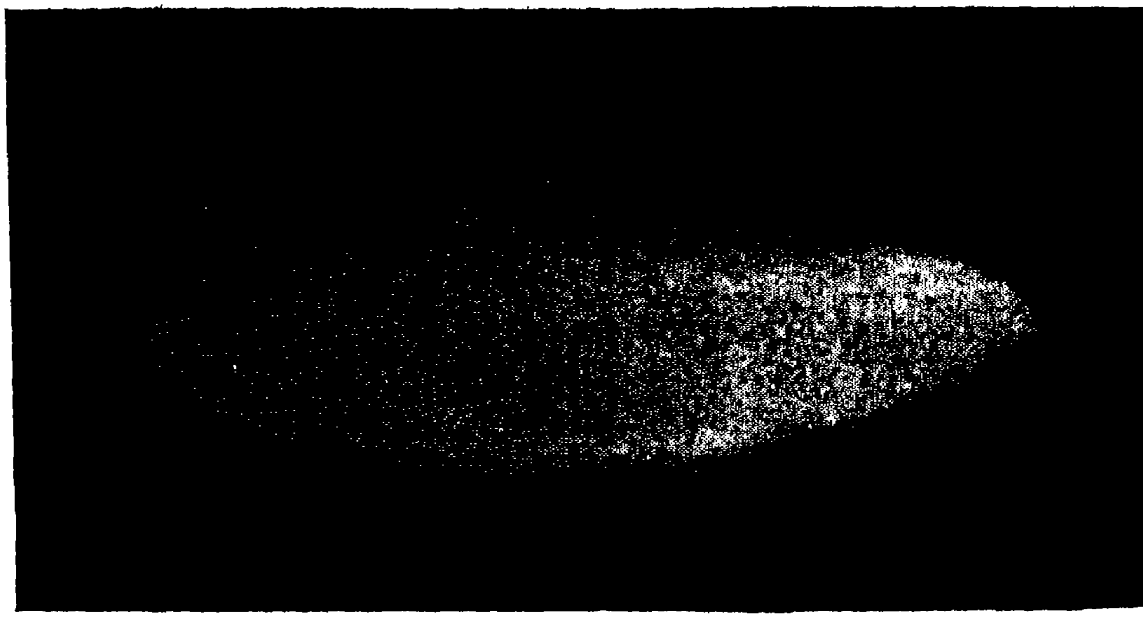


FIG. 47*a* (5).

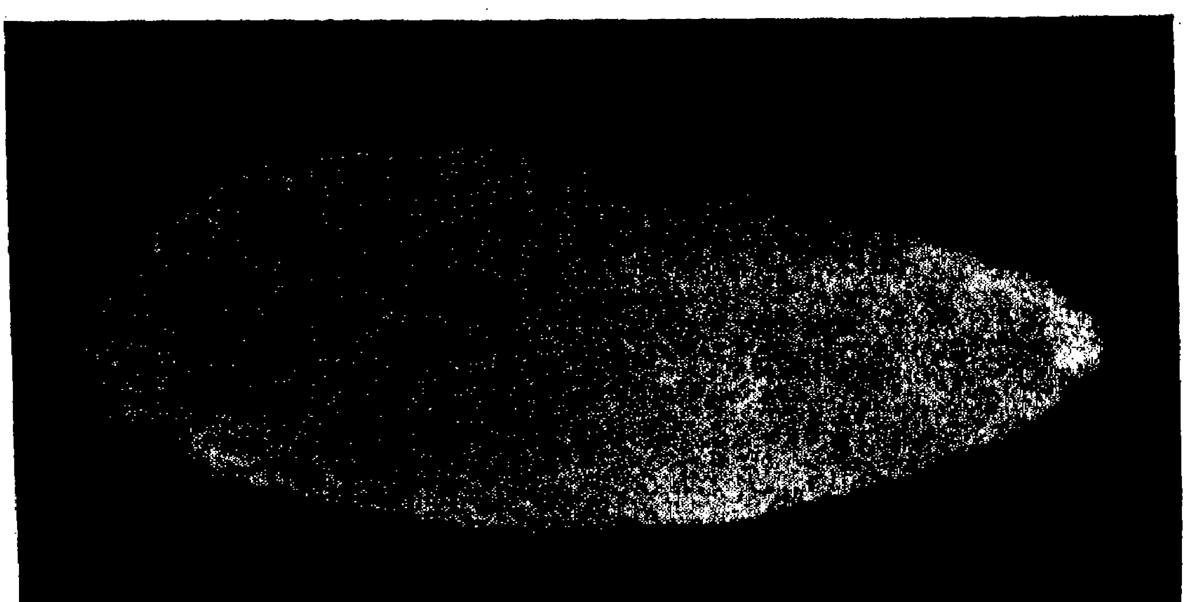


FIG. 47*a* (6).

The two lines thus found are called focal lines, and play an important part in the theory of oblique pencils.

We thus arrive at an important result, which can be shown to hold generally :—

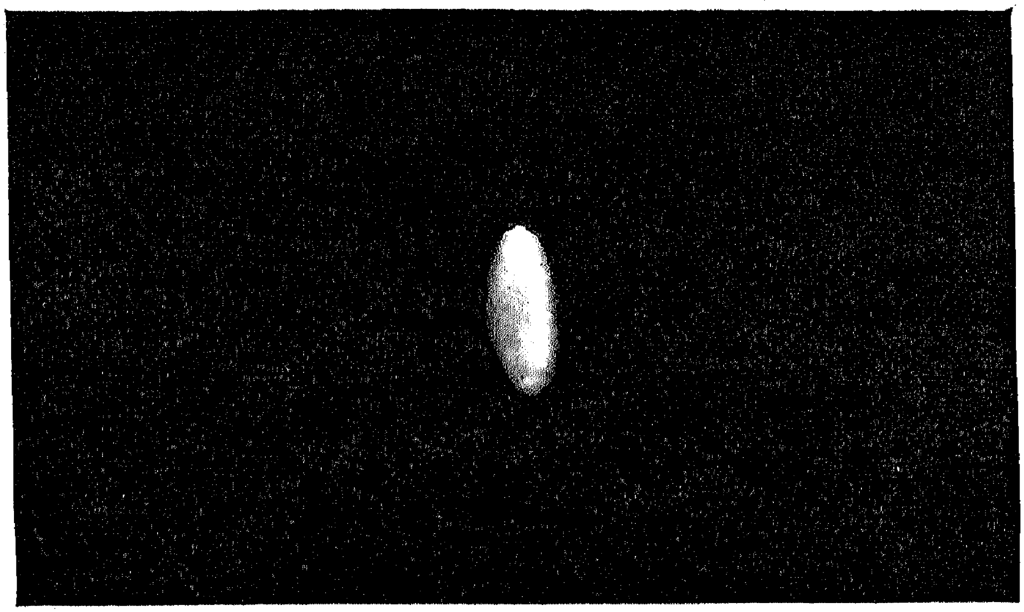


FIG. 47*b* (1).

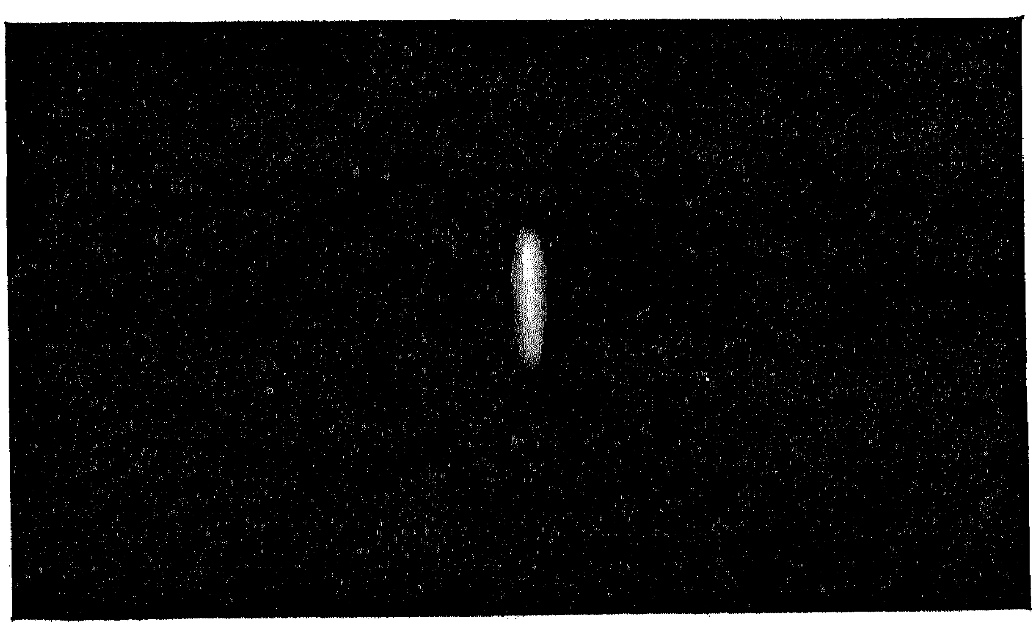


FIG. 47*b* (2).

If a small pencil proceeding from a luminous point pass obliquely through any refracting surfaces, the rays after any number of refractions pass

approximately through two straight lines at right angles.

FIG. 47*b* (3).

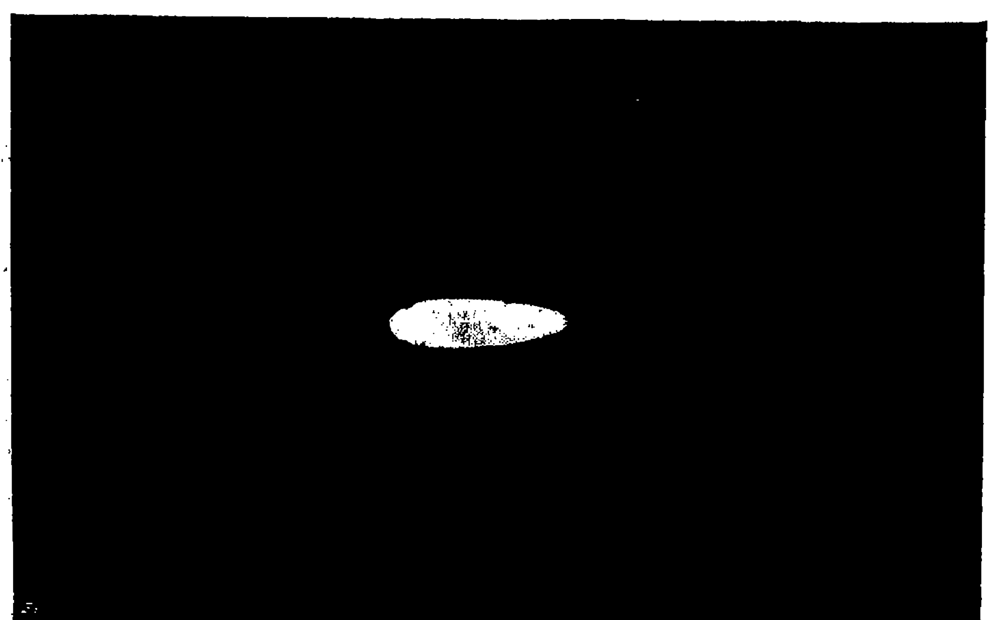


FIG. 47*b* (4).

It is not altogether easy to form a clear idea of the shape of the pencil after refraction, but the imagination may be aided by a model which is not hard to con-

struct. Take a cardboard box, and on opposite sides of it mark out two lines, at right angles ; in these lines

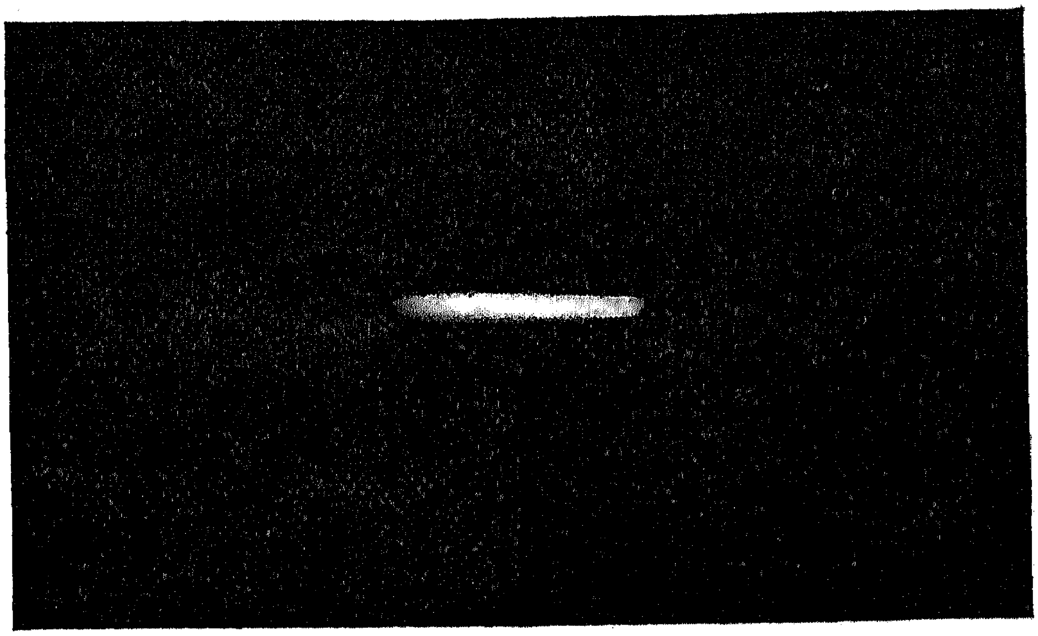


FIG. 47*b* (5).

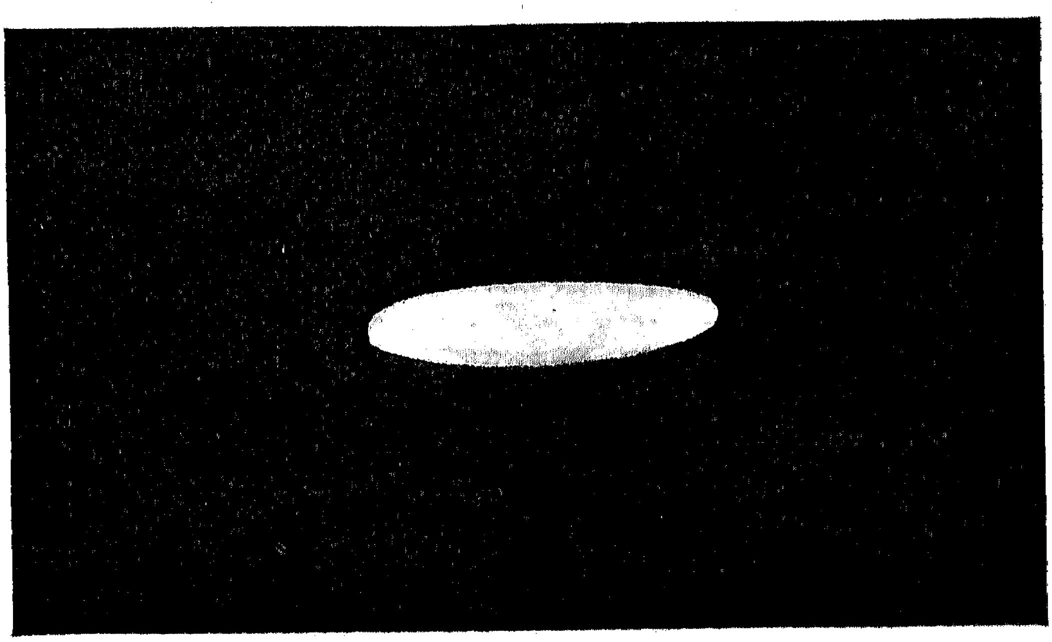


FIG. 47*b* (6).

pierce holes at convenient distances apart (five in each will be enough), and pass a thread from every hole of one set to every hole of the other ; if the sides of the

box are now pulled apart until the threads are taut, we shall have a fair model of the pencil.

The circular-shaped patch of light at a point between the lines is called the *circle of least confusion.*

71. Let us now examine how such an arrangement of the rays originates. Since the characteristics of all small pencils of rays proceeding from a point, or parallel, are, after refraction, the same, let us take a case which is rather more simple than an oblique central pencil, though not so easy to realize experimentally.

Consider a small pencil of rays parallel to the axis, but striking the lens at some distance from the centre of the lens; let the lens be placed with its axis horizontal, with the pencil vertically above the axis.

Looked at from the side the pencil would present the appearance of Fig. 47*c*, which may be obtained from Fig. 47 by erasing all the rays not required. The rays retained form a small part of the caustic surface near H, and those in the plane of the paper pass very nearly all through that point. It therefore at first sight looks as if H is the image, or the focus of the pencil; but this is not the case.

The pencil if seen from above would appear as in Fig. 47*d*, in which the letters correspond with those of Fig. 47*c*.

We have to see how the focal lines arise; in Fig. 47*c* we have a section only of the pencil; the pencil itself has of course sensible thickness. We may imagine the pencil itself to be produced by rotating Fig. 47*c* through a small angle about the axis of the lens; H will remain always at the same distance from the axis, and therefore will trace out the arc of a circle, which, being short, may be regarded as a straight line.

Hence all the rays of the pencil pass approximately through this line, which is therefore a focal line. On the other hand, let the central ray of the pencil cut

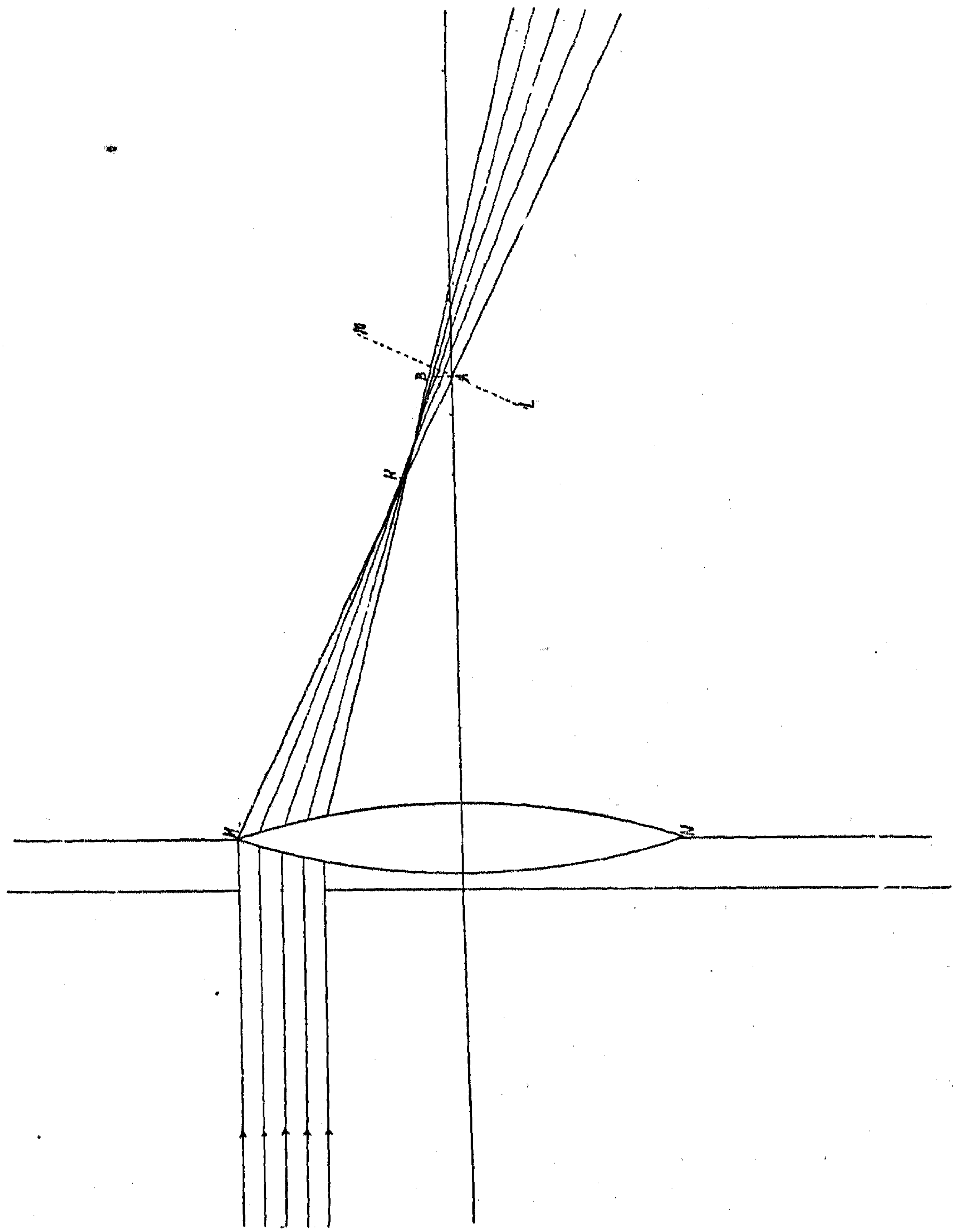


FIG. 47c.

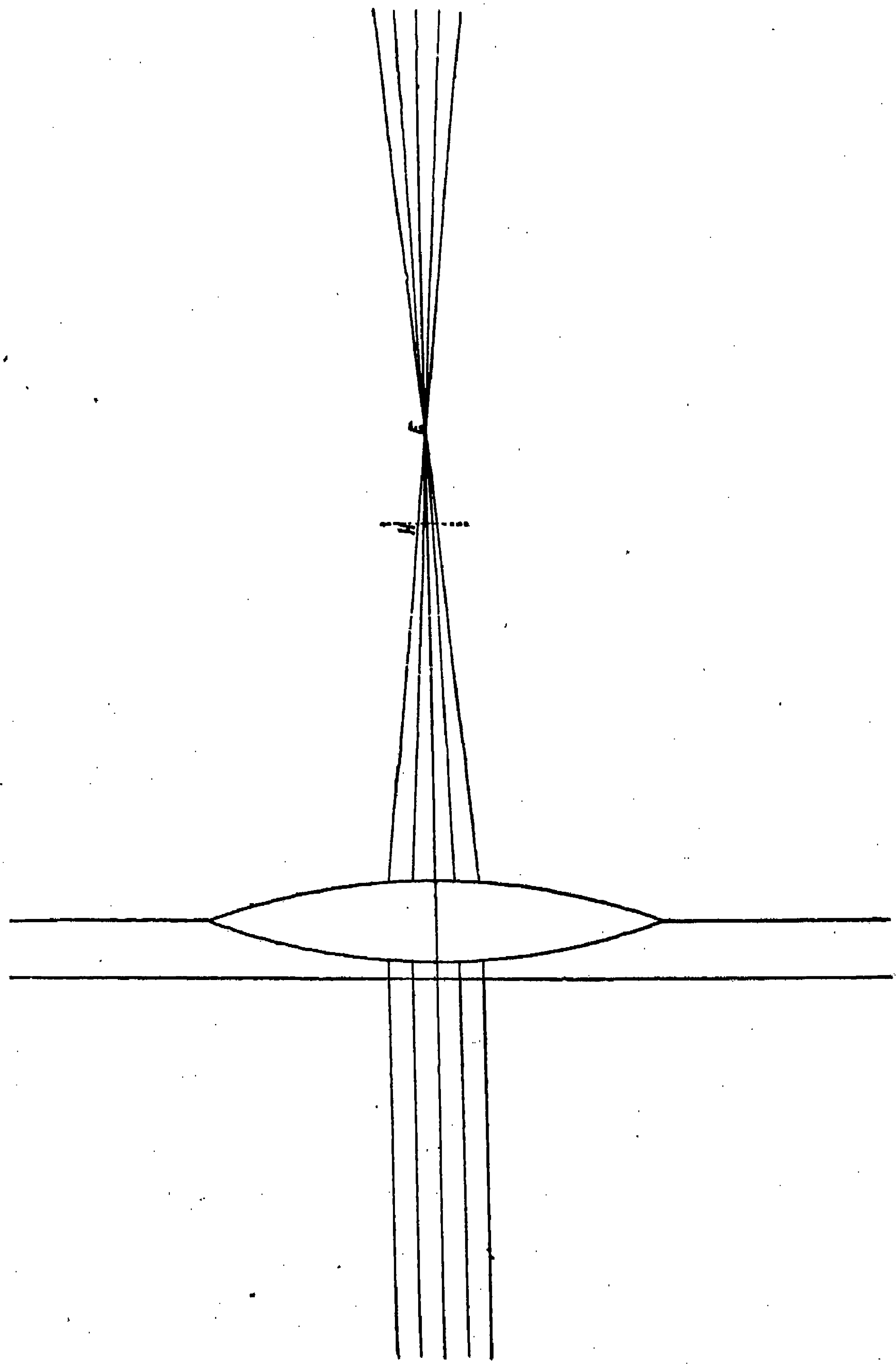


FIG. 47*d*.

the axis in A, and draw N A M perpendicular to the general direction of the rays; the revolution will turn this line through a small angle, and it will trace out a figure like that in Fig. 48, a sort of rough figure of 8, through which all the rays pass; being slender this may roughly be taken as a line, and is the other focal line. Thus all the rays of the pencil pass very nearly through two straight lines at right angles.

As we pass along the pencil from H to A it will contract horizontally and extend vertically so that at some intermediate point the pencil is of the same breadth in both directions and is roughly a circle, the circle of least confusion.

A similar course of reasoning will show the existence of focal lines in the case of a small oblique pencil striking the lens at its centre.

72. General Theory of Focal Lines.—The general theory rests on the following two proportions:—

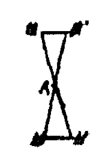


FIG. 48.

(*a*) If a pencil, to begin with, is such that its rays are all normal to some surface, then after any number of reflections and refractions, there will still be some surface to which they are normal.

A simple example of this is a pencil of rays from a point reflected at a plane surface; before reflection the rays are all normals to spheres whose centre is the source of light, and after reflection they are normal to spheres whose centre is the image of the source.

This is shown in Fig. 49, where the dotted circles are sections of the spheres; here the effect of the reflection is simply to reverse the surface without changing its nature, but in most cases a reflection or refraction will totally change the nature of the surface.

The surface in question is that which in Physical Optics is called the Wave Surface, and is in fact the shape of the wave starting from the given source, after

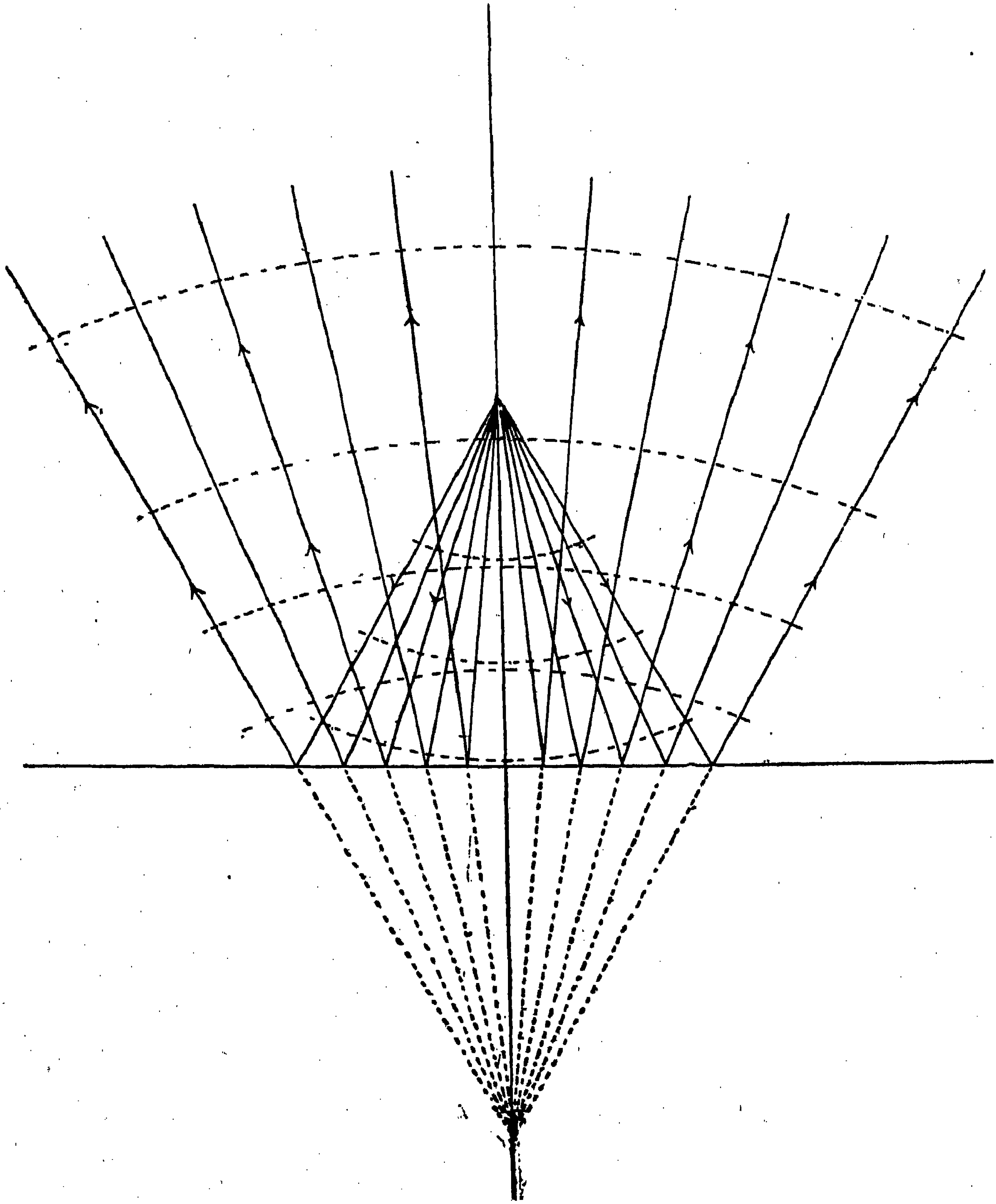


FIG. 49.

a certain time; the effect of reflection or refraction being to bend the wave into various shapes.

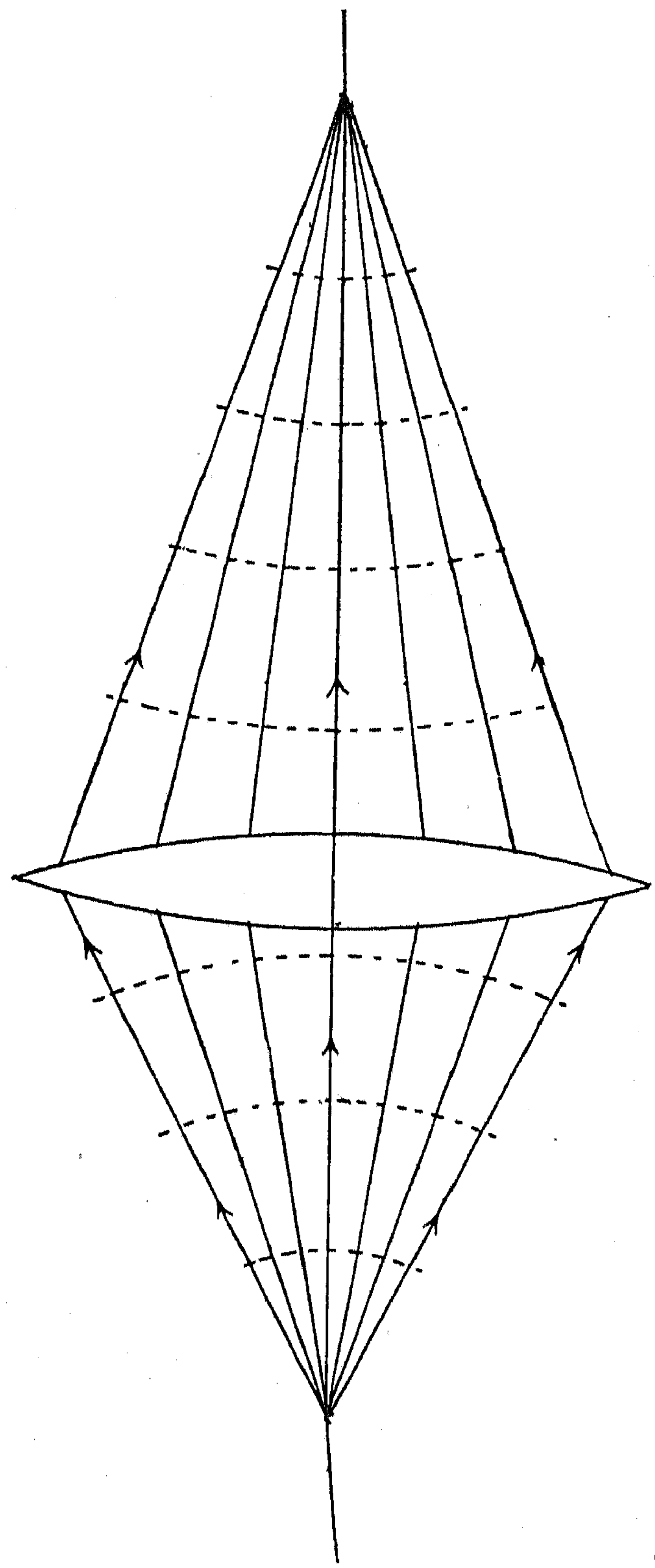


FIG. 50.

The well-known effect of throwing a stone into a still pond in causing circular waves to travel outwards is an example of this.

The action of a lens in bringing rays proceeding from one point, to a focus at another, may be explained by saying that the lens twists the wave surface from one sphere to another, as in Fig. 50; which is due to the fact that light does not travel as fast in glass as in air. The central portions of the wave have to travel through a greater or less thickness of glass (according to the nature of the lens) than the extreme portions, and thus are either overtaken by or overtake the extreme portions, from which the change in the shape results.

The second proposition, which is given in books as Analytical Solid Geometry,[1] is—

(*b*) The lines normal to a small area of any surface (provided there is no edge or point in the area) pass approximately through two straight lines at right angles.

From (*a*) we learn that the rays in the pencils with which we have to do, since they are, to begin with, normal either to a sphere or a plane (because they either proceed from a point or are parallel), are always normal to a surface, called the wave surface.

And since they are normals to a small portion of the wave surface, we learn from (*b*) that they pass approximately through two straight lines at right angles.

This proves the statement, made above, that all small pencils which are oblique to refractory surfaces have focal lines after refraction.

When the pencil strikes the surfaces symmetrically the focal lines coincide and reduce either to a small circle or point, as already described in § 64.

73. Central Oblique Pencil.—Consider now a small pencil striking the lens obliquely at its centre; in this case we can give the expressions for the distances of the focal lines from the lens.

Take the first refraction of the pencil through one spherical surface; this is reproduced in Fig. 51; in which the size of the pencil is purposely very much

[1] Frost's *Solid Geometry*, 1875, p. 388, § 588.

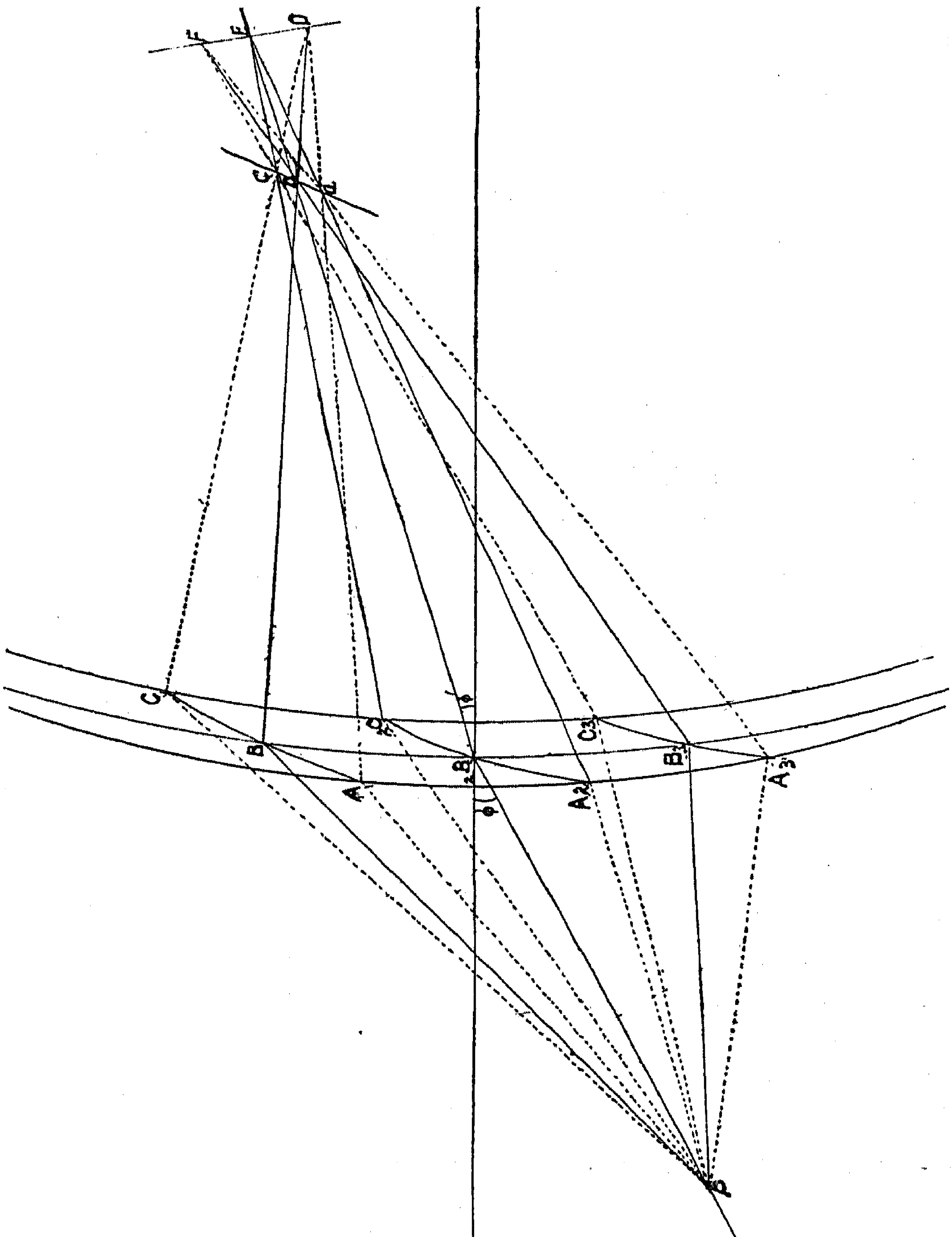


FIG. 51.

exaggerated to avoid confusion ; the pencil is supposed to be divided up by vertical and horizontal planes.

The rays in the vertical planes $P A_1 A_2 A_3$, $P B_1 B_2 B_3$, $P C_1 C_2 C_3$ converge after the refraction to points a, b, c, respectively, which form the horizontal focal line; and the rays in the planes $P A_1 B_1 C_1$, $P A_2 B_2 C_2$, $P A_3 B_3 C_3$, at right angles to the former, to D, E, F respectively, which form the other focal line.

Let ϕ be the angle which the axis P B of the pencil makes with the axis of the lens, and ϕ' the corresponding angle after refraction; let $w_1 w_2$ be the distances of b and E from B (the positive and negative directions being reckoned as before). Then it can be proved that the relations between u, w_1, w_2, are[1]—

$$\frac{\mu \cos^2 \phi'}{w_1} - \frac{\cos^2 \phi}{v} = \frac{\mu \cos \phi' - \cos \phi}{r}$$

$$\frac{\mu}{w_2} - \frac{1}{v} = \frac{\mu \cos \phi' - \cos \phi}{r}$$

Both these relations reduce to that already found for a spherical surface if we put $\phi = 0$, $\phi' = 0$, and are in fact an extension of the previous formula. Now let the pencil strike a second surface of radius s, and let v_1 and v_2 be the distances of the focal lines from the surface, the lens being taken as thin.

At this second refraction, the horizontal focal line of the first refraction, being due to rays in vertical planes, will evidently give rise to the horizontal focal line after the second refraction, and similarly the remaining focal lines in the two cases will correspond.

The axis of the pencil, which passes undeviated, will after refraction at the second surface be inclined to the axis at an angle ϕ, and the refractive index for this second refraction is $1/\mu$; hence the formulæ for the second refraction may be got from those for the first refraction by interchanging ϕ, and ϕ' and putting $1/\mu$ for μ, we thus get

[1] Aldis, *Geometrical Optics*, ed. 3, Arts. 46, 73.

$$\frac{\frac{1}{\mu}\cos^2\phi}{v_1} - \frac{\cos^2\phi'}{w_1} = \frac{\frac{1}{\mu}\cos\phi - \cos\phi'}{s}$$

$$\frac{\frac{1}{\mu}}{v_2} - \frac{1}{w_2} = \frac{\frac{1}{\mu}\cos\phi - \cos\phi'}{s}$$

or multiplying throughout by μ,

$$\frac{\cos^2\phi}{v_1} - \frac{\mu\cos^2\phi'}{w_1} = -\frac{\mu\cos\phi' - \cos\phi}{s}$$

$$\frac{1}{v_2} - \frac{1}{w_2} = -\frac{\mu\cos\phi' - \cos\phi}{s}$$

Adding these to the former equations we get for the relations between v_1, v_2 and u for a thin lens—

$$\frac{\cos^2\phi}{v_1} - \frac{\cos^2\phi}{u} = (\mu\cos\phi' - \cos\phi)\left(\frac{1}{r} - \frac{1}{s}\right)$$

$$\frac{1}{v_2} - \frac{1}{u} = (\mu\cos\phi' - \cos\phi)\left(\frac{1}{r} - \frac{1}{s}\right)$$

These relations reduce to those already found for a thin lens if ϕ and ϕ' vanish.

When the incident pencil consists of parallel rays we must put $1/u = 0$, and we get

$$\frac{\cos^2\phi}{v_1} = \frac{1}{v} = (\mu\cos\phi' - \cos\phi)\left(\frac{1}{r} - \frac{1}{s}\right)$$

which shows us that v_1 can never be numerically greater than v_2, for

$$v_1 = v_2\cos^2\phi$$

and the cosine of an angle cannot be greater than unity. Hence the focal line which is perpendicular to the plane containing the axes of both the pencil and the lens is nearer to the lens than the other focal line.

73*a*. Construction for Focal Lines.—When the pencil incident on a single spherical surface is composed of parallel rays we may find the position of the focal lines as follows—

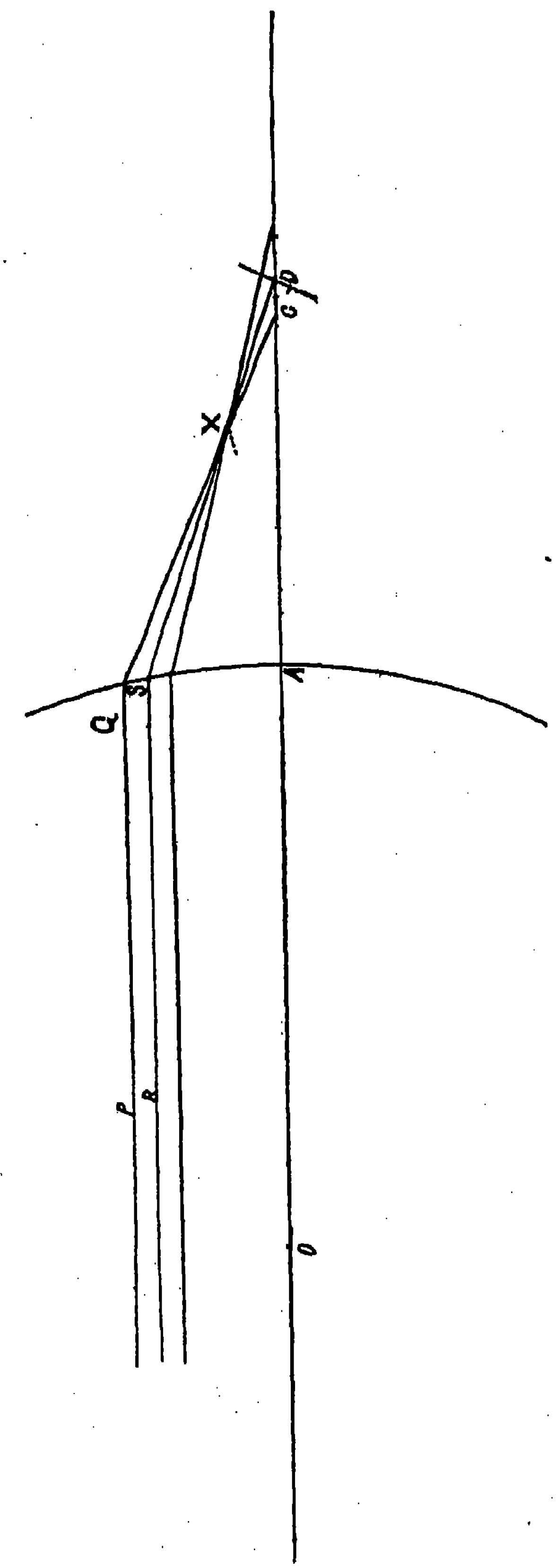


FIG. 51*a*.

Let P Q be the mean ray of the pencil (Fig. 51*a*) and R S a near ray; through the centre O of the surface

draw O A parallel to the incident pencil, and let it meet P Q, R S, after refraction, in C and D, and let Q C, S D intersect in X.

If now the figure be imagined to be rotated through a small angle round O A, X will trace out the focal line perpendicular to the paper; the other focal line will be at C, for all the rays intersect O A near this point.

Thus to get the position of the focal line lying in the plane of the paper we must draw a radius parallel to the incident pencil and find the point where this is cut by the mean ray after refraction.

74. Distance between Focal Lines after Refraction at the First Surface.—We shall consider in this article the case only of pencils with parallel rays; here we must put $1/u = 0$, and the distances of the focal lines from the surface are given by—

$$\frac{\mu \cos^2 \phi'}{w_1} = \frac{\mu}{w_2} = \frac{\mu \cos \phi' - \cos \phi}{r}$$

Now $\sin \phi = \mu \sin \phi'$, and therefore $\mu = \sin \phi / \sin \phi'$

$$\therefore \mu \cos \phi' - \cos \phi = \frac{\sin \phi}{\sin \phi'} \cos \phi' - \cos \phi =$$

$$\frac{\sin \phi \cos \phi' - \cos \phi \sin \phi'}{\sin \phi'} = \frac{\sin (\phi - \phi')}{\sin \phi'}$$

Substituting this we get easily

$$w_1 = r \frac{\mu \cos^2 \phi' \sin \phi'}{\sin (\phi - \phi')}, \; w_2 = r \frac{\mu \sin \phi'}{\sin (\phi - \phi')}$$

$$\therefore w_2 - w_1 = r \frac{\mu \sin \phi' (1 - \cos^2 \phi')}{\sin (\phi - \phi')} = r \frac{\mu \sin^3 \phi'}{\sin (\phi - \phi')}$$

If the angle of incidence is *small* we can replace $\sin \phi$ and $\sin (\phi - \phi')$ by the circular measure of the angles, and we get

$$w_2 - w_1 = r \frac{\mu \phi'^3}{(\phi - \phi')} \text{ and this becomes } \frac{r \phi^2}{\mu (\mu - 1)}$$

for the relation $\sin \phi = \mu \sin \phi'$ reduced to $\phi = \mu \phi'$.

This shows us that the distance $w_2 - w_1$, between the focal lines, increases with the inclination φ of the axis of the incident pencil to the axis of the lens, if ϕ is small; this can be proved to be true also when the inclination is considerable.

A separation of the focal lines after the first refraction will obviously tend to produce a separation of the focal lines after the second refraction; we see therefore that with a lens the greater the obliquity the greater is the distance between the focal lines.

74*a*. Effect of Spherical Aberration on the Picture.—It is hardly necessary to point out that the spherical aberration of a lens tends to destroy the sharpness and clearness of the picture.

Each luminous point of the object, near the axis of the lens, instead of being represented in the picture by a luminous point, as by the elementary theory it should be, is represented by a patch of light which no amount of focussing can reduce in size beyond a certain limit. If these patches are small enough not to subtend an angle of more than one minute (§ 90) when viewing the picture at a convenient distance, the effect produced is the same as if they were points; but if they are larger than this they have a visible size and overlap, thus confusing the images of points near together and causing indistinctness.

In Fig. 42, C D is the section of least area, and it can be shown that the distance of C D from F is three-fourths of A F, and the diameter of the circle at C D is one-half the lateral aberration F B.[1]

75. Astigmatism. The effect of aberration on pencils which pass obliquely is called *astigmatism*.

We have seen that when the pencil is oblique and the aperture of the lens is large its section is nowhere even approximately a point, but is a considerable patch of light varying in size at different places.

But if the pencil be restricted by a diaphragm to be

[1] Coddington's *Optics*, pp. 11, 12.

fairly small, then the rays pass, not through a point but very approximately through the straight lines at right angles, and the nearest approach to a focus is at some point between them, where the section of the pencil is nearly circular, called the *circle of least confusion*. If the aperture used is small, this circle may be made small enough to be practically a point, which can be regarded as the focus.

If such foci were all in one plane perpendicular to the axis of the lens and the plate were placed in this plane the picture would be in focus all over; but in most cases the circles of least confusion lie on a curved surface, passing through the principal focus and usually concave towards the lens, producing *curvature of the field*.

When this is the case there is no position in which the plate can be placed that the foci of all the pencils may lie on it; if it be placed to bring the central portion of the picture into focus, it will cut very oblique pencils either at or near a focal line, giving rise to a line or elongated patch of light.

Let us consider the nature of the picture formed by such lines or elongated patches; let the object be a cross (Fig. 52, 1). If the cross be placed at right angles to the focal line on the screen, every point on the vertical line will be broadened out, each point of the object giving rise to a line or elongated patch; the effect of this is to represent the cross by an indistinct broad blur (Fig. 52, 2), the only distinct portion being the crossbar, for here the patches of light overlap and lengthen, but do not broaden it; if the cross be parallel to the focal line it is not hard to see that the general form of the image will be that in Fig. 52 (3), and if the screen be placed to receive the circles of least confusion, then the effect is shown in Fig. 52 (4), in which the size of the circle is exaggerated to show the nature of the phenomenon clearly.

Careful inspection of these figures will show that the

general effect of using the section of oblique pencils near focal lines is to produce fairly sharp images of straight lines parallel to the focal line used, but only diffuse images of lines at right angles to these, so that practically a picture is produced only of lines parallel to the focal line used.

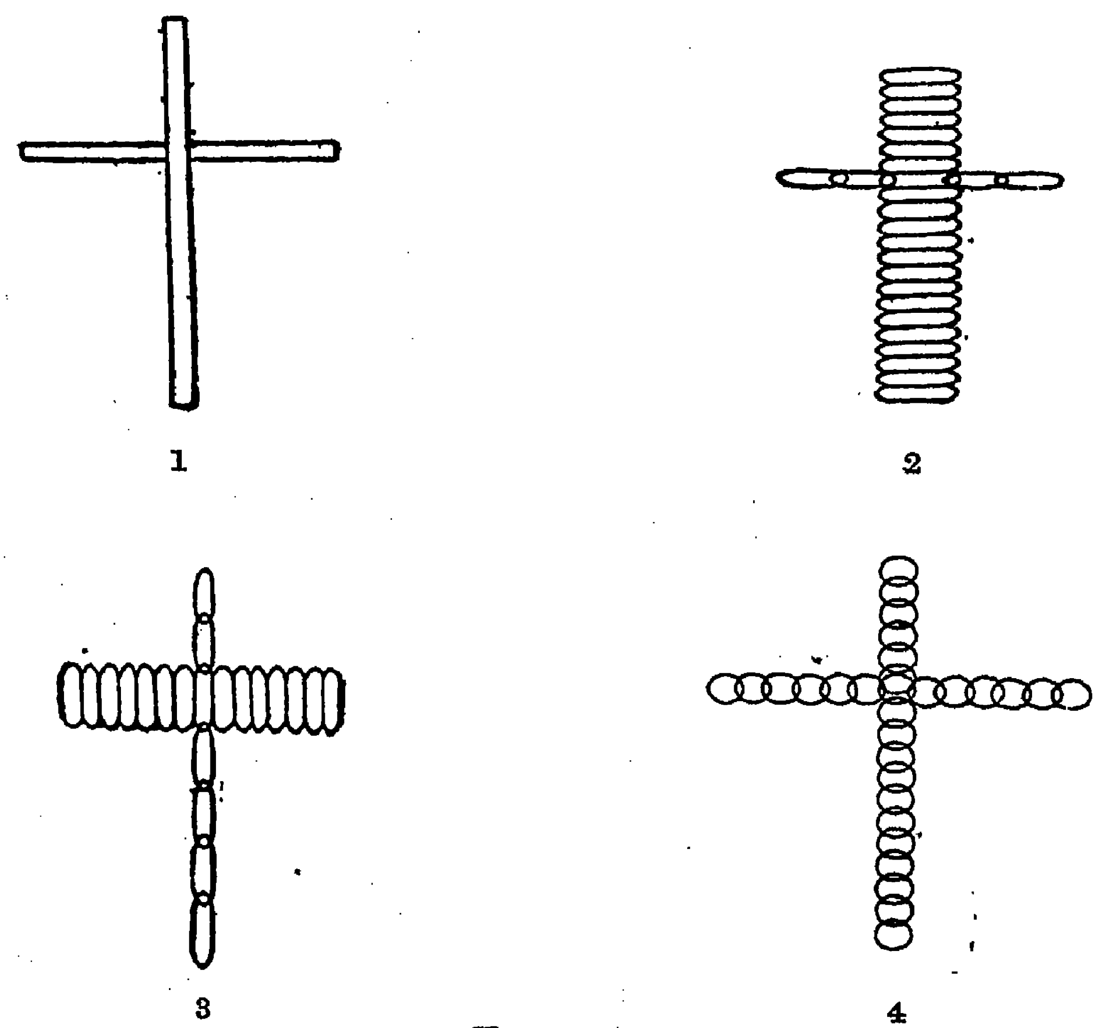


FIG. 52.

This effect of producing an image of lines in one direction but not of those at right angles is *astigmatism;* it occurs not only in photographic lenses but is a not uncommon defect of sight produced by some deformation of the lens of the eye.

In photographic lenses since the surface containing the circles of least confusion bends towards the lens, it is usually the focal line furthest away from the lens which falls on the plate; this line is that one which is

radial or points towards the axis of the lens, hence if the astigmatism is sensible the images of lines pointing towards the centre of a picture will be sharper than those of lines at right angles to them; but in most cases the phenomenon will not be clearly marked, a general indistinctness at the edges of the picture being mainly observable.

The distance between the focal lines of the pencil is generally taken to be the measure of its astigmatism.

76. Experiment on Astigmatism.—The remarks of the preceding section can be illustrated by a simple experiment.[1]

Place upright in front of a lamp or gas flame a piece of wire gauze, the wires being horizontal and vertical, and behind this a single lens (fitted with a diaphragm if necessary), not parallel to the gauze but twisted about a vertical axis through an angle of about 45°; if the image be received on a vertical screen it will be found that the horizontal wires are in focus in one position of the screen and the vertical wires in another.

If the lens be at a considerable distance from the gauze so that $1/u$ can be neglected, and v_1, v_2 be the distances of the focal lines from the lens, we have (§73)

$$\cos^2 \phi = \frac{v_1}{v_2}$$

hence if the inclination of the lens to the direction of the light can be measured we can roughly verify the formulæ given.

77. Distortion due to the Use of a Diaphragm.—A diaphragm is used with a lens to reduce the size of the pencil, thus diminishing the size of the circle of least confusion, and improving the definition.

The best effect is produced when the diaphragm is not in contact with the lens, but at a short distance from it, so that the pencils strike the lens less obliquely, and there is less astigmatism; in fact the arrangement

[1] Glazebrook and Shaw's *Practical Physics*, 4th Ed. p. 354.

combines the advantages of the use of a small pencil with incidence as nearly direct as possible.

But this arrangement, though it produces a beneficial effect on the sharpness of the picture, introduces another defect which would be noticeable when the lens is used for copying or architectural work; a distortion is produced, the nature of which we proceed to examine.

This distortion must be clearly distinguished from that which we have already considered, which was due to the wrong position of the plate relative to the object, causing lines which should be parallel to run together, which can be corrected by the use of the swing back.

Here the effect is not to make lines converge, but to bend them, making lines near the edge of a picture *curved* instead of straight. The phenomenon is best illustrated experimentally, but as the effect is usually small it must be looked for carefully, and the straight lines on the picture tested with a straight edge.

In Fig. 53 is given a horizontal section of the arrangement, through the centre of the lens. In front of a screen to receive the image the lens is placed upright, both being movable; in front of the lens is placed a movable diaphragm—a piece of cardboard pierced with a small clean circular hole will do—the hole being on the axis of the lens; to one side a straight-edged rule is placed vertically, and behind it a lamp with a large plain globe. If a lamp with a globe is not available a piece of ground glass may be used instead of the globe to make a uniform background to a considerable length of the vertical rule.

The diagram is not drawn to scale, the lamp and rule being considerably nearer to the lens than they should be in the actual experiment.

If the diaphragm is near the lens the image of the straight edge, formed on the screen, is straight, but if the diaphragm be moved backwards the image will

become curved, the line being concave towards the axis of the lens.

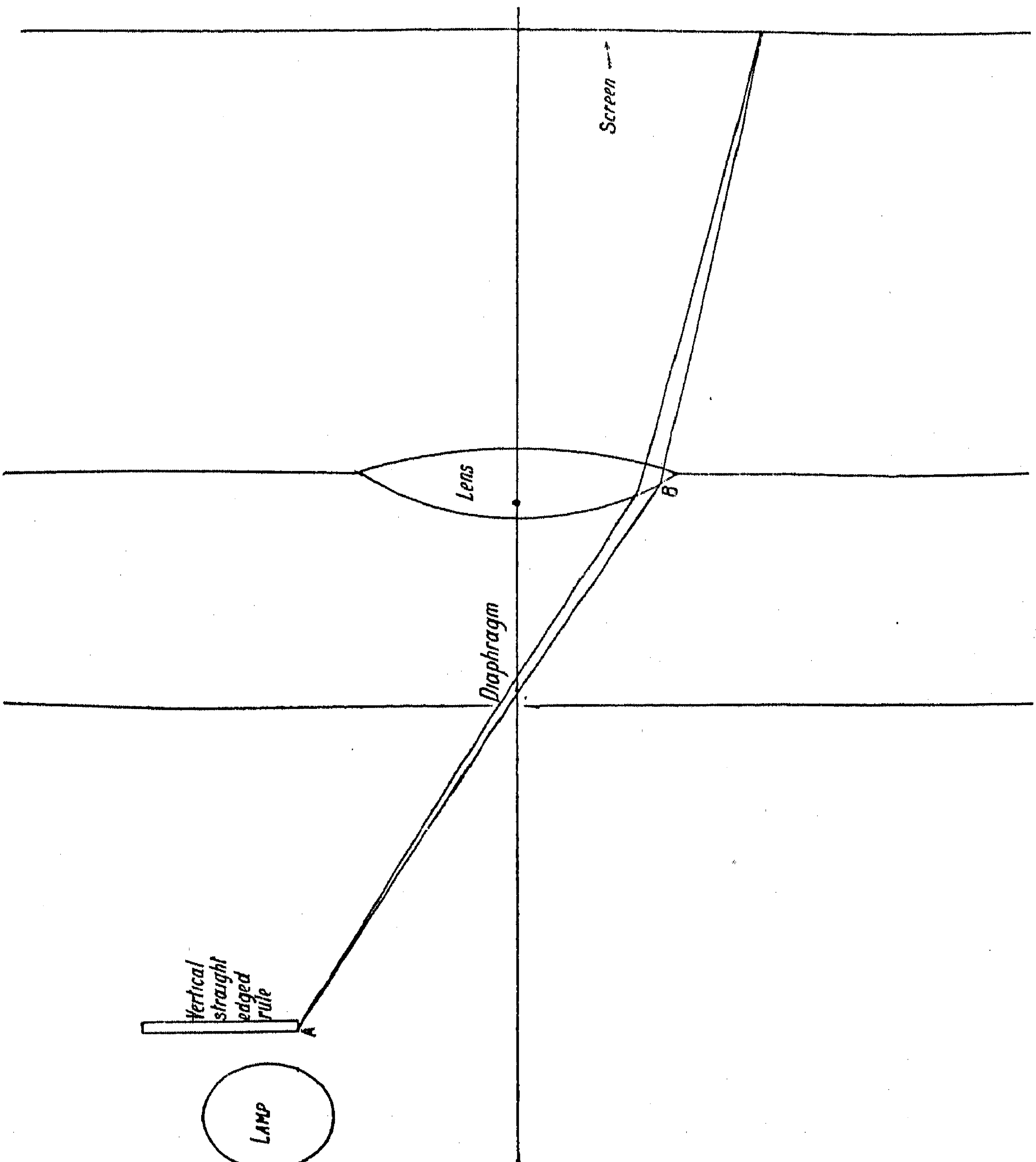


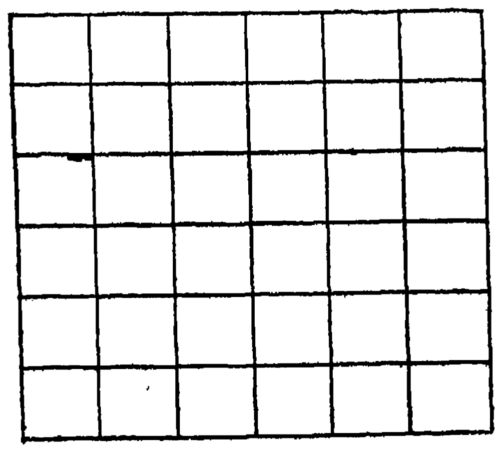
FIG. 53.

In the diagram the pencil coming from A the edge of the rule is traced, it falls on the *edge* of the lens,

and it is to this that the phenomenon is to a large extent due.

It will be noticed that the curvature of the image increases when the diaphragm recedes from the lens and decreases when it approaches it.

It should be noticed that as the diaphragm recedes



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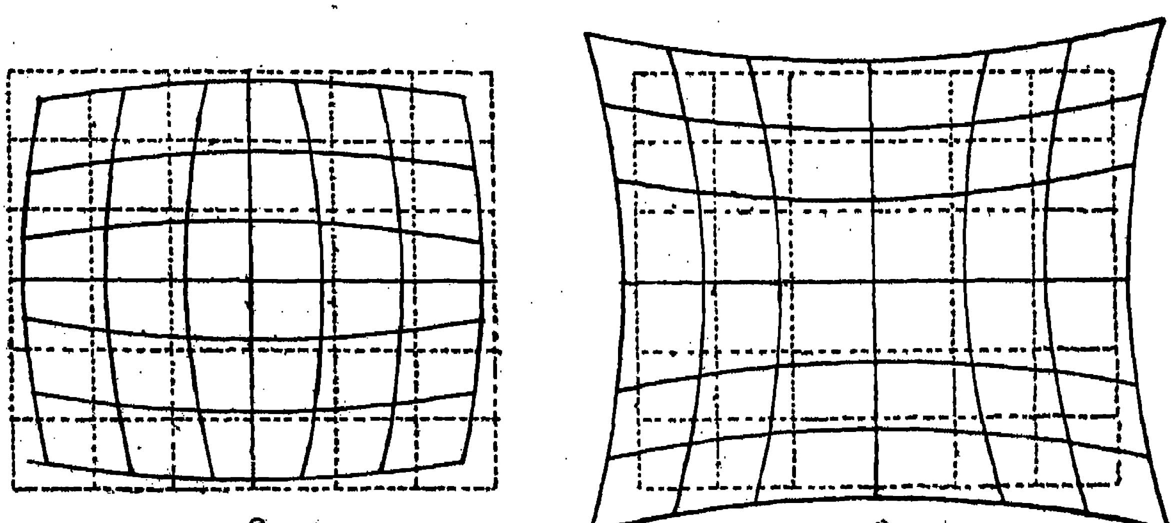

2 3

FIG. 54.

the pencil strikes the lens nearer and nearer to its edge; we thus conclude that the further from the axis the pencil strikes the lens the greater is the displacement of the image from its proper position.

If the diaphragm is placed behind the lens the curvature is in the opposite direction, the line being convex to the axis of the lens.

In practical work we do not often have to photograph two sets of lines at right angles, but it will be interesting to see the effect produced on such a set by the distortion; Fig. 54 (1) represents the object, a square grating; the image of this formed with the diaphragm in front of the lens is given in Fig. 54 (2), and the image when the diaphragm is behind the lens in Fig. 54 (3); in the last two diagrams the dotted lines show the size of the image when not distorted.

It should be noticed carefully that in (2), which is called barrel-shaped distortion the image is smaller than it should naturally be, and also that the curvature is caused, not by the lines being bulged outwards at their middle points, but by every point of the line being displaced inwards, the ends of the lines more so than the middle points.

It thus appears that the effect of the distortion is to displace every point towards the centre of the picture, those points furthest away being displaced most; the parts of the picture at the edge are therefore unduly crowded. On the other hand, in (3), sometimes called pin-cushion distortion, the effect is just the opposite; every point of the picture is displaced outward from its true position, those points at the edge being most displaced, causing a spreading out instead of a crowding of the edge of the picture.

This view of the phenomenon is illustrated in Fig. 55; here a series of circles (1) at equal distances apart are (2) crowded towards the centre when the diaphragm is before the lens, being most crowded at the edge, while (3) when the diaphragm is behind the lens the circles are extended, the extension being most marked at the edge; in (2) and (3) the dotted circle represents the true size of the outer circle.

The diagrams are very much exaggerated to bring out clearly the points they illustrate.

It should be remarked that in (2) the displacement of different circles towards the centre is *not* propor-

tional to their radii, for then the effect would be tc reduce the size of the diagram merely without altering

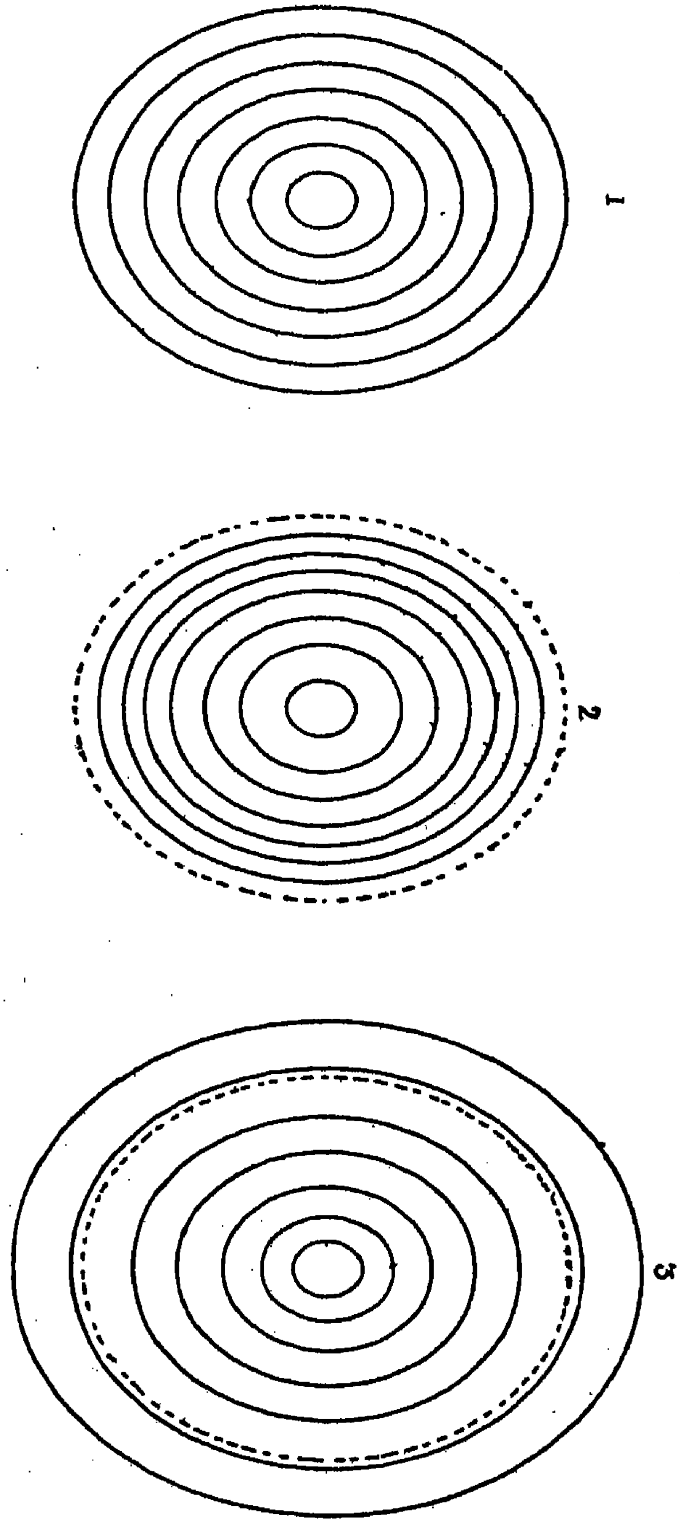

FIG. 55.

the relative spacing of the circle, and no distortio would be produced; a straight line would then remai

a straight line, but would be shortened. Similar remarks will apply to the magnification of the different circles in (3).

The phenomenon of distortion is seen when a large magnifying glass is held near such a rectangle as that in Fig. 54 (1); the iris of the eye here acts as the diaphragm, and the appearance is that of Fig. 54 (3); if instead a diverging lens be used the appearance produced would be that of Fig. 54 (2).

78. Cause of the Distortion.—It is usually stated in a vague way that the distortion is due to the difference between the thickness of the glass traversed by rays at the centre and at the edge of the lens; but by the aid of the properties of small pencils a more detailed and satisfactory account can be given, though a complete algebraical proof is beyond our present range. We must first return to the question of perspective already treated in § 63 (Fig. 42).

We saw that according to the elementary theory the picture retains its proper relative proportions if an object has an image lying on the secondary axis passing through the object.

In Fig. 42 take for example the object A, $A N_1$ is the line joining it to the nodal point of incidence, and the image a lies on the line through N_2, the nodal point of emergence, parallel to $A N_1$.

Similarly $N_2 b$, $N_2 c$, etc. are parallel respectively to $B N_1$, $C N_2$, etc.

In this case the various lengths in the picture have the same ratio to each other as the corresponding lengths in the object have; for instance

$$F c : F d = X B : X A.$$

If, however, it should happen that the images of A, B, etc. do not lie at a b, etc., but at points either nearer to or further from the axis than these, and at distances from it not proportional to $F a$ and $F b$, then the lengths in the picture will no longer be proportional to those in the object, and distortion will arise.

We have therefore to find the reason why the lines joining points in the image to the nodal point of emergence are not parallel to the lines joining the corresponding points of the object to the nodal point of incidence; and also to show that the deviation is such as will produce the effects already described. As the object is usually distant let the incident pencil from any point of the object be taken to consist of parallel rays (this will simplify future work without altering the nature of the phenomenon).

Let A B (Fig. 56) be the aperture in the diaphragm, and let the rays in the plane of the paper meet after refraction at P.

Let N_1 and N_2 be the nodal points of incidence and emergence, and $X N_1$ parallel to the incident ray; draw $N_2 Y$ parallel to $X N_1$.

Then the focal line perpendicular to the plane of the paper is at P, that in the plane of the paper at some point Q, which experiment shows to be in most cases further from the lens than P.

For central oblique pencils the position of P is usually as shown in the figure between $N_2 Y$ and the axis of the lens.

The circle of least confusion is at C about midway between P and Q; this point may be taken as the image. Draw lines through P and Q perpendicular to the axis to meet $N_2 Y$ in R and S, and C D parallel to them to meet $N_2 Y$ in D.

Then the image, instead of being on $N_2 Y$ as it should be by the elementary theory, is at a distance C D below it, and the picture is therefore crowded towards the centre, as it should be when the diaphragm is in front of the lens.

If we know the positions of P and Q we can find C D, for $2\,CD = PR + QS.$

The form of the expression for the aberration shows that it is proportional to z^2, the *square* of the distance from the axis at which the pencil strikes the lens, and

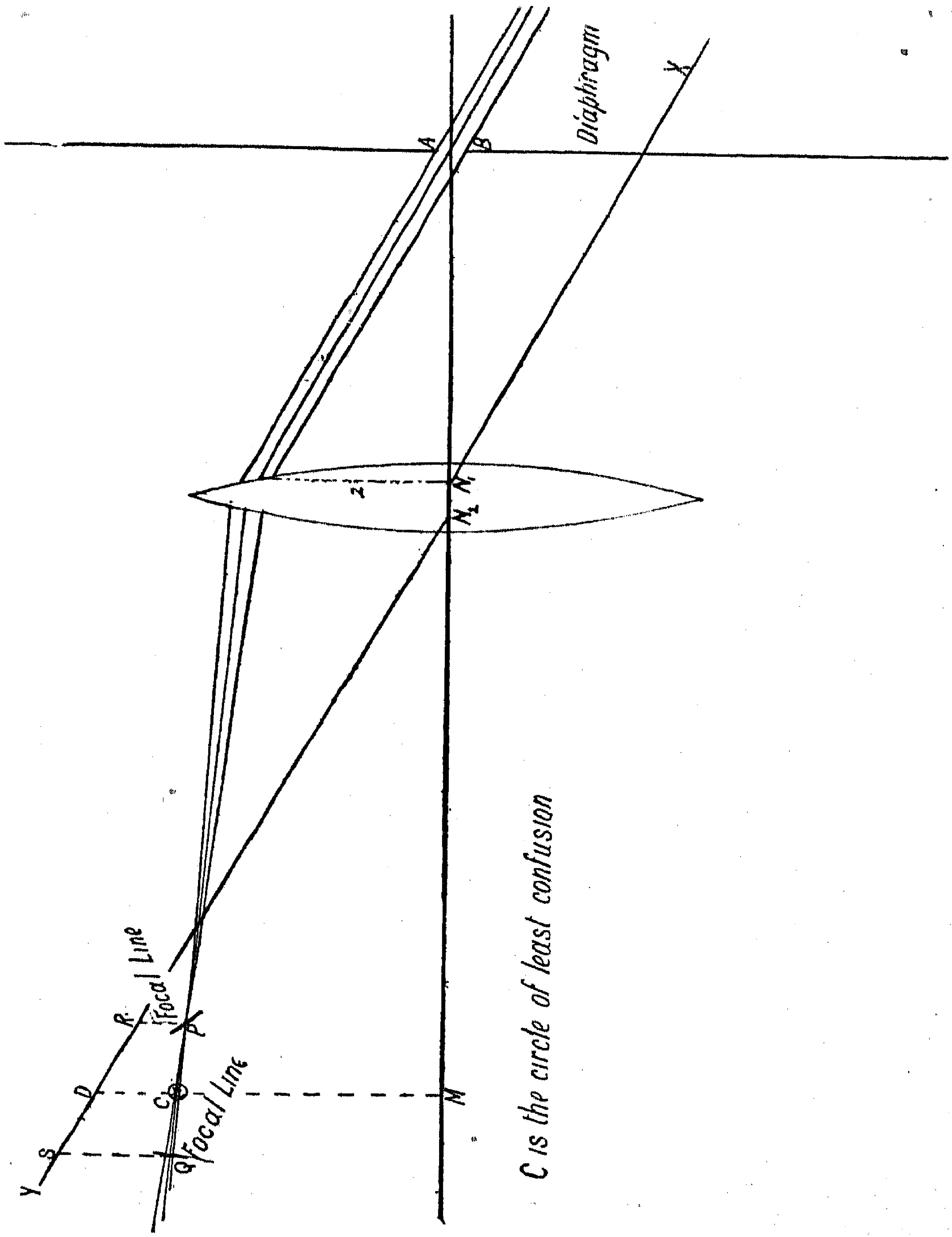

FIG. 56.

in Fig. 56 it is evident that the distances of different points from the axis are very nearly proportional to

the distances from the axis at which the corresponding pencils strike the lens. Now the expression for the aberration is the basis of all calculations, and by means of it together with that for *tan* η, where η is the inclination of the emergent ray from the axis, we can solve any problem we wish.

The first power of z does not occur in either of the expressions, and none of the processes of the calculation can introduce it.

We conclude therefore that the expression for any small variation from the elementary theory contains not the first, but the second power of z; hence C D is not proportional to z, but to z^2.

We see therefore that the distortion C D diminishes the distance of every point from the axis, and also that it is proportional to the square of the distance of the object from the axis (higher powers of z being neglected).

We have therefore got all the particulars necessary for the explanation of the barrel-shaped distortion. We shall take the quantity C D as the measure of the distortion, reckoning it positive when C is further from the axis than D (in Fig. 56 C D is negative).

The explanation of distortion of the opposite kind, when the diaphragm is behind the lens, is very similar to that already given, and can be understood from an inspection of Fig. 57, which is lettered to correspond with Fig. 56; here P lies on the side of N_2 Y remote from the axis of the lens.

79. Calculation of Distortion. Numerical Example.—As the subject of distortion does not appear to have been fully treated except in Coddington's *Optics*, which is long since out of print, we proceed to give a numerical example which will to some extent serve as a justification for the statements made in the last article.

Take the case of a plane convex lens, with its plane face turned towards the light, for which the calculations

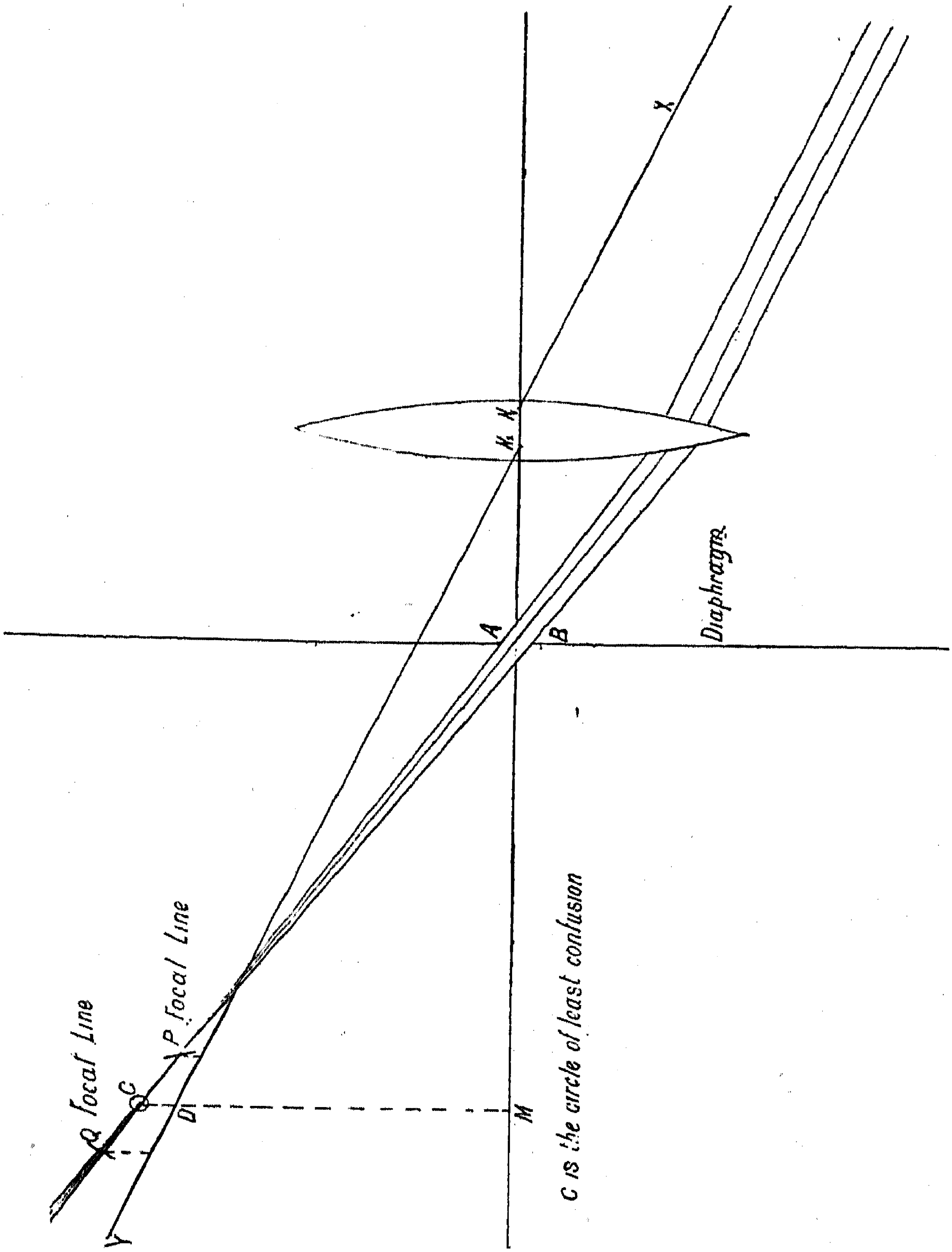

FIG. 57.

are much simplified ; for here the optical centre and therefore the nodal point of emergence are at the point

where the curved surface meets the axis (§ 49, end). Let the dimensions of the lens be as follows, where the symbols have the meanings previously (§ 49) given to them.

[1] $r = \infty$ or $1/r = 0$, $s = 3$ in., $e = \cdot 2$ in. $\mu = 1{\cdot}5$.

If u be the distance of the object from the front surface, and v the distance of the image from the back surface; ϵ the inclination of the incident ray, η of the emergent ray to the axis, the formulæ of § 65 (*Note*) become, since $1/r = 0$—

$$\frac{1}{v} = \frac{1}{u} - \frac{\mu - 1}{s} - \frac{e}{\mu u^2} - \frac{(\mu - 1) z^2}{2}\left(A - \frac{B}{u} + \frac{C}{u^2}\right)$$

$$\frac{\tan \eta}{\tan \epsilon} = \frac{u}{v}\left[1 + \frac{e}{\mu u} - \frac{(\mu - 1) z^2}{2 \mu s}\left(\frac{1}{s} - \frac{1}{v}\right)\right];$$

substituting the values of μ and s we get

$$\frac{1}{v} = \frac{1}{u} - \cdot 167 - \frac{\cdot 133}{u^2} - z^2\left(\cdot 021 - \frac{\cdot 152}{u} + \frac{\cdot 361}{u^2}\right)$$

$$\frac{\tan \eta}{\tan \epsilon} = \frac{u}{v}\left[1 + \frac{\cdot 133}{u} - \frac{z^2}{18}\left(\cdot 333 - \frac{1}{v}\right)\right].$$

As it is impossible to give every step of the substitution, the reader should verify these formulæ for himself. They are the fundamental formulæ for the lens, and by means of them we can find the position of the emergent rays.

Let X be the middle point of the aperture in the diaphragm (Fig. 58), A and N_2 the front and back surfaces of the lens; then N_2 is the nodal point of emergence.

Take the pencil parallel to X H such that

$$A\,X = 1 \text{ inch}, \; A\,H = \cdot 5 \text{ inch, or}$$
$$u = 1 \text{ inch}, \quad z = \cdot 5 \text{ inch}.$$

[1] From the mathematical point of view a flat surface may be regarded as the surface of a sphere of infinite radius or of zero curvature.

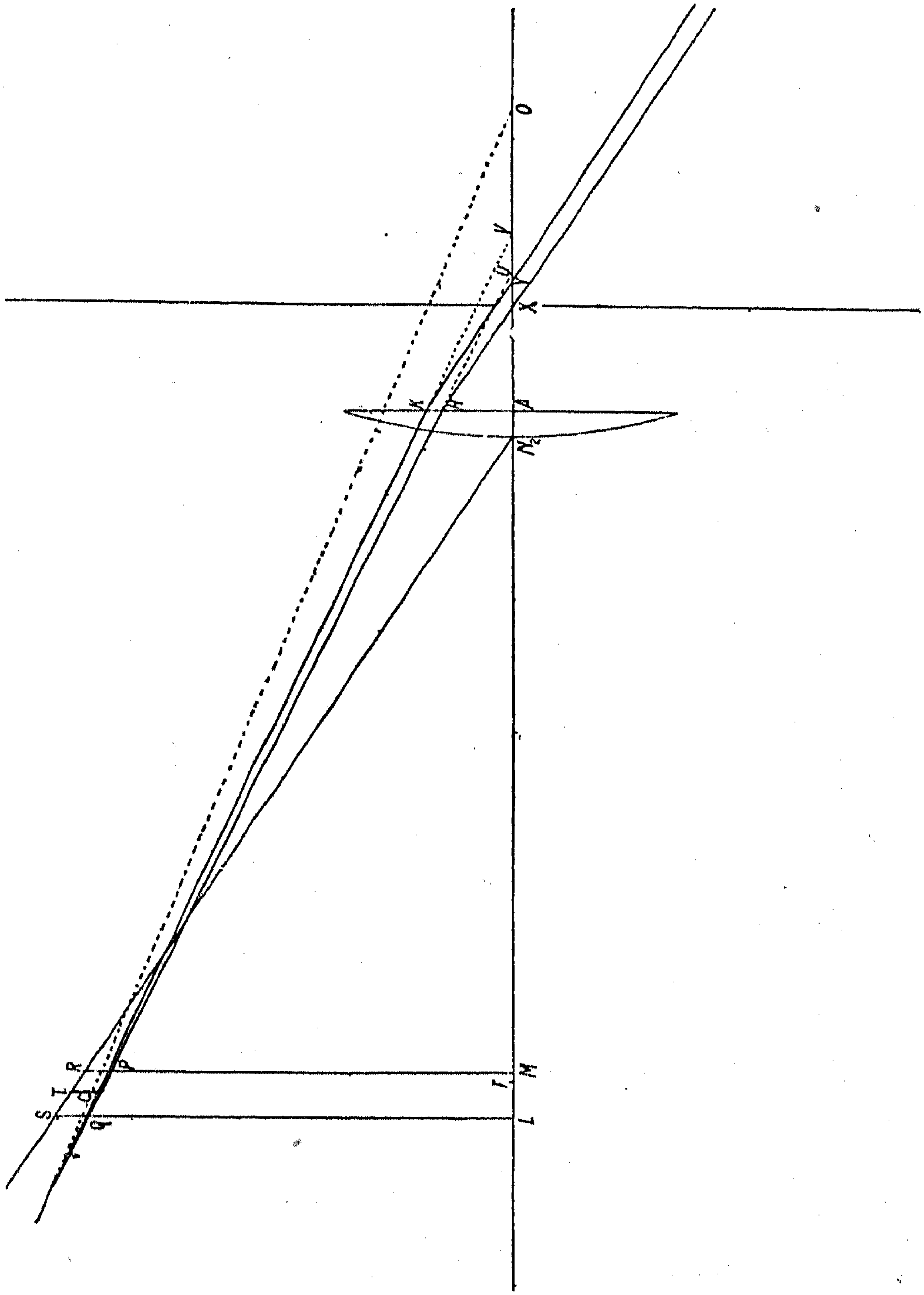

FIG. 58.

Let the second ray taken be K Y parallel to X H where X Y = ·2 inch; then by similar triangles

$KH : XY = HA : AX = 1 \cdot 2 \quad \therefore KH = \cdot 1$ inch;

$\therefore u' = AY = 1\cdot 2$ inch, $z' = KA = \cdot 6$ inch.

Let PU and PV be the two emergent rays and angle $PUA = \eta$, angle $PVA = \eta'$, also angle $HXA = \epsilon$; then $tan\ \epsilon = HA/AX = 1/2 \quad \therefore \epsilon = 26^\circ\ 34'$.

For the ray HX we have

$$\frac{1}{v} = 1 - \cdot 167 - \cdot 133 - \frac{1}{4}(\cdot 021 - \cdot 152 + \cdot 361)$$

$$= 1 - \cdot 300 - \cdot 057 = \cdot 643$$

$$\therefore v = 1\cdot 55 \text{ inch.}$$

$$\frac{tan\ \eta}{tan\ \epsilon} = \cdot 643\left[1 + \cdot 133 - \frac{1}{72}(\cdot 333 - \cdot 643)\right]$$

$$= \cdot 643\,[1 + \cdot 133 + \cdot 004] = \cdot 7311;$$

$$\therefore tan\ \eta = \cdot 3655,\ \eta = 20^\circ\ 5'.$$

For the ray KX we have $u' = 1\cdot 2$ in., $z' = \cdot 6$ in.

$$\therefore \frac{1}{v'} = \cdot 833 - \cdot 167 - \cdot 092 - \cdot 36\,(\cdot 021 - \cdot 127 + \cdot 251)$$

$$= \cdot 833 - \cdot 167 - \cdot 092 - \cdot 052 = \cdot 522$$

$$\therefore v' = 1\cdot 91 \text{ inch.}$$

$$\frac{tan\ \eta'}{tan\ \epsilon} = 1\cdot 2 \times \cdot 522\left[1 + \cdot 111 - \frac{\cdot 36}{18}(\cdot 333 - \cdot 522)\right]$$

$$= \cdot 626\,[1 + \cdot 111 + \cdot 004] = \cdot 6980;$$

$$\therefore tan\ \eta' = \cdot 3490 \qquad \eta' = 19^\circ\ 14'.$$

Now, $PM = PV\ sin\ PVM =$

$$UV\,\frac{sin\ PUM}{sin\ UPV}\ sin\ PVM$$

$$= (v - v')\,\frac{sin\ \eta\ sin\ \eta'}{sin\ (\eta - \eta')} = \cdot 36\,\frac{sin\ 20^\circ\ 5',\ sin\ 19^\circ\ 14'}{sin\ 51'}$$

$= 2\cdot 745$ inches.

Also it can easily be shown that

$$VM = (v' - v)\,\frac{sin\ \eta\ cos\ \eta'}{sin\ (\eta - \eta')} = \cdot 36\,\frac{sin\ 20^\circ\ 5',\ cos\ 19^\circ\ 14'}{sin\ 51'}$$

$= 7\cdot 868$ inches.

$\therefore N_2 M = V M - V N_2 = 7{\cdot}87 - 1{\cdot}91 = 5{\cdot}96$ in.

Let the perpendicular from P on the axis meet the secondary axis through N_2 on R; then—

$R M = N_2 M \ tan\ R N_2 M = N_2 M \ tan\ H X A = 5{\cdot}96 \times {\cdot}5 = 2{\cdot}98$ inches.

$\therefore P R = R M - P M = 2{\cdot}98 - 2{\cdot}74 = {\cdot}24$ inch.

We have thus found the distance below the secondary axis of the focal line perpendicular to the plane of the paper.

We must now find the position of the other focal line.

The incident rays being refracted at a plane surface will be parallel after refraction, and we have a pencil of parallel rays striking the curved surface; hence (§ 73*a*) the focal line required will be at the point where a line through O, the centre of the surface, parallel to the pencil striking the curved surface, meets the mean emergent ray U P. Let Q be this point.

Let θ and ϕ be the angles of incidence and refraction at the plane surface, then—

$$tan\ \theta = \tfrac{1}{2},\ \therefore \theta = 26^\circ\ 34'$$

$$sin\ \phi = \frac{sin\ \theta}{\mu} = \frac{sin\ 26^\circ 34'}{1{\cdot}5}\ \therefore \phi = 17^\circ\ 21'$$

Hence the angle Q O M must be $17^\circ\ 21'$.

Draw Q L perpendicular to the axis, and produce it to meet N_2 R in S; then—

$$Q L = O U \frac{sin\ P U A\ .\ sin\ Q O M}{sin\ U Q O} =$$

$$1{\cdot}45\ \frac{sin\ 20^\circ\ 5'\ .\ sin\ 17^\circ\ 21'}{sin\ 2^\circ\ 44'} = 3{\cdot}08 \text{ inches.}$$

$$L U = O U \frac{cos\ P U A\ .\ sin\ Q O M}{sin\ U Q O} =$$

$$1{\cdot}45\ \frac{cos\ 20^\circ\ 5'\ sin\ 17^\circ\ 21'}{sin\ 2^\circ\ 44'} = 8{\cdot}53 \text{ inches.}$$

$\therefore L N_2 = L U - N_2 U = 8{\cdot}53 - 1{\cdot}55 = 6{\cdot}98$ inches.

Then, $S L = L N_2 \tan S N_2 L = L N_2 \tan H X A = 6{\cdot}98 \times {\cdot}5 = 3{\cdot}49$ inches.

$$\therefore Q S = S L - Q L = 3{\cdot}49 - 3{\cdot}08 = {\cdot}41$$

If C be the circle of least confusion midway between P and Q, and C T be drawn perpendicular to the axis to meet N_2 R on T, we get

$$C T = \tfrac{1}{2}(P R + Q S) = \tfrac{1}{2}({\cdot}24 + {\cdot}41) = {\cdot}32 \text{ inch.}$$

$$\therefore \text{Distortion} = - {\cdot}32 \text{ inch.}$$

The principal focal length of the lens is 6 inches, hence the principal focus F is very near to M on the side remote from the lens.

Mathematical readers will easily find that the image is curved away from the lens, and that its radius of curvature is very nearly 9 inches.

II.—Chromatic Aberration

80. We have seen (§ 16) that when white light is refracted rays of light of different colours are differently deviated, the deviation being greatest for violet and least for red rays; and the angle between a standard ray and any other, after refraction, was called the dispersion of that ray.

Since lenses act by refraction, they will exhibit the phenomenon of dispersion, with the result that the rays of different colours will not always come to a focus at the same point.

This can be seen from the consideration of the expression for the focal length of a lens—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

In this μ, the refractive index, is different for different rays, being greater for violet than for red, showing that the focal length is less for violet than for red rays.

The effect of this is that a single lens does not form one picture, but several of different colours at different

distances, the violet one being the nearest to the lens. This can be easily tested by exploring the cone of rays

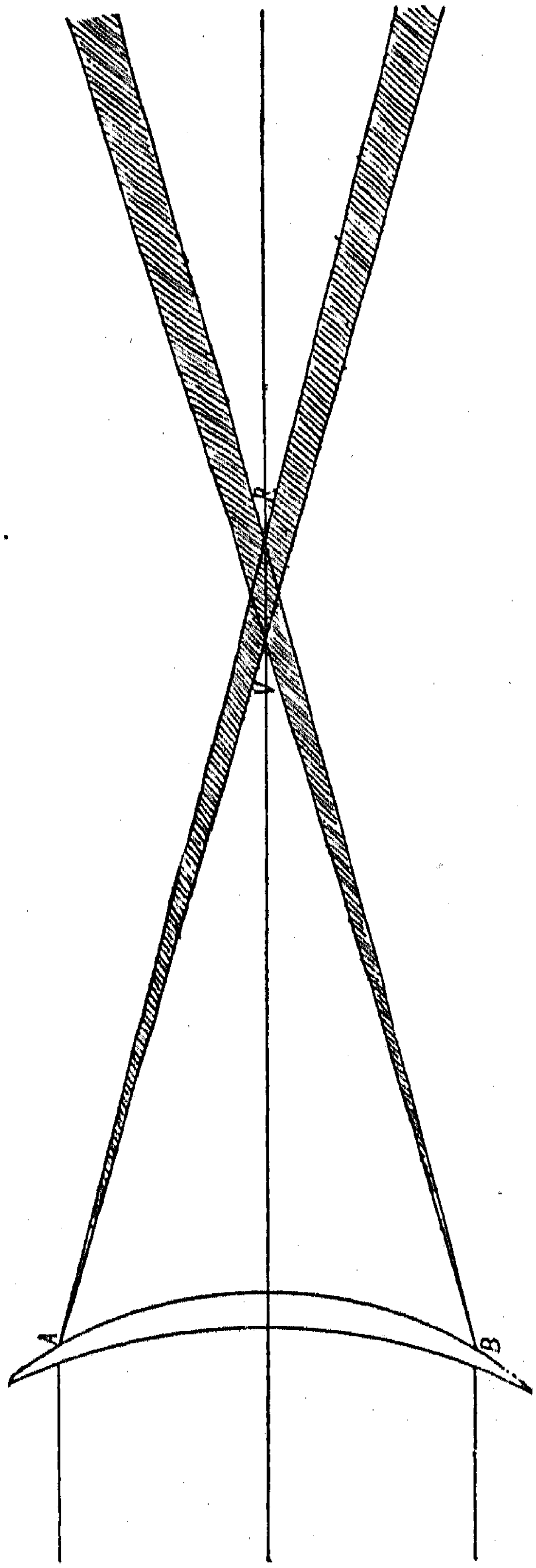

FIG. 59.

formed by a totally uncorrected lens, when white light from a point on its axis falls on it.

If the screen be placed near the lens the circular patch of light is white in the middle, but edged with red; as the screen is moved further away the patch contracts, and careful observation shows evidences of the foci for the different colours; and beyond the foci the circular patch is white in the centre, but edged with violet.

An examination of Fig. 59 will show that this is what we should expect. The cones formed by the violet and red rays are shown; V is the geometrical focus for violet light, R that for red light.

For points nearer to the lens than V, the cone of red rays is the larger, giving a white patch on the middle where the colours overlap with an edge of red; beyond R on the other hand the violet cone overlaps, giving a white centre with violet margin.

The state of affairs between V and R cannot be accurately stated, as it is complicated by spherical aberration, but careful observation with a particular large lens showed that when the screen was moved away from the lens through V and R, small discs of various colours appeared on it.

In this description, for the sake of simplicity two colours only have been considered, but the reader will have no difficulty in imagining the actual state of affairs when the intermediate rays are taken into account; it will not be very different from that described.

In the early days of photography most of the lenses used showed Chromatic Aberration, and as the rays affecting the eye are different from those most effective on the photographic plate, it was necessary, after obtaining the visual focus, to give the plate a slight shift to make the resulting picture sharp.

This was of course an inconvenience, but so long as plates were sensitive only to a very small part of the solar spectrum it was not an insuperable objection; but now that plates have been made sensitive to a much

wider range of rays the adjustment will no longer do what is required.

81. Irrationality of Dispersion.—The problem in hand is to destroy, by the use of two or more lenses, the dispersion of the various rays, without at the same time destroying their deviation; or in other words to make the rays of all colours, coming from one point of the object, coincide after their passage through the lens without at the same time destroying the converging or diverging power of the lens. Newton, misled by an experiment, believed that this was impossible, and his opinion being generally accepted, long hindered the improvement of lenses, and turned the attention of opticians from refracting to reflecting telescopes.

The mistake was discovered by Dollond, who produced the first achromatic lenses; it is said to have been previously discovered by a Mr. Hall of Worcester, but this is doubtful.

To understand the difficulty, consider a ray of light passing successively through two prisms of the same material and of equal angles, turned in opposite directions with their adjacent faces parallel (Fig. 60);[1] let the ray of white light P Q fall on the first prism, and let S S′ and T T′ be the emergent red and violet rays respectively; the angles of incidence of these rays on the second prism will be the same as their angles of emergence from the first. The second prism will therefore produce in each ray a deviation equal to that caused by the first prism, but in an opposite direction, and the rays will emerge parallel, but parallel also to P Q, and though the dispersion is corrected, the deviation is destroyed also.

The ratio of the deviation to the dispersion is the same in both lenses, so that whenever we destroy the one we also destroy the other.

The mistake Newton made was thinking that the ratio of dispersion to deviation is the same for all

[1] See Glazebrook's *Physical Optics*, 2nd Ed., p. 219.

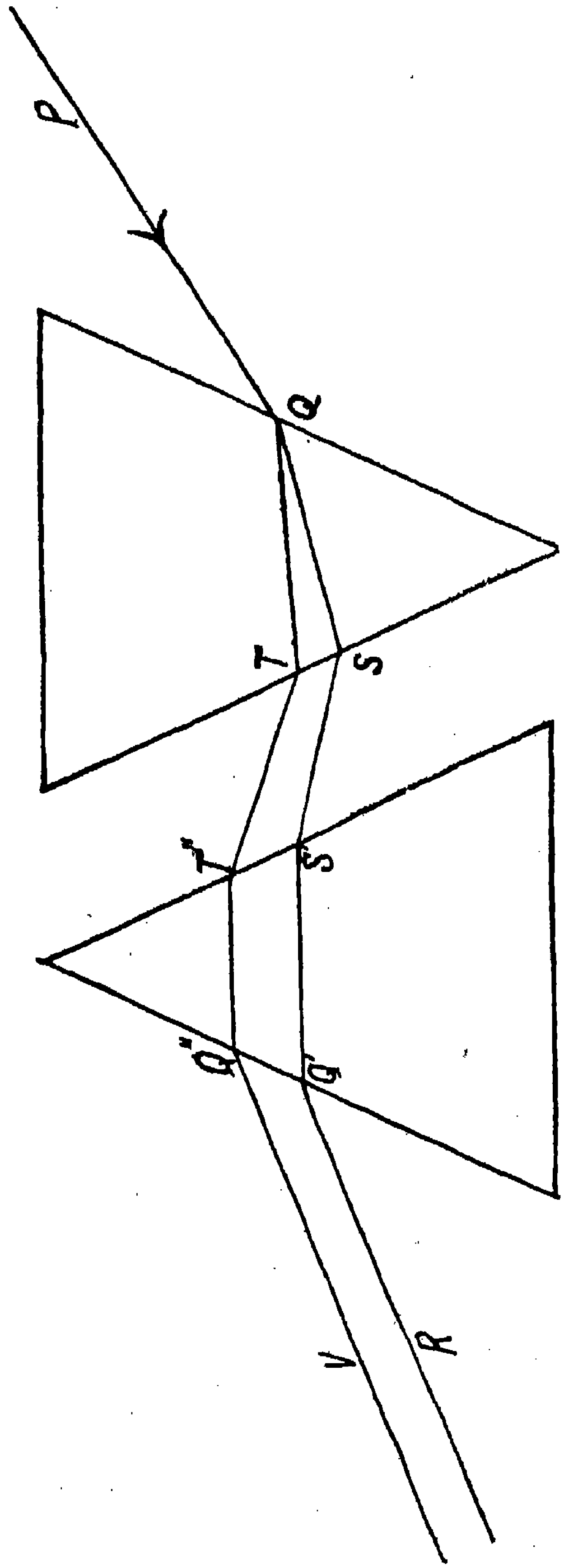

FIG. 60.

substances. Dollond showed that by using two prisms of different materials, for instance of crown glass and

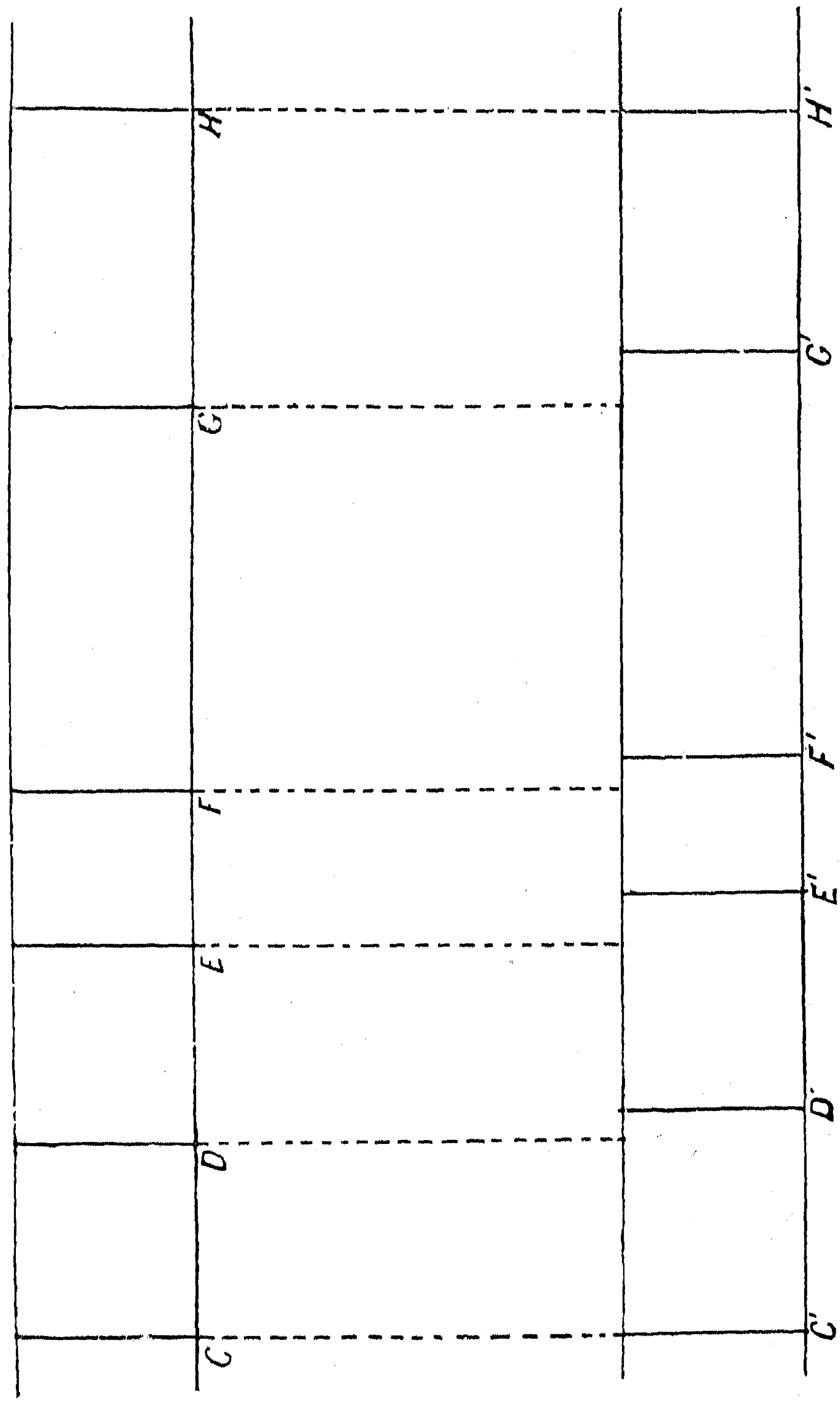

FIG. 61.

flint glass, the dispersion could be destroyed but a considerable deviation left.

Flint glass has a much higher ratio of dispersion to deviation than crown glass ; a prism of crown glass of an angle 60° will produce the same dispersion between the red and violet rays as a prism of flint glass with an angle of 37°, but the deviations are by no means the same in the two cases.

If prisms of crown and flint glass be arranged to produce spectra placed one over the other, for comparison (Fig. 61), so that the lines C and H of the solar spectrum coincide, the remaining portions of the spectra will not be found to be at all identical ; for instance, the lines D and E in each will not coincide. Thus, if two lines of the spectra coincide the intermediate lines do not do so too.

This phenomenon is called the Irrationality of Dispersion.

The effect of this irrationality is, that although the rays of two particular colours may be made to have the same deviation, yet rays of other colours will have different deviations, and the emergent light will be tinted and exhibit what is called a Secondary Spectrum.

If three prisms be used, rays of three different colours may be made to coincide, and the emergent light will be much less coloured than when two only are employed.

Coddington in his treatise on optics gives a table (p. 181) of refractive indices for different lines of the Solar Spectrum extracted from a paper of Fraunhöfer, and this will serve as an illustration of the statements made above.

The absence of regularity in the dispersion of these substances is illustrated by Coddington in the following table (p. 182), which contains the differences of the numbers in the last, exhibiting the intervals between the fixed lines in the several spectra.

The last column, it should be noticed, gives the difference between the refractive indices for the extreme lines, B and H, considered.

Refracting Medium	μ						
	B	C	D	E	F	G	H
Water	1·3309	1·3317	1·3336	1·3358	1·3378	1·3413	1·3442
Solution of Potash	1·3996	1·4005	1·4208	1·4056	1·4081	1·4026	1·4164
Spirit of Turpentine	1·4705	1·4715	1·4744	1·4783	1·4817	1·4882	1·4939
Crown Glass No. 13	1·5243	1·5253	1·5280	1·5314	1·5343	1·5399	1·5447
,, ,, ,, 9	1·5258	1·5268	1·5296	1·5330	1·5360	1·5416	1·5465
,, ,, letter M	1·5548	1·5559	1·5591	1·5631	1·5667	1·5735	1·5794
Flint Glass No. 3	1·6020	1·6038	1·6085	1·6145	1·6200	1·6308	1·6404
,, ,, ,, 30	1·6236	1·6255	1·6306	1·6373	1·6435	1·6554	1·6660
,, ,, ,, 23	1·6266	1·6284	1·6336	1·6405	1·6468	1·6588	1·6697
,, ,, ,, 13	1·6277	1·6297	1·6350	1·6420	1·6482	1·6603	1·6710

Refracting Medium	$\mu-1$	BC	CD	DE	EF	FG	GH	BH
Water	3309	8	19	22	20	35	29	133
Solution of Potash .	3996	9	23	28	25	45	38	168
Spirit of Turpentine	4705	10	29	39	34	65	47	234
Crown Glass . . 13	5243	10	27	34	29	46	48	204
,, . . 9	5258	10	28	34	30	56	49	207
,, . . M	5548	11	32	41	36	68	59	246
Flint Glass . . 3	6020	18	47	60	55	108	96	384
,, . . 30	6236	19	51	67	62	119	106	424
,, . . 23	6266	18	52	69	63	120	109	431
,, . . 13	6277	20	53	70	62	121	107	433

The action of prisms has been considered here in place of that of lenses, for the nature of the phenomenon is similar in the two cases, and prisms are easier than lenses to think about.

82. Chromatic Aberration of a Thin Lens.—In this case we shall consider only a central pencil of rays, parallel to the axis; we have seen that the principal focal length f of the lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

Now let μ_1 and μ_2 be the refractive indices for red and violet rays respectively, and let f_1 and f_2 be the corresponding focal lengths, then

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{r} - \frac{1}{s}\right) \text{ and } \frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

For brevity denote

$$\frac{1}{r} - \frac{1}{s} \text{ by } \frac{1}{\rho}$$

$$\therefore f_1 = \frac{\rho}{\mu_1 - 1}, \quad f_2 = \frac{\rho}{\mu_2 - 1}$$

$\therefore$ The distance between the two foci, or the chromatic aberration of the lens, is

$$f_1 - f_2 = \rho\left(\frac{1}{\mu_1 - 1} - \frac{1}{\mu_2 - 1}\right) = \rho\,\frac{\mu_2 - \mu_1}{(\mu_1 - 1)(\mu_2 - 1)}$$

Now if μ represent the mean value of the index of refraction, we may without serious error assume that

$$(\mu_1 - 1)(\mu_2 - 1) = (\mu - 1)^2$$

Hence

$$f_1 - f_2 = \rho \frac{\mu_2 - \mu_1}{(\mu - 1)^2} = \frac{\rho}{\mu - 1} \cdot \frac{\mu_2 - \mu_1}{\mu - 1}$$

The quantity $\frac{\mu_2 - \mu_1}{\mu - 1}$ is called the *Dispersive Power* of the medium, and is often denoted by $\hat{\omega}$, and $\frac{\rho}{\mu - 1}$ is evidently $\frac{1}{f}$ where f is the mean focal length.

Hence we get

Chromatic Aberration = Mean focal length $\times$ dispersive power,

$$\text{or } f_1 - f_2 = \hat{\omega} f.$$

This expression is of great importance.

The quantity $\hat{\omega}$, the dispersive power of the substance, is the form in which the difference between the refractive indices for the different rays enters into the calculations.

83. Chromatic Aberration for a Thick Lens.—In this case the calculation is not quite so simple; and we must here remember that the positions of the nodal points depend on the refractive index, and hence vary for different rays.

We must therefore measure our distances from the surfaces of the lens; let E be the distance of the principal focus from the back surface, and let the symbols have their usual meanings (§ 44); then

$$\frac{1}{\mathrm{E}} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) - \frac{e}{\mu} \cdot \frac{(\mu - 1)^2}{r^2}$$

From this it can be proved, that if $\hat{\omega}$ is the dispersive power, the value of the chromatic aberration is

$$\hat{\omega}\left\{\mathrm{E} + \frac{e\,\mathrm{E}^2(\mu - 1)^3}{\mu^2 r^2}\right\}$$

Examples.—Find the Chromatic Aberration for a thin lens where $r = -7$ inches, $s = 5$ inches, $\mu = 1{\cdot}524$, $\hat{\omega} = {\cdot}0102$; also for a lens of thickness $e = {\cdot}2$ inch.

(*a*) Thin lens.

$$\text{Here } \frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = {\cdot}524 \left(-\frac{1}{7} - \frac{1}{5}\right)$$
$$= -{\cdot}179, \therefore f = -{\cdot}5{\cdot}58 \text{ inches.}$$

$\therefore$ Aberration $= \hat{\omega} f = -{\cdot}0102 \times 5{\cdot}58 = -{\cdot}057$ inch.

(*b*) Thick lens.

$$\text{Here } \frac{1}{\mathrm{E}} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) - \frac{e(\mu - 1)^3}{\mu r^2} = -{\cdot}179$$
$$-{\cdot}001 = {\cdot}180 \therefore \mathrm{E} = -5{\cdot}56$$

$$\therefore \text{Aberration} = {\cdot}0102\left\{-5{\cdot}56 + \frac{{\cdot}2 \times 30{\cdot}9 \times {\cdot}143}{2{\cdot}39 \times 49}\right\}$$
$$= {\cdot}0102\left\{-5{\cdot}56 + {\cdot}007\right\} = -{\cdot}056 \text{ inch.}$$

Comparing these results we see that they differ only by one thousandth of an inch, and thus the aberration is practically the same for both cases.

When e the thickness is small compared with the radii of curvature of the faces we may neglect its effect and take the aberration to be the same as that for a thin lens with the same radii of curvature, with a sufficient approach to accuracy for all practical purposes.

84. Condition that two Lenses in Contact should form an Achromatic Combination.—Let r_1 s_1 be the radii of the first lens, and μ_1, μ_1' its refractive indices for the two rays which are to be combined, and let r_2, s_2, μ_2, μ_2' be the corresponding quantities for the second lens, and let

$$\frac{1}{\rho_1} = \frac{1}{r_1} - \frac{1}{s_1}, \frac{1}{\rho_2} = \frac{1}{r_2} - \frac{1}{s_2}$$

Then the focal lengths of the lenses are (§ 82)

$$\frac{\rho_1}{\mu_1 - 1} \text{ and } \frac{\rho_2}{\mu_2 - 1}$$

and (§ 40) the focal length F of the combination is given by

$$\frac{1}{\mathrm{F}} = \frac{\mu_1 - 1}{\rho_1} + \frac{\mu_2 - 1}{\rho_2} .$$

Let u and v be the distances of object and image from the lens for rays of both colours; we therefore get

$$\frac{1}{v} - \frac{1}{u} = \frac{\mu_1 - 1}{\rho_1} + \frac{\mu_2 - 1}{\rho_2} \text{ also}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{\mu_1' - 1}{\rho_1} + \frac{\mu_2' - 1}{\rho_2}$$

Hence we must have for achromatism

$$\frac{\mu_1' - 1}{\rho_1} - \frac{\mu_1 - 1}{\rho_1} + \frac{\mu_2' - 1}{\rho_2} - \frac{\mu_2 - 1}{\rho_2} = 0$$

$$\text{or } \frac{\mu_1' - \mu_1}{\rho_1} + \frac{\mu_2' - \mu_2}{\rho_2} = 0$$

Let ν_1, ν_2 be the mean refractive indices, then

$$\frac{\mu_1' - \mu_1}{\nu_1 - 1} \times \frac{\nu_1 - 1}{\rho_1} + \frac{\mu_2' - \mu_2}{\nu_2 - 1} \cdot + \frac{\nu_2 - 1}{\rho_2} = 0$$

and if f_1, f_2 are the mean focal lengths this becomes

$$\frac{\hat{\omega}_1}{f_1} + \frac{\hat{\omega}_2}{f_2} = 0 \quad . \quad . \quad . \quad (a)$$

$\hat{\omega}_1$, $\hat{\omega}_2$ being as before the dispersive powers of the glass of which the lenses are made.

This determines the ratio $f_1 : f_2$ of the mean focal lengths of the lenses; if the focal length of the required lens is to be F, we have also

$$\frac{1}{\mathrm{F}} = \frac{1}{f_1} + \frac{1}{f_2} \quad . \quad . \quad . \quad . \quad (b)$$

by themselves to determine the *four* radii, and another condition can therefore be introduced to provide the remaining equation required.

The condition most usually imposed is that of Clairaut, who made the radii of the two adjacent lenses the same, so that they could be cemented; d'Alembert proposed to use it to improve the definition at points not on the axis; while J. W. Herschel used it to correct the aberration for rays coming from a point on the axis at a finite distance, as well as for rays parallel to the axis.

When the four conditions are obtained the four radii can theoretically be found, though the algebraical work is heavy.[1]

92. Indirect Method.—Another method is sometimes adopted; a rough approximation to the lens required is taken as the starting-point. The course of a ray parallel to the axis, and cutting the lens at a given distance from the axis, is calculated by trigonometry, the ray taken being intermediate in colour between the extreme rays, and the point where this ray cuts the axis is found. The corresponding point for rays very near the axis is then found; if these two cut the axis in very near points, similar calculations are then made for rays of the extreme colours used. If all the points thus found are very close together the lens is satisfactory; if not the computer judges what kind of changes must be made in the curvatures of the surfaces, and repeats the calculations.

This process is continued until by successive approximations a satisfactory result is arrived at.

This method appears long and cumbersome, but it is probably not so arduous as it appears, for a computer who has an instinct for his work acquires to a remarkable degree a knowledge of all parts of the calculation, which enables him to estimate the nature of the change

[1] See Martin's paper quoted above.

in the result which various changes in the data would produce.

93. Three or more Lenses.—When more than two lenses are employed there will be more quantities to be determined, thus making it possible to satisfy more conditions, such as rendering the field of view flat, correcting aberration at points not on the axis, and so on.

An arrangement recommended by M. Martin is to make the nodal points of the lens which is equivalent to the system coincide, so that the combination shall act like a single thin lens.

The details of the calculations are very intricate, and can be properly grasped only by one well used to such work; no general method of procedure can be given, as each case must be considered on its own merits.

94. Materials.—Besides the calculations to which we have alluded, the lens designer must also take account of the various kinds of glass which he can obtain, for by the use of suitable glasses many results can be attained which were impossible by the choice of curvatures alone.

Of late years Schott of Jéna has succeeded in producing a series of glasses, giving a very wide range of properties, and it has in consequence become possible to construct lenses which can be used with a much larger aperture than formerly.

Dr. Paul Rudolph of Jéna has published a very interesting fact about Jéna glass.

In an achromatic combination made of two lenses, it is necessary, to secure convergence, that the convergent lens should have the least dispersive power; for in § 84 we found that if f_1 f_2 are the focal lengths of the component lenses, $\hat{\omega}_1$ $\hat{\omega}_2$ their dispersive powers, and F the focal length of the combination,

$$\frac{\hat{\omega}_1}{f_1} + \frac{\hat{\omega}_2}{f_2} = 0 \quad (a), \quad \frac{1}{\mathrm{F}} = \frac{1}{f_1} \times \frac{1}{f_2} \quad (b)$$

From (a) we see that since the dispersive powers are positive, f_1 and f_2 must be of opposite signs, or one lens convergent, the other divergent; let f_2 represent the convergent lens, it will then be negative. If the system is to be convergent F must be negative, and hence we must have $f_1 > f_2$ numerically; and hence from (a) $\hat{\omega}_1 > \hat{\omega}_2$ or the convergent lens has the least dispersive power.

On inspection of the tables in § 81 it will be seen that the refractive index and the dispersive power increase together, for if we calculate the dispersive powers for the lines B, H for the glasses mentioned in these tables we get

Refracting Medium.		μ — I.	B, H.	Dispersive Power.
Crown Glass	13	·5243	·0204	·0390
Ditto	9	·5258	·0207	·0394
Ditto	M	·5548	·0246	·0443
Flint Glass	3	·6020	·0384	·0638
Ditto	30	·6236	·0424	·0680
Ditto	23	·6266	·0431	·0688
Ditto	13	·6277	·0433	·0690

where the quantity given in the B, H column is the difference between the refractive indices for the lines B and H.

Hence the convergent lens when made with the old materials must have not only the lesser dispersive power, but also the lesser refractive index.

But the most favourable arrangement for correcting astigmatism in a doublet, is that in one of the two elements the convergent lens should have the greater index of refraction; this was impossible, as we have seen, with the old glasses, but has been rendered possible by the use of Jéna glass, in which the dispersive power does not necessarily increase with the index of refraction.

95. Flare Spot.—This defect consists of a bright

patch of light in the centre of the field, and it is due to reflections at the surfaces.

It is a fact, well known by experiment, that when a ray of light passes from one medium to another, both reflection and refraction take place, the quantity of light reflected being as a rule greater the larger the angle of incidence.

Since the incident pencil is limited by the diaphragm the flare spot may in some sense be regarded as the image of the diaphragm ; an example of this is shown in Fig. 62. Here C D is the aperture in the diaphragm ; the incident rays being parallel converge to the principal focus F, and the plate E H cuts the axis at this point ; the reflected rays are shown by dotted lines, these, after two reflections inside the lens are refracted out, giving a cone the section of which by the plate is a circle with A B as diameter. The patch represented by A B is therefore the flare spot.

The figure represents the case of a single lens, but since with a combination there are more reflecting surfaces the danger of a flare spot will be greater, and there may be more than one such spot.

The intensity of illumination of a flare spot can never be very great, because it is formed by two reflections at least, at each of which a great deal of the light is refracted ; still it may be enough, specially if the spot be small, to spoil the picture.

96. Correction of the Flare Spot.—In the case of the thin lens (Fig. 62), it is obviously of no use to move the diaphragm, the incident rays being parallel ; but in a compound lens, where the diaphragm is between the two elements, some alteration can be effected by moving it, for the rays incident on the second lens are not parallel to the axis.

But when a lens exhibits a bad flare spot, very little can be done to get rid of it, and the design must be reconsidered.

Since reflections always take place inside the lens, the

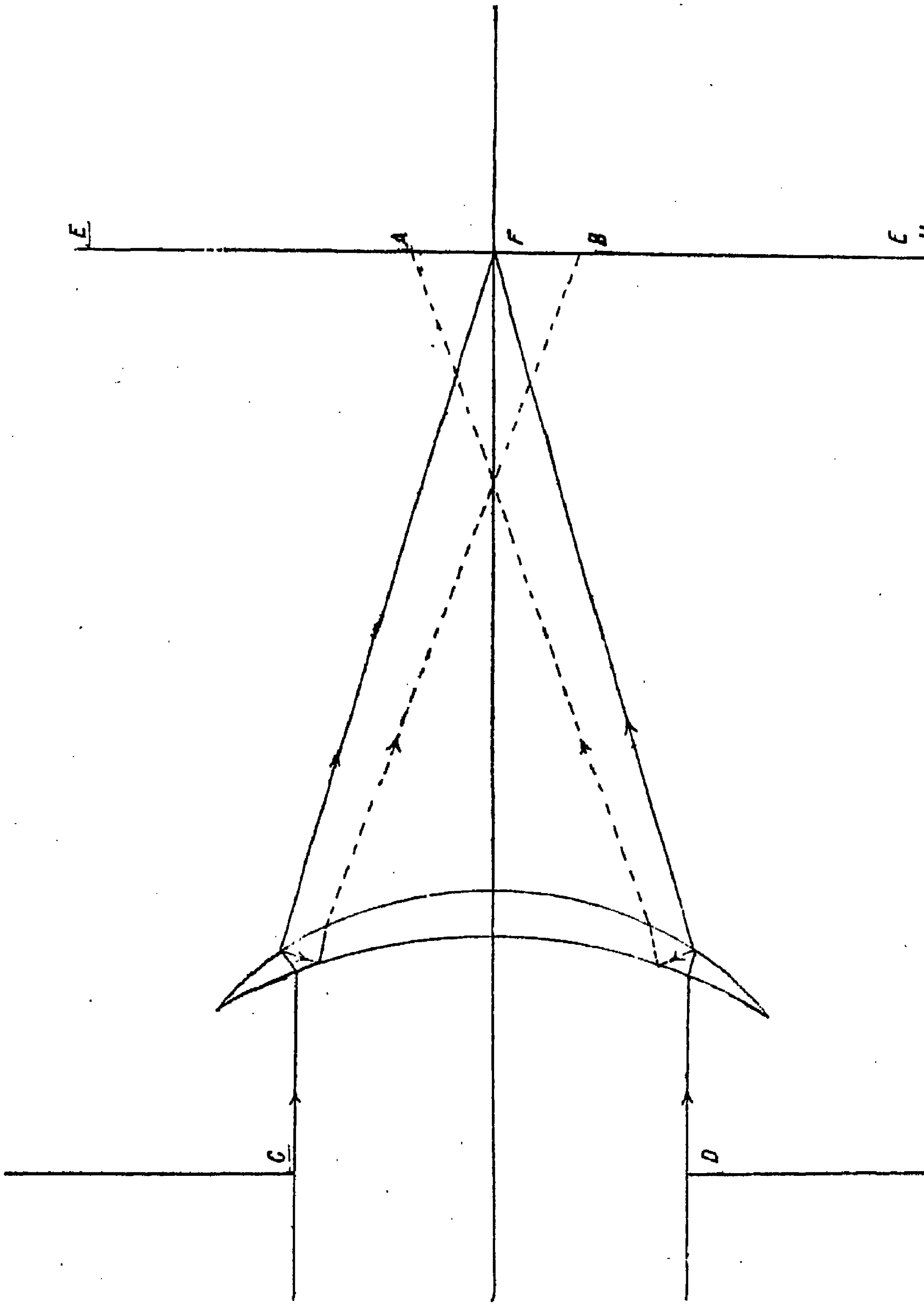

FIG. 62.

formation of a flare spot cannot be avoided, but it may be possible so to construct the lens, that the spot may

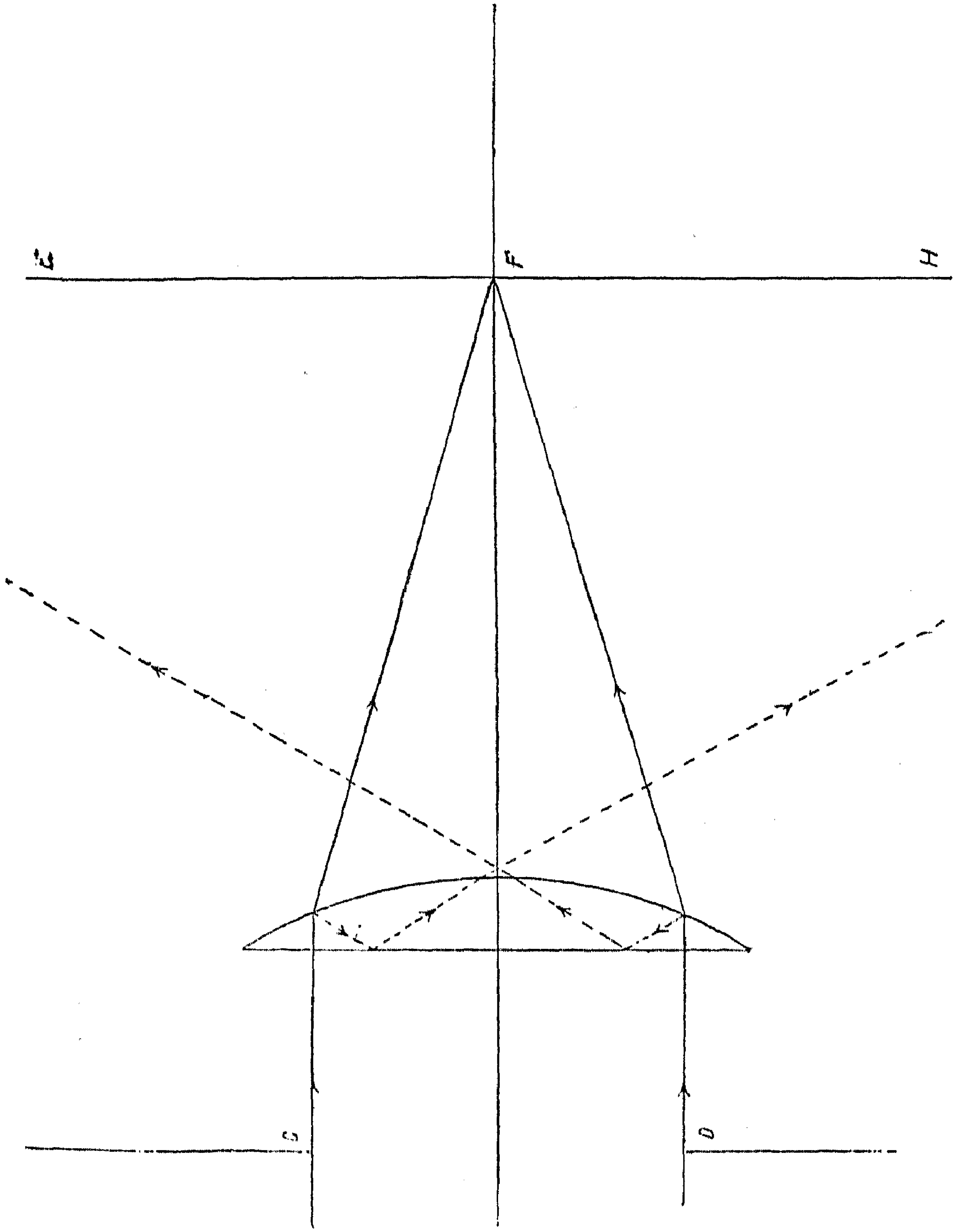

FIG. 63.

be very large. The effect of this is two-fold :—Firstly, the flare spot covers the whole plate so that all parts are

affected equally, and secondly, since the light is spread over a large area, its intensity is much diminished. An example is given in Fig. 63, which corresponds to Fig. 62; the flare spot is here so large that it cannot be indicated in the figure.

97. Correction of Spherical Aberration and Astigmatism by means of a Diaphragm.—The aberration and astigmatism of a lens already constructed may be lessened by means of a diaphragm. It has been shown that as long as the greatest breadth of the section of a pencil of light by the plate does not exceed a length which we have called 2ϵ, the resulting patch of light will appear to be a point. The section of such a pencil if anywhere too broad can obviously be reduced by reducing the size of the incident pencils by a diaphragm. The proper position of the diaphragm needs some consideration; in most cases this can be found only by experiment or calculation, but a few general remarks may be made.

When the lens is to be used with a very small angle, the diaphragm may be put close to the lens, which has the effect of making all but the central portion of the lens ineffective; but if the lens have a fairly large angle, a diaphragm close to it will give very bad definition for pencils at all oblique. This is to be expected, for we have remarked (§74), that the greater the inclination of the incident ray to the normal to the surface, the greater will be the astigmatism. It will therefore be of advantage to place the diaphragm at a little distance from the lens, for a little examination will show that the angle of incidence of the pencil at the first surface is always less than in the former case.

In the particular case of a meniscus lens, the concave surface should be turned to receive the incident light, and the diaphragm should be placed at the centre of curvature of the surface (Fig. 64); the effect of this is that all the incident pencils strike the first surface

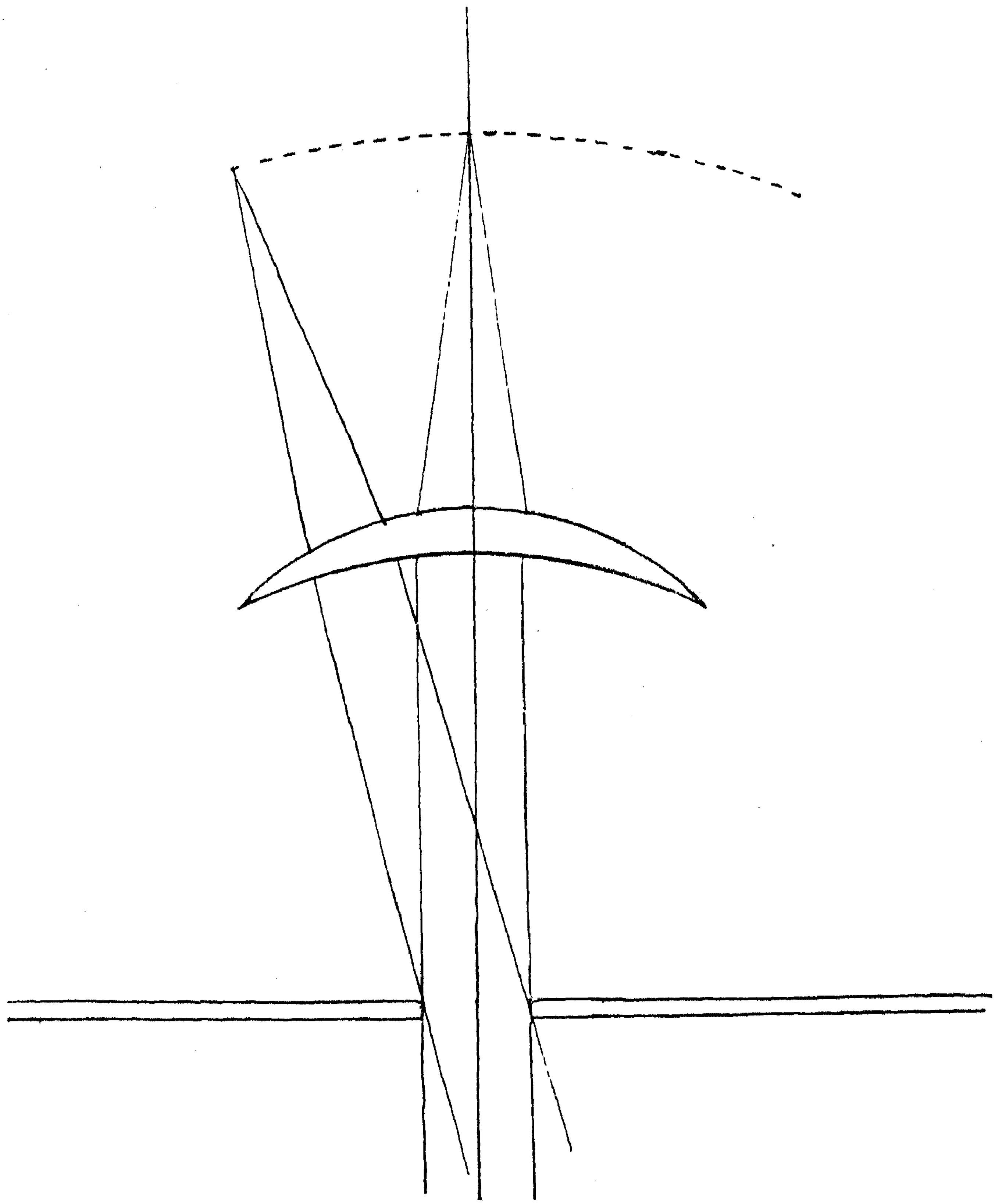

FIG. 64.

normally, and since the pencils are small, very little aberration is caused by the first refraction.

98. Depth of Focus.—When a diaphragm is used to reduce the size of pencils, if the most oblique pencils are small enough, those less oblique will be smaller than they need be. If any one of these latter pencils fall on a screen which is moved about, there will be a considerable length in which the greatest breadth of the cross section is less than 2ϵ, the greatest breadth permissible for definition.

Imagine now that along every secondary axis are marked off the extreme points at which the greatest breadth of the section is less than 2ϵ; if this is done for all the pencils the two series of points will lie on two surfaces, which will enclose a volume at all points of which the definition will be good enough.

Hitherto we have practically assumed that to get a good picture the circles of least confusion of all the small pencils must lie on the plate, but now we see that if the plate lie within the focal volume, the picture will appear sharp all over, even if the surface containing the circles of least confusion is not nearly a plane. Besides this, since the picture is sharp as long as the plate is within the focal volume, we shall, if the plate is well inside this volume, be able to move it about inside it, and thus have a certain latitude of position in focussing; or in other words, the use of the diaphragm gives depths of focus.

In Fig. 65, N_1, N_2 are the nodal points (the lens not being shown), C A D and C B D are the sections of the surfaces on which lie the extreme positions on the secondary axis as defined above, the included area is the section of the focal volmme; C F D is the section of the surface on which lie all the circles of least confusion.

If a plate occupy the position indicated by the dotted line, and its diagonal is not greater than P L, it will lie entirely within the focal volume, and the picture will be sharp all over it. If the diagonal of the plate be shorter than P L, the plate can be moved slightly

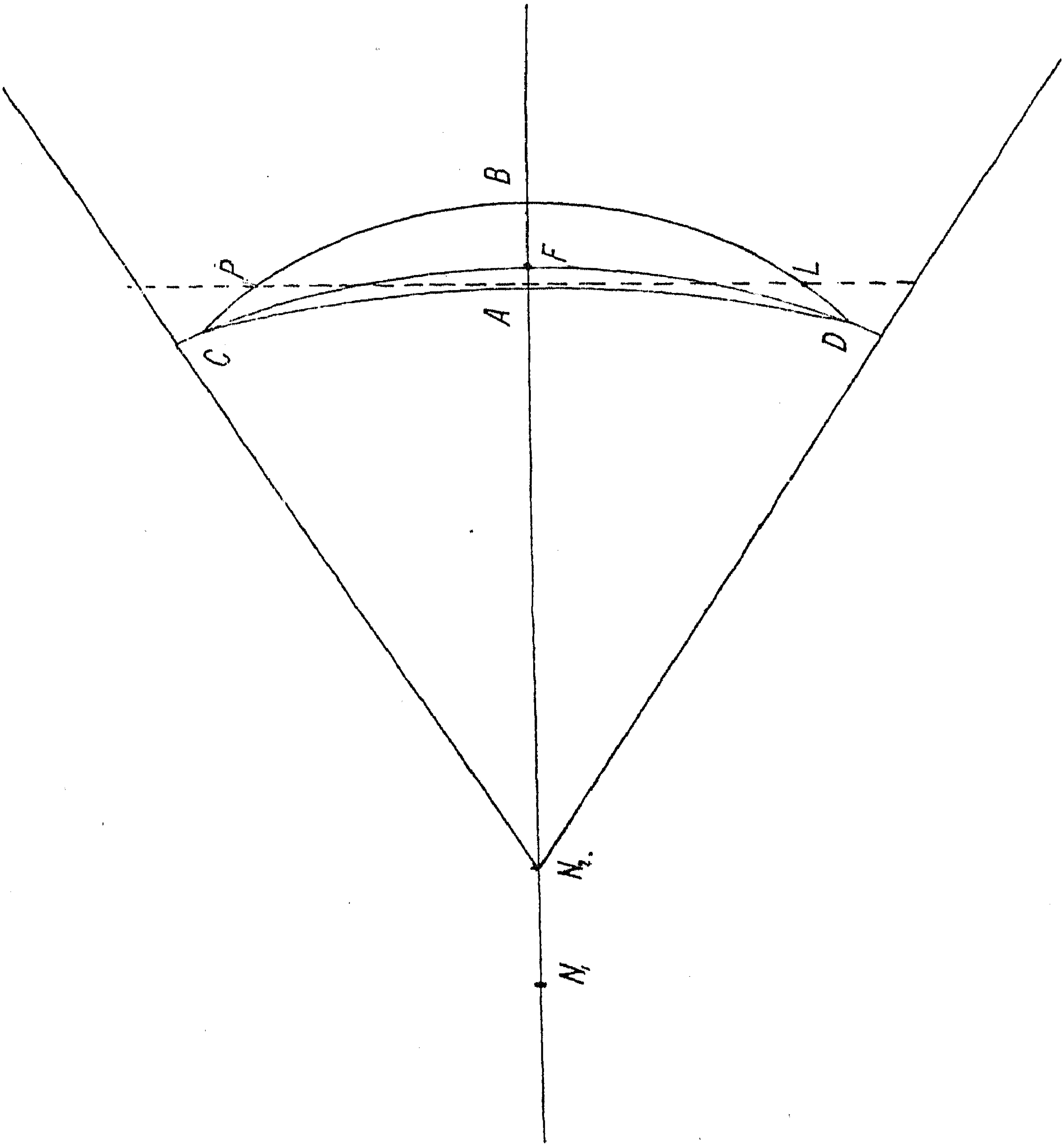

FIG. 65.

backwards and forwards without any portion emerging from the focal volume.

99. Correction of Distortion.—In § 77 it has been

explained at length that the use of a diaphragm gives rise to distortion; if the diaphragm be placed in front of the lens, straight lines near the edge of the portion of the object are represented by curved lines, which are *concave* towards the axis; if the diaphragm be placed behind the lens, the lines are curved and *convex* towards the axis.

To correct this defect two or more groups of lenses are used, the diaphragm being placed between two of the groups; the result of this is that the diaphragm being behind the front element, tends to make lines convex to the axis, but since it is in front of the other element, it tends to make them concave to the axis.

Hence, if the diaphragm be properly placed, the two tendencies correct each other, and the resulting lines are straight.

100.—We have now glanced briefly at the main points to be considered in correcting aberration and in designing lenses. It is impossible, within our present limits, to give any adequate idea of the subject; those who wish for more information should consult M. Martin's paper, previously quoted, on "la détermination des courbures des objectifs," where there is a history of the development of the subject, and many references to original papers; besides this, M. Martin gives the actual work in a particular case.

The chapter in Wallon's *L'Objectif Photographique* on the correction of aberrations may also be read with advantage.

CHAPTER V

LENS TESTING

101.—The examination of a lens falls naturally into two divisions: first, the determination of what may be called the constants of the lens; and secondly, testing for faults of workmanship and insufficient correction of the aberrations.

The most complete system of testing is that devised by Möessard,[1] who has invented for this purpose an instrument called the tourniquet; but within the last two or three years a system of lens testing has been established at Kew Observatory, the apparatus employed being a modification of the tourniquet. In Möessard's system no account was taken of the expense or of the time required to make the tests, the object being to examine a lens completely irrespective of other considerations, while, on the other hand, the object of the Kew system is to provide a really useful though not elaborate test for a reasonable charge.

Both systems will be described in turn, but first we shall give several methods for finding the most important constant of a lens, the principal focal length.

102. Measurement of Principal Focal Length.—We have defined the principal focal length as the distance between the principal focus of the lens and the nodal point of emergence; but this is not always what is given

[1] *Étude des Lentilles et Objectifs Photographiques*, par P. Möessard. Gauthier-Villars, Paris, 1889.

by the makers. Sometimes the distance from the principal focus to the diaphragm is given, sometimes the distance from the principal focus to the back surface of the lens, called the *back focus.*

For some purposes the distance between the principal focus and the diaphragm is near enough to the true focal length, but it will not do for calculations in connection with enlargements or reductions where accuracy is required, and for most purposes the back focus is quite misleading.

In the case of single lenses, which may be regarded as thin, and of symmetrical combinations, which may be regarded as equivalent to thin lenses placed at the diaphragm, either of the two following methods may be adopted.

(1) Focus a distant object on the ground glass, and then measure the distance between the ground glass and the lens, or in the case of a combination between the ground glass and the diaphragm; this is the principal length.

(2) Place upright in the front of the lens a foot rule or other divided scale, then by trial place and adjust the camera so that the image of the scale on the ground glass is of the same size as the object, which can be tested by measuring with a scale similar to that used as object. [It is of course not meant that the image of the *whole* of the scale must be got on the ground glass, but that the sizes of the divisions in the object and image should be equal.] The ground glass and scale are now conjugate foci, and since the sizes of the object and image are equal, they must be at equal distances from the centre of the lens.

The relation connecting u and v, the distances of object and image from the centre of the lens, we know to be

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

and the object and image being at equal distances on *opposite* sides of the lens,

$$v = -u$$

hence we get from the previous relation

$$-2/u = 1/f \text{ or } f = -u/2$$

but the distance apart of the ground glass and scale is $2u$, hence the focal length is one quarter of the distance between the ground glass and the scale when the object and image are equal.

In performing the experiment, it should be remembered that the object must be distant from the lens twice its focal length, or no sharp image can be produced; trouble and loss of time are often caused by placing the object too near the lens to begin with.

In many cases, the extension of the camera is not enough to allow the method to be adopted as described, but it is not hard to carry it out without the camera. Fix the lens in a suitable firm support which can be moved about on a table, also fix vertically in movable supports two similar divided scales; place one scale behind the lens, and then with an eye-lens such as watchmakers use, look for the image of the scale. When this image is found, place the other scale alongside of it, and examine to see if the divisions in the image are of the same lengths as those on the scale. If the two sets of divisions are not of the same length, they can be made so by moving the lens and scales about. When this adjustment has been made, remove the lens and measure the distance between the scales; one quarter of this is the focal length.

This method may seem harder than the former, in which the camera was employed, but it does not prove so in practice, and it is more satisfactory, for it is easier to measure the distance between two scales than between a scale and the ground glass.

(3) Another method which has the advantage of giving the true focal length, measured from the nodal

point, is to fix a scale in front of the camera as before, and to arrange it so that the image on the ground glass is of the same size as the object, then move the ground glass to focus up sharply some distant object; *the focal length required is equal to the distance through which the ground glass has been moved.*

For, in the first case, when object and image were equal, the ground glass was distant twice the focal length

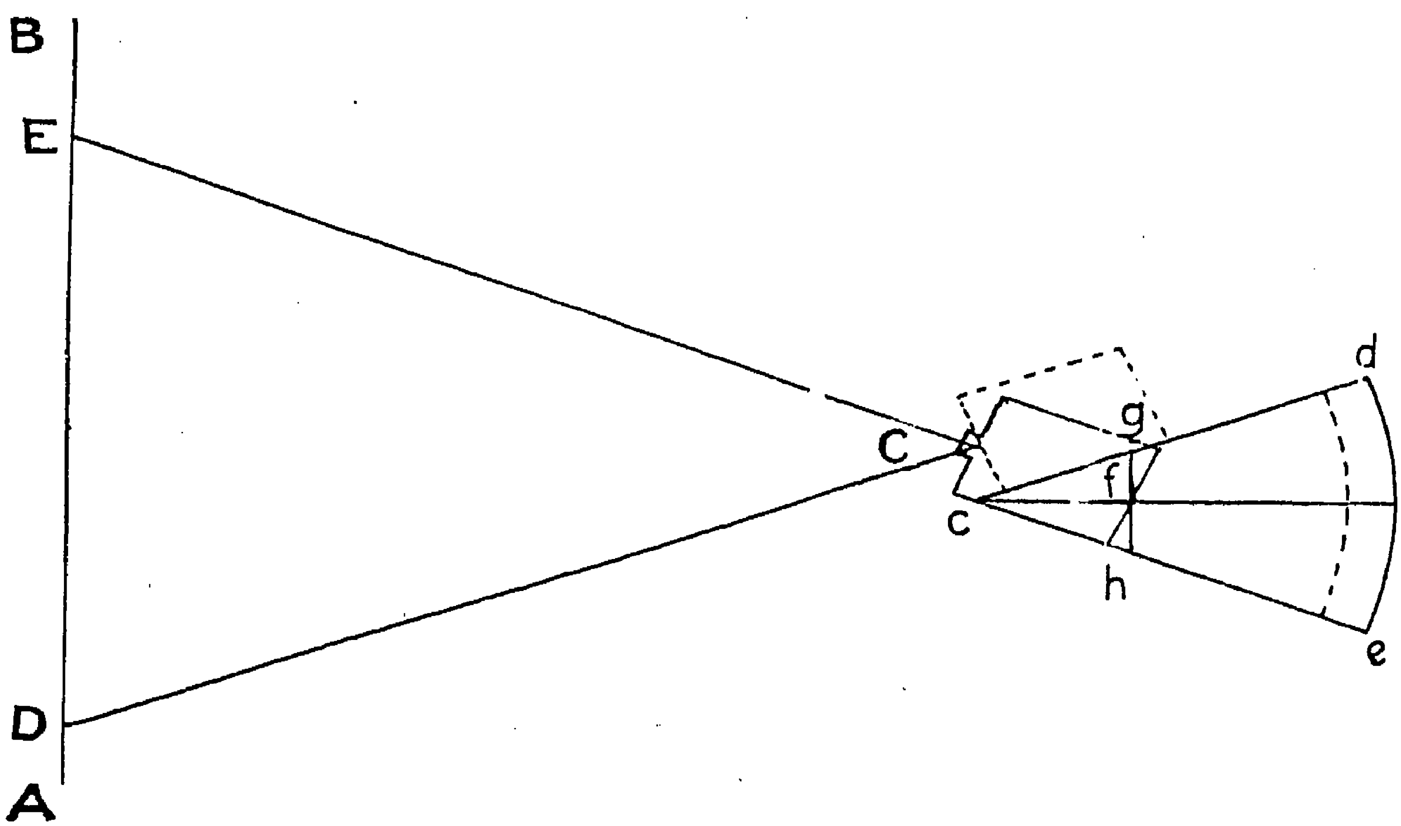

FIG. 66.

from the nodal point; and, in the second case, when the distant object was in focus, it was at the principal focus.

(4) The following method, due to Grubb, is quoted from Monkhoven's *Photographic Optics*:—

"Let A B, Fig. 66, be objects widely separated situated on the horizon; C the objective, screwed on to a camera placed on a well-levelled table. On bringing them to a focus on the ground glass, we find that the

objects D and E form the limits of the image on the ground glass. D C E is the angle included by the lens. Draw on the middle of the ground glass a vertical right line, and turn the camera until the point E falls on this line. With a pencil pressed against the side of the camera draw the right line *c e*. Turn the camera towards the point D until this point falls on the line traced on the ground glass. Draw the right line *c d* in the same way as *c e* was done. If this line does not cut *c e*, prolong it until it does. It is clear that the angle *e c d* is equal to D C E. Therefore, by placing the centre of a protractor at *c*, the number of degrees, *e d*, is read off; that is, the angle included by the lens.

"Its *absolute* focal length is thus obtained :—

"Measure on the ground glass the distance of the points with a compass, and take half of them. Bisect the angle *e c d* by a straight line *c f*, against which place a square rule. Carry the half-distance, D E (measured on the ground glass), on the square rule, and make *f g* equal to this half-distance, and perpendicular to *c f*. Then *f c* will be the true focal length of the objective, which, when once known, permits the size of images to be calculated."

(5) Several other methods are given in Wallon's *L'Objectif Photographique*, pp. 129—140, ed. 1891.

I. M. Möessard's System and the Tourniquet.

103. Desiderata.—The quantities determined and tests made are as follows :—

(1) The principal focal length and positions of nodal points.

(2) The form of the principal focal surface.

(3) The depth of focus, or the principal focal volume.

(4) Astigmatism.

(5) Distortion.

(6) The field of the lens.
(7) Brightness and transparency.
(8) Achromatism.

The first five and the last of these quantities have already been fully treated; the remaining two need a little explanation. By the *field of the lens* is meant in general the angle of the cone enclosing the largest space over which the objective will furnish a sharp picture; this cone has its vertex at the nodal point of emergence.

The *brightness* of the image depends on the *transparency* of the lens, which may be defined to be the ratio of the quantity of light which actually gets through the lens, to the quantity which would get through if the glass were removed, and the mounting and stop left unaltered; it is an important matter, for the brighter the image, the shorter will be the exposure. All lenses waste some of the light which falls on them, because of the reflection and scattering at the surfaces, and sometimes, if the glass is not of good quality, a considerable amount of light is absorbed. Möessard divides his operations into five experiments.

(1) Determination of the nodal points and principal focal length.

(2) Determination of the principal focal surfaces, depth of focus, astigmatism, maximum flat field.

(3) Measurement of distortion and of the field free from distortion.

(4) Measurement of transparency and field of equal brightness.

(5) Test for achromatism and determination of visual and chemical foci.

In the course of these five experiments, the faults of construction, such as bad mounting and centering, irregularities in the lenses, etc., are detected.

104. Description of the Tourniquet.—This apparatus resembles an ordinary camera in appearance; it consists (Fig. 67) of a carrier H, which can be worked with

rack and pinion, and is connected to a cubical box in front by bellows. On the carrier is a small ground

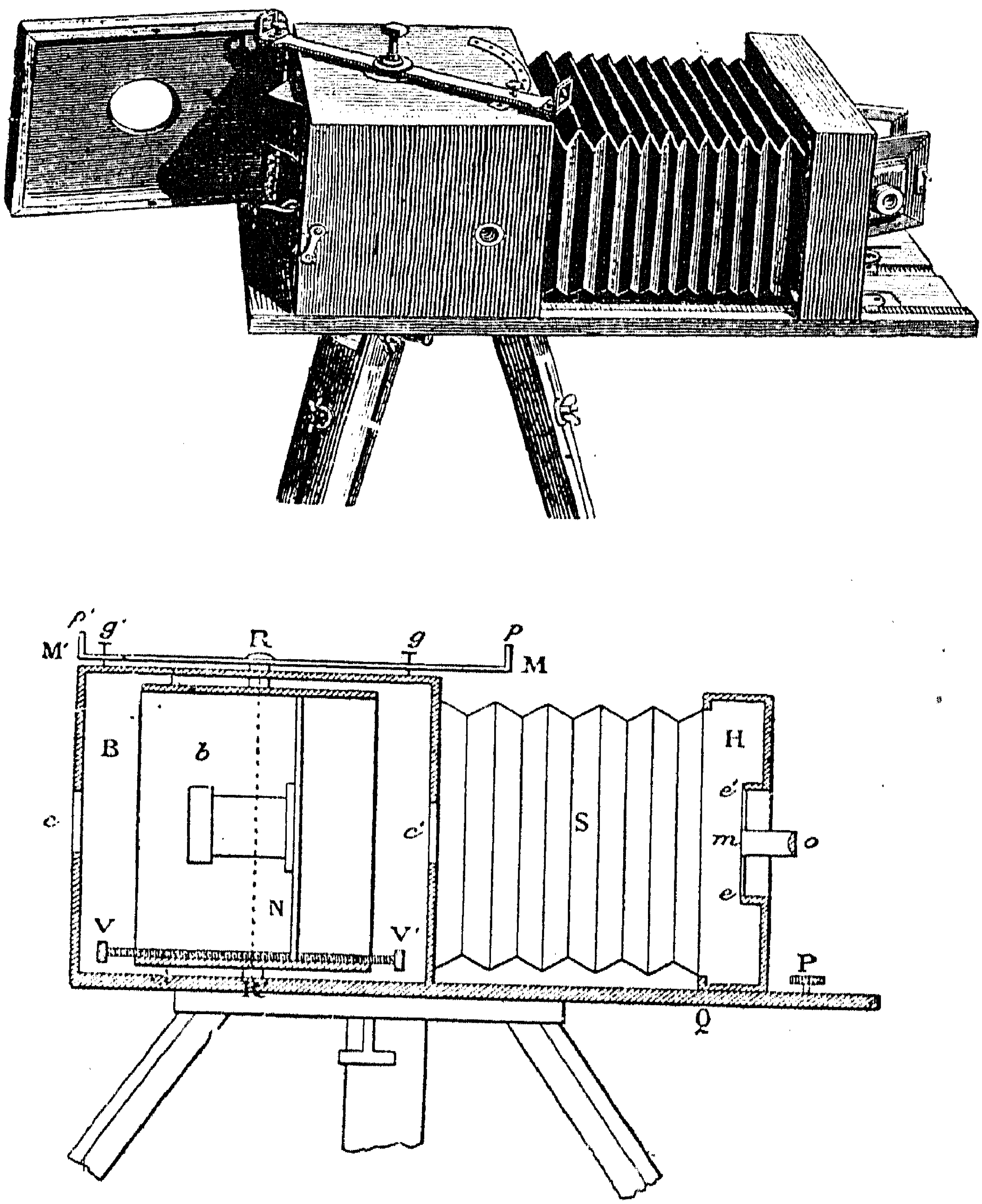

FIG. 67.

glass $e\,e'$, in a frame hinged at the side so that it can be thrown back; there is also hinged to it and opening

on the opposite side, a panel, carrying at its centre a micrometer scale m, divided to tenths of a millimetre, and furnished with a simple microscope to read it. Thus when the ground glass is folded back, and the micrometer is in place, the image can be viewed by the microscope, and its size accurately measured against the scale; the scale can be twisted through an angle of 90°, so that lines in all directions can be measured.

The cubical box in front has two small circular openings; one, e, in front, the other e', into the bellows; the front of the box can open, as shown in the figure, being hinged at the side. Inside the cubical box is a smaller one, open at two sides, which can turn about a vertical axis R R', and the dimensions are so chosen that it can turn right round. The axis R R' projects through the top of the outer fixed box, and can be moved from the outside by the metal arm M M'. Inside the movable box is a vertical panel, movable forward and backward by the screw V V'; to the centre of this panel is fixed the lens to be tested. To enable lenses of all sizes to be tested, several panels with various flanges are kept; also the panel is not quite as broad as the box, allowing some side-play, which is useful to centre the lens correctly.

The metal arm M M' carries at its ends sights by means of which it can be directed to any object required, and it is pierced at $g\,g'$ by two holes into which fit pins by which it can be fixed in the zero position parallel to the length of the apparatus. On the top of the box is fixed a circular arc with holes at equal angular distances, into which the pins through g can fit, which enables the lens to be set with its axis making various known angles with the axis of the tourniquet.

Lastly the base board along which the carrier slides is fitted with a scale and vernier to measure the distances through which it moves; the zero of the scale is at the centre of the axis R R'.

The whole apparatus is supported on a substantial tripod.

To centre the lens and make its axis pass exactly through the axis of rotation, the tool shown in Fig. 68 is used; this consists of a tube $t\,t'$ which fits closely the axis R, which is hollow; at the lower end is fixed a rider with its sides equally inclined to the vertical. This is pushed down near the lens, and the lens is adjusted till it touches A B and B C at the same time.

105. Experiment 1. Determination of the Nodal Points and Principal Focal Length.—To find the position of the nodal points we must make use of the property of the nodal point of emergence proved in § 65—*i. e.* if the lens be rotated about an axis through this nodal point at right angles to the axis of the lens, the picture on the ground glass will remain stationary, provided the angle of rotation be not very large.

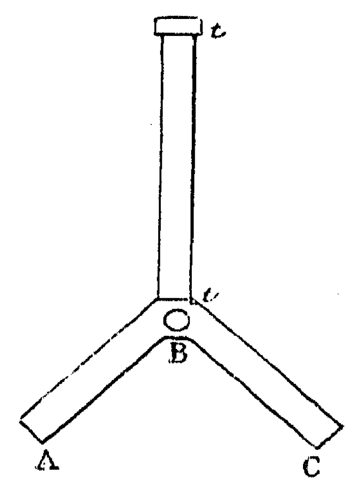

Fig. 68.

The picture of a distant object is focussed on the ground glass, and the lens is then moved from side to side by the arm M M′; if the picture moves the same way as the handle, then the nodal point is between the image and the axis, but if it moves the other way the point is beyond the axis. The lens is then adjusted as required, by the screw V V′, and the test repeated, and so on till the image stays still. For greater accuracy the ground glass may be removed and the tests repeated, while the image is viewed through the microscope against the micrometer scale.

When the adjustment is complete the nodal point of emergence lies on the axis R R′, and the focal length, being the distance from the axis to the ground glass, is read off on the scale along the base board of the instrument.

The position of the nodal point is marked on the mounting of the objective, by passing down the tube $t t'$ a tool which when tapped makes a **V** mark; the angle of the **V** coincides exactly with the axis and is therefore the position of the nodal point.

To find the other nodal point, turn the lens right round with the inner box and repeat the experiment, marking the nodal point as before; the focal length thus found should be the same as that in the first case.

If an examination of the component lenses is required, these can be tested in a similar manner by screwing out the lenses in turn from the mounting.

106. Experiment 2. Determination of the Principal Focal Surface, Depth of Focus, Astigmatism, Maximum Flat Field.—This experiment must be performed with every diaphragm to be tested.

The nodal point of emergence is placed on the axis of rotation as before and the focal length obtained; the lens is then turned through a definite angle by the arm M M′ and fixed in that position. The object is again focussed and the distance of the screen from the nodal point again noted; this process is repeated at regular angular intervals, on either side of the mid position till the extremity of the sharp field of the lens is reached. The angle at which the image ceases to be sharp is noted, and also the angle at which the light is just all shut off by the mounting, giving the angles of the cone of sharpness and cone of illumination (§ 114).

The results of the observations are plotted on a diagram of which Fig. 69 is a reduced copy; the distances are measured off from N, along the lines corresponding to the angles through which the lens is turned. Suppose, for instance, that the lens is of 5 inches focal length, if the points found are joined by a continuous line we shall get a curve such as A F B, which is the section of the principal focal surface by the plane of the diagram.

To find how far the field is practically flat, the tangent at F is drawn to A F B, and the length R R′

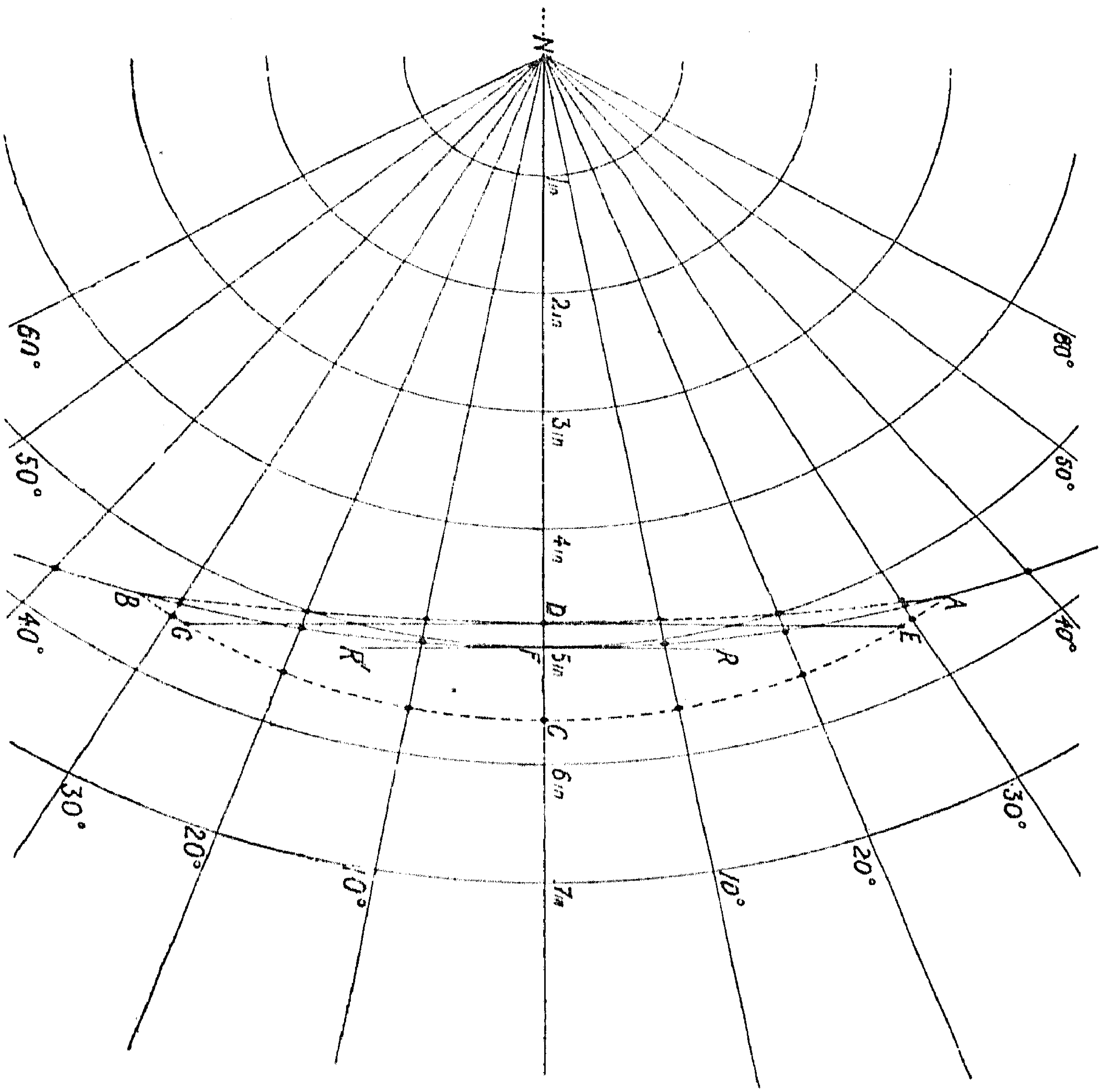

FIG. 69.

is marked off so that no point on it is distant more than one fiftieth of an inch from the curve A F B; points on R R′ will then be practically indistinguishable

from the curve, or the corresponding portion of the curve is practically a straight line.[1]

To test the symmetry of the lens, twist it through any required angle about its axis and repeat the measures; if the lens is symmetrical about the axis, the values now obtained should be identical with the former.

Depth of Focus.—For this measure a diagram (Fig. 70) composed of black and white triangles is employed; it is placed at a fair distance and focussed so that the breadth of the image *a b* or *b c* of the triangles across some line *a b c* is less than one two-hundredth of an

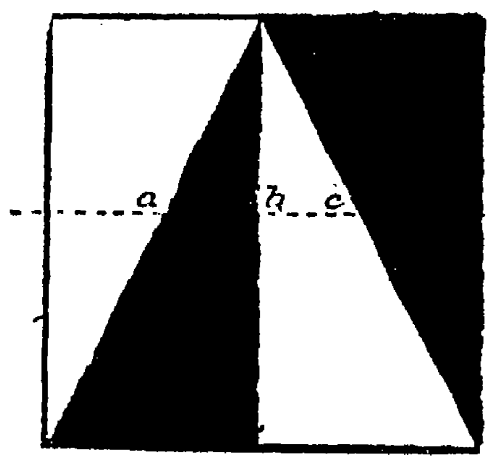

FIG. 70.

inch. In default of the diagram the lens may be focussed on some dark objects, distant chimneys for instance, so that the breadth of some detail is one two-hundredth of an inch.

This done, the ground glass is moved backwards and forwards and the positions are noted at which the points *a* and *b* become indistinguishable or the detail of the distant object disappears; this gives the depth of focus along the axis of the lens.

[1] In this description and elsewhere English measures have been substituted for French measures, the nearest convenient approximation being taken.

The lens is then turned and fixed at various inclinations and the observations repeated.

The lengths so obtained are plotted on the same diagram as that on which the focal surface was plotted (Fig. 69); the continuous curves A C B, A D B drawn through these points are sections of the bounding surfaces of the principal focal volume. To find the largest possible plane picture, draw within the area bounded by the curves the longest possible straight line perpendicular to the axis; the line will clearly be E D G touching the curve A D B at D. The length of the line will be the length of the diagonal of the largest plate which can be covered properly by the lens with the diaphragm used; the experiment can if required be performed with other diaphragms.

Astigmatism.—To determine this use a fairly distant object marked with horizontal and vertical lines. Place the lens at the angle at which it is required to determine the astigmatism. Move the ground glass about till in one position the horizontal lines are sharp and the vertical ones blurred, and in the other the vertical lines are sharp while the horizontal ones are blurred. In these positions the ground glass receives the horizontal and vertical focal lines of the pencil; the distance between the two positions of the ground glass is therefore the distance between the focal lines, and this we have taken as the measure of the astigmatism.

107. Experiment 3. Measurement of Distortion, and of the Field free from Distortion.—This measure depends on the fact that when a lens exhibits distortion, the displacement of the picture when the lens is rotated in Experiment 1 is due not only to the nodal point not being on the axis, but also to the distortion; for suppose, for example, that the nodal point of emergence is by some means placed on the axis of rotation, then, according to the elementary theory, the picture should not move as the lens is rotated, which arises from the

property that the lines joining the corresponding points of object and image to the corresponding nodal points are parallel. But we have seen (§ 78) that the distortion arises from the displacement of the image, by aberration from the place assigned to it by the elementary theory. A little consideration will then show that even if the nodal point of emergence is on the axis of rotation, the picture will at points not on the axis be displaced by the rotation; this will in most cases be too small to detect (at any rate near the centre of the picture if the rotation is small), but will become noticeable if the angle is large.

In most cases then the nodal point can be placed on the axis as in Experiment 1, by moving the lens through a small angle; when this adjustment is made, the lens should be displaced through a large angle, and the distance noted through which the point in the picture, originally in the centre of the field, is displaced along the micrometer scale.

The length thus found is the distortion, with the stop used, for a pencil making an angle with the axis equal to that through which the lens was displaced.

The procedure necessary, if the distortion is large enough to interfere with the first adjustment, is explained by Möessard in his book; it would occupy too much space to give it here.

The extent of the field free from distortion is found by observing the angle through which the lens can be turned without the central point of the picture moving more than one two-hundredth of an inch.

108. Experiment 4. Measurement of Transparency and Field of Equal Brightness.—Möessard has given a method of estimating the transparency of a lens, but it has the disadvantage of giving it only for visual and not for actinic rays; this is practically useless for photographic purposes, as the actinic rays are much cut off by a yellow tinge in the glass which produces very little visual effect.

We shall therefore omit the account of this experiment, referring those who wish to read of it to Möessard's or Wallon's book. In practice the comparison of two lenses is best made photographically; a method for this is described in § 125.

The field of equal brightness is considered among the Kew tests, § 115, No. 16.

109. Experiment 5. Test for Achromatism and Determination of Visual and Chemical Foci.—In a sheet of cardboard a rectangular slit is made, about 2 inches long and half-an-inch broad, across which are fastened two threads at right angles, along and across the slit. The card is placed in a window 8 or 10 feet distant from the tourniquet, so as to be projected against the sky or a white wall, and the threads are carefully focussed with the micrometer.

The micrometer is then replaced by a direct vision spectroscope, so placed that the image found is at the distance of distinct vision. The image of the slit seen in the spectroscope becomes a band of colour, composed of the rays of the solar spectrum; if the lens is well achromatized, the edges of the slit will be quite sharp from one end to the other.

If the lens is not achromatic the positions of the foci for different colours can be found by moving the spectroscope, and observing the positions in which the various lines become sharp.

This method, though theoretically satisfactory, is not practically convenient, as it requires the use of a direct vision spectroscope, which few photographers possess; the following method is more convenient.

110. Photographic Test of Achromatism.—The simplest way to test the achromatism of a lens, or in other words the coincidence of the foci of the visual and chemical rays, is to photograph numbered slips of cardboard placed at slightly different distances from the lens.

A strip of wood is taken (a convenient size is $\frac{3}{8}$ inch

square by $1\frac{1}{2}$ inches in length), on this transverse cuts are made at distances of $\frac{1}{4}$ inch—seven will be enough; seven thin strips of card about 1 inch long by $\frac{3}{8}$ inch broad are numbered at their extremities from 1 to 7. These strips of card are then stuck into the slits in the wood, being arranged fan-wise (Fig. 71), so that when viewed from the front all the numbers at the extremities are visible.

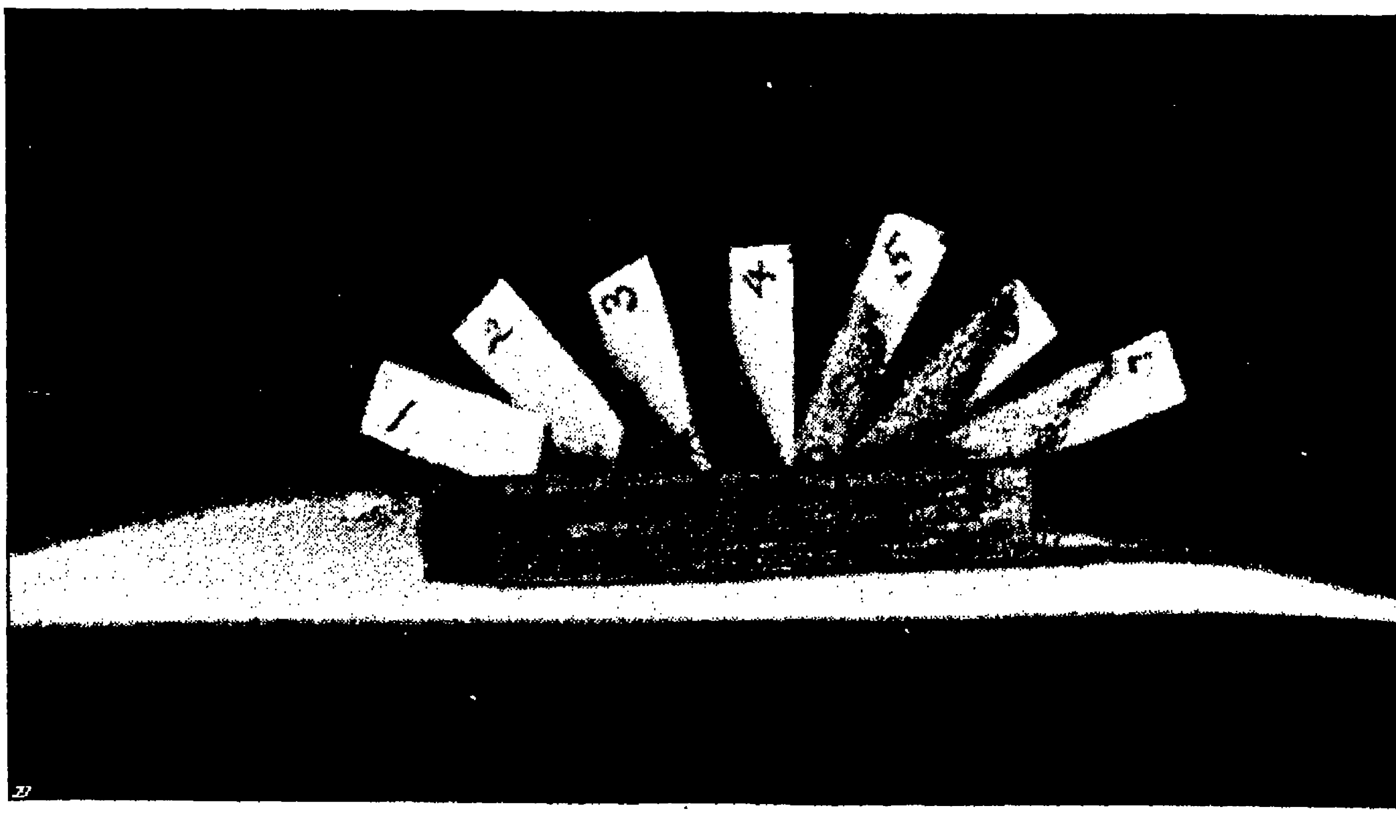

FIG. 71.

The strip of wood is then placed one or two yards in front of the camera, with its length along the axis of the lens; the middle number 4 of the series is clearly focussed and a photograph taken. The negative when developed is examined to see which number has come out the sharpest; if 4 is the sharpest, then the visual and chemical foci coincide, but if they do not it will show their relative positions.

We can estimate the difference between the principal focal lengths for the two kinds of rays as follows.

Let f_1 be the focal length for the visual rays, $f_1 + a$ that for the chemical rays, also let u and v be the distances of object and image for visual rays, and $u + x$ and v similar quantities for the chemical rays; here x and a are small quantities compared with u, v and f. We have then—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \qquad \frac{1}{v} - \frac{1}{u+x} = \frac{1}{f+a}$$

Subtracting we get

$$\frac{1}{u} - \frac{1}{u+x} = \frac{1}{f+a} - \frac{1}{f}. \quad \text{or} \quad \frac{x}{u(u+x)} = \frac{-a}{f(f+a)}$$

$$\therefore \; xf(f+a) = -au(u+x)$$

or

$$f^2x + xaf = -u^2a + axu$$

But a and x are small, hence we may neglect the product xa, compared with either x or a, and the relation becomes

$$^1 \; f^2x = -u^2a \qquad \text{or} \qquad a = -\frac{f^2}{u^2}x.$$

Hence, if we know the visual focal length, the distance from the lens of the card which appears clearly focussed, and x the distance from this card of that which photographs most clearly, then we can calculate the difference between the visual and chemical focal lengths.

Example.—The card numbered 4, which is focussed sharply, is 2 feet from the lens, $\therefore u = 2$ feet, the visual focal length is 6 inches, or $f = +\,1/2$ foot (convex lens); it is found that the card marked 2 is sharpest in the photograph when 4 is sharpest on the ground glass. Find the difference between the visual and chemical focal lengths.

$x = -\,1/2$ inch $= -\,1/24$ foot, the cards being $1/4$ inch apart.

[1] Readers acquainted with the differential calculus will easily get this result by differentiation.

Hence

$$a = -\frac{f^2}{u^2}x = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{24} = \frac{1}{384} \text{ feet} = \frac{1}{32} \text{ inch.}$$

Hence the chemical focal length in inches is

$$f + a = -6 + 1/32 = -(6 - 1/32)$$

or the chemical focal length is numerically less than the visual one by one thirty-second of an inch.

If we put the relation found above in the form

$$x = -\frac{u^2}{f^2}a$$

we see that the larger is u the distance of the object from the lens, the larger will be x the distance between the cards for a given value of a, the difference between the focal lengths.

Hence the further the numbered cards are placed away from the lens the more sensitive will be the method.

In practice it is usually required only to test the achromatism of a lens, and one which is not achromatic would be rejected for ordinary work. But in scientific experiments it is sometimes necessary to use a single uncorrected lens for taking a series of photographs of objects at a fixed distance; if so the method given is useful for finding the proper relative distances of object and image.

111. Examination of the Faults of Construction.—The faults which should be looked for are in the centering of the lenses and the working of the surfaces.

Centering.—The tourniquet supplies a method for testing the centering of the component lenses of an objective, for if they are not concentric with their mounting, the nodal points will not lie on the axis of the mounting; it will not then be in general possible to put the nodal points on the axis of rotation, and this makes it impossible to make the picture stationary, as

in Experiment 1. But there are evidently two positions in which the nodal point is either immediately above or below the axis of the mounting, into which the lens can be twisted in which the nodal point examined is on the axis of rotation.

The method of conducting the test is to fix the lens in the tourniquet, and to proceed to find the nodal point as already explained. If the picture cannot be rendered stationary, then the centering is bad, but if the picture can be rendered stationary, twist the lens through a right angle, and repeat the test. If the picture is still stationary the centering is good, if not it is bad.

The Working of the Surfaces.—If the centering is good the correctness of the surfaces may be tested by finding the position of the nodal points; twist the lens through an angle and find them again, and so on until two or three sets are found; mark these as before on the mounting. If the surfaces are all symmetrical about the axis, the marks so made will lie on a circle whose plane is perpendicular to the axis of the lens.

This test will also show if there is any great want of homogeneity in the material of the lenses.

Faults in the Glass.—By this are meant defects in the glass itself, such as striæ, veins, and colour; they can be detected by examining the lens in a good light, turning it about in all directions.

II.—The Kew System.

112.—In 1891–2 the Kew Committee of the Royal Society decided to establish a series of tests for photographic lenses; a system was accordingly devised by Major Darwin and the late superintendent of the observatory, Mr. Whipple.

As this series of tests is especially interesting to English readers, some considerable account of it will be

given; the details are quoted from a paper by Major Darwin.[1]

The object in view is best quoted from the paper:—

"The object of the Committee was to organize a system by which any one could obtain, on payment, an impartial and authoritative statement of the quality of a lens to be used for ordinary photographic purposes, and that the fee, which had to cover the cost of the examination, should be moderate. This latter consideration acted as a serious restriction, and it was consequently necessary that all the tests should give results of undoubted practical value to the practical photographer; the certificate of examination must be recorded in the way most generally useful, and in language which could not fail to be understood. A complete scientific investigation of a lens from every point of view would occupy so long a time as to make the necessary fee quite prohibitive, and, moreover, the results would contain much information which would be quite useless to the ordinary user of the lens.

"There are undoubted advantages in testing a lens by the examination of negatives made by it, but it may be here stated, once for all, that the question of expense rendered it impossible, for the present, to adopt any photographic method; eye observations alone have to be relied on."

In most cases a lens is designed for a particular kind of work, and for use with a plate of a particular size, so to shorten the examination, the person entering the lens is asked to state these particulars, and the examination is made for them only.

113.—The list of tests made is best given by quoting

[1] "On the Method of Examination of Photographic Lenses at Kew Observatory." By Leonard Darwin, Major late Royal Engineers. *Proceedings of the Royal Society*, No. 318, December 1892. Vol. 52, p. 403.

Major Darwin has kindly given permission to quote from the paper and to use the diagrams.

a certificate of examination which will afterwards be explained in detail; the part in italics represents the results of the tests.

KEW OBSERVATORY, RICHMOND, SURREY.

Certificate of Examination of a Photographic Lens.

1. Number on lens, *3876*. Registered number, *95*.
2. Description, *landscape lens*. Diameter, *1·5* inches.
3. Maker's name, *A. B.*
4. Size of plate for which the lens is to be examined, *6·5* inches by *8·5* inches.
5. Number of reflecting surfaces, *4*.
6. Centering in mount, *good*.
7. Visible defects—such as striæ, veins, feathers, &c., *nil*.
8. Flare spot, *nil*.
9. Effective aperture of stops—

Number engraved on stop.	Effective aperture. Inches.	*f*/number.	C.I. No.
No. *7·5*	*1·32*	*f/8·6*	*1/1·38*
No. *10*	*1·19*	*f/9·5*	*1/1·12*
No. *15*	*0·97*	*f/11/·7*	*1·35*
No. *25*	*0·75*	*f/15·1*	*2·26*
No. *50*	*0·49*	*f/23*	*5·3*
No......	....	....	....
No......	....	....	....

10. Angle of cone of illumination with largest stop = *68°*, giving a circular image on the plate of [1] *13·2* inches diameter.
   Angle of cone outside which the aperture begins to be eclipsed, with stop C.I. No. *1/1·38*, = *20°*, giving a circular image on the plate of *4·0* inches diameter.
   Diagonal of the plate = *10·7* inches, requiring a field of *51°*.
   Stop C.I. No. *5·3* is the largest stop of which the whole opening can be seen from the whole of the plate.
11. Principal focal length, [1] = *11·28* inches. Back focus, or length from the principal focus to the nearest point on the surface of the lenses, = *10·4* inches.

[1] The lens is focussed on a very distant object.

12. Curvature of the field, or of the principal focal surface. After focussing[1] the plate at its centre, movement necessary to bring it into focus for an image *1·5* inches from its centre = *0·02* inch.

Ditto for an object *3* inches from its centre = *0·04* inch.

,,	*4·5*	,,	= *0·10*	,,
,,	*5*	,,	= *0·15*	,,

13. Definition at the centre with the largest stop, *excellent.* C.I. stop No. *1·35* gives *good* definition over the whole of a *6·5* inch by *8·5* inch plate.
14. Distortion. Deflection or sag in the image of a straight line which, if there were no distortion, would run from corner to corner along the longest side of a *6·5* inch by *8·5* inch plate = + 0·*01* inch.[2]
15. Achromatism. After focussing[1] in the centre of the field in white light, the movement necessary to bring the plate into focus in blue light (dominant wave-length, 4420), = + 0·*04* inch.[3] Ditto in red light (dominant wave-length, 6250) = − 0·*01* inch.[1]
16. Astigmatism.[4] Approximate diameter of disc of diffusion[1] in the image of a point, with C.I. stop No. —— at —— inches from the centre of the plate = 0·—— inch.
17. Illumination of the field. The figures indicate the relative intensity at different parts of the plate.[1]

With C.I. stop No. *1/1·38.*			With stop No. *5·3.*	
At the centre..................	100	:	Ditto	100
At *3* inches from the centre	*67*	:	Ditto	*82*
At *5·35* ,,	*28*	:	Ditto	*66*

General Remarks.—*An excellent medium angle rapid objective, practically free from distortion.*

Date of issue ________________

W. HUGO, Observer.
G. M. WHIPPLE, Superintendent.

[1] The lens is focussed on a very distant object.

[2] The sag or sagitta here given is considered positive if the curve is convex towards the centre of the plate.

[3] Positive if movement towards the lens, negative if away from it.

[4] The lens is supposed to be perfect in other respects.

Note.—The following is the scale of terms used: excellent good, fair, indifferent, bad.

"In considering and in recording the results of examinations, it has been found convenient to give more exact meanings to certain expressions than have

as yet been assigned to them. The following definitions have therefore been adopted at Kew :—

"*A narrow angle lens* means one covering effectively not more than 35°.

"*A medium angle lens* means one covering between 35° and 55°.

"*A wide angle lens* means one covering between 55° and 75°.

"*An extra wide angle lens* means one covering more than 75°.

"*The C.I. No. of a stop* means the number which indicates the intensity of illumination produced by it on the plate according to the system proposed at the International Photographic Congress of 1889 (see § 123).

"The *largest normal stop* means the largest stop that can be used with the lens so as to produce definition up to a selected standard of excellence all over a plate of given size, the objects whose images are seen being all equally distant.

"*A slow lens* means one of which the largest normal stop has a less diameter than has C.I. No. 6.

"*A moderately rapid lens* is one of which the largest normal stop is C.I. No. 6, or larger than that size and less than C.I. No. 2.

"*A rapid lens* is one of which the largest normal stop is C.I. No. 2, or larger than that size and less than C.I. No. 2/3.

"*An extra rapid lens* is one of which the largest normal stop is C.I. No. 2/3, or larger than that size."

114. Headings of Certificate, 1—8.—The first four headings refer only to the numbering of the lens, the maker's name, etc., and need not be further considered.

5. *Number of Reflecting Surfaces.*

"In most cases the number of reflecting surfaces of glass is known at once from the type of lens, but, if in

doubt, a simple experiment will settle the point; the room is darkened, and the reflection of a lamp is observed in the lenses; each of the surfaces of the lenses will give one direct reflected image, and the number can thus easily be counted.

6. *Centering in Mount.*

"Two different errors might be described under this heading: either (1) the optical axis of a perfect lens may not coincide with the axis of the mounting, or (2) the axes of the different lenses of a doublet or triplet may not all be in the same straight line. As to the first of these errors, we believe it would never be sufficient to have any appreciable effect on the practical value of a lens, and therefore no test for it is considered necessary. With regard to the second error, Wollaston's test is the only one applied; this consists of looking at the flame of a lamp or candle *through* a compound lens, and noting if all the different images of the light as seen by successive reflections from the surfaces of the glass can be brought into line by a suitable movement of the whole lens, which should be the case if the component lenses are arranged about a common axis.

7. *Visible Defects, such as Striæ, Veins, Feathers, etc.*

"Under this heading any faults detected by a careful inspection are given."

8. *Flare Spot.*

The nature of this defect has been explained in § 95: to detect it the lens is placed in an ordinary camera, which is pointed to the sky; if the ground glass is brought to the principal focus, the flare spot is then readily visible.

115. Headings of Certificate, 9—16.—For these tests,

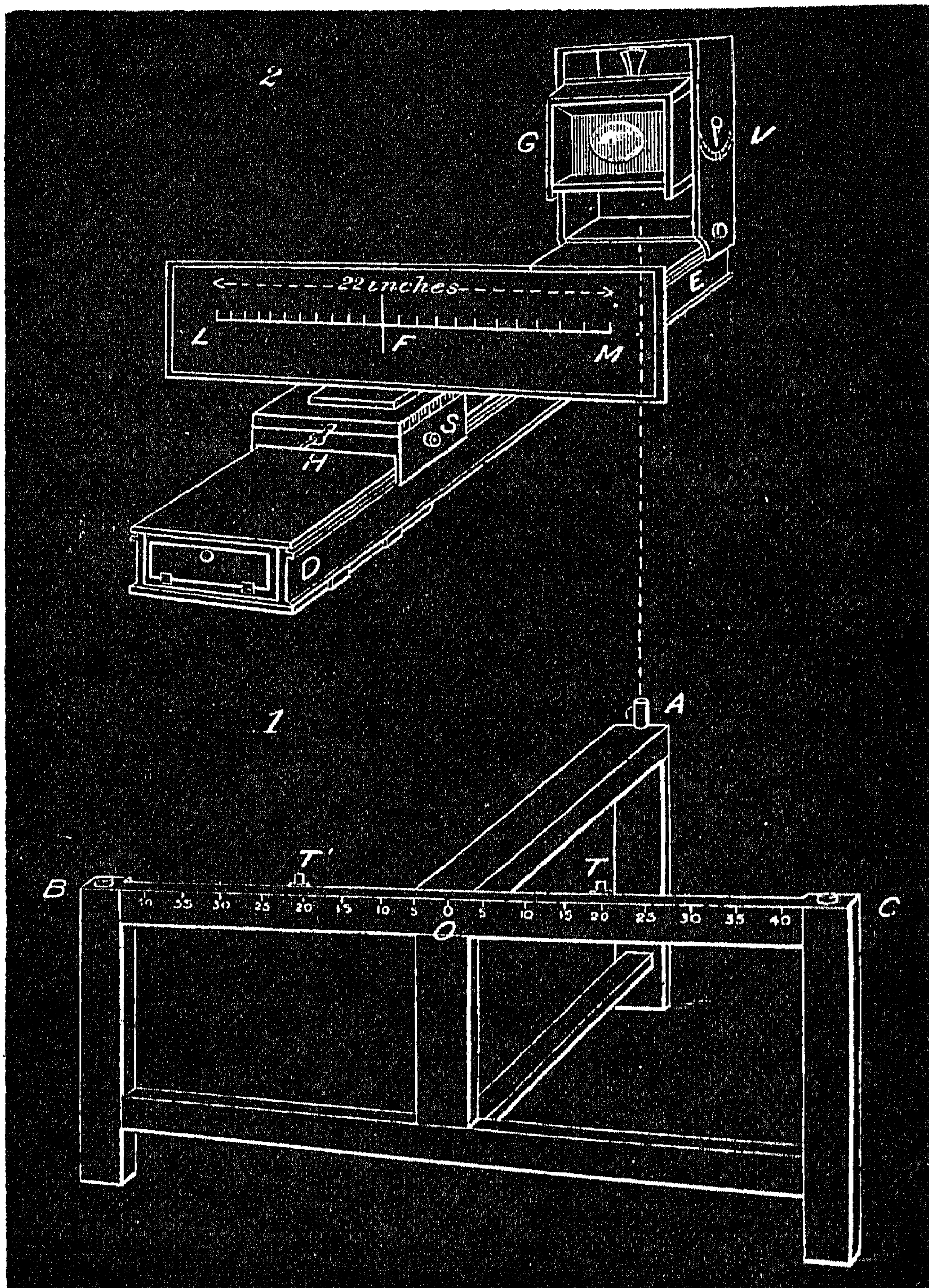

FIG. 72.

a testing camera or apparatus has been designed by Major Darwin (see Fig. 72).

"The three-legged stool or bench, seen in 1, represents the legs of the camera, and 2 shows the apparatus that takes the place of the body; G is the lens-holder, and L M the ground glass, both of which are capable of independent movement backwards and forwards on the hollow wooden beam D E, called the 'swinging beam.' There is a conical brass cap or pivot, not shown in the sketch, under the upper plank of the swinging beam, underneath where the lens-holder G is shown in the sketch. The whole of the apparatus shown in 2 is placed on the top of the three-legged stool, the round-headed iron pin A passing loosely through a hole in the lower plank of the swinging beam, and fitting into the conical brass cap or pivot. The swinging beam, being thus supported by the pin A and by the long arm B C of the stool, is capable of being revolved round A as a centre. On the ground glass is engraved a horizontal line, which is accurately divided into fiftieths of an inch; this line passes through the centre of the ground glass (or through the point where the perpendicular from the lens-holder cuts the glass), and is also parallel to B C, the top of the stool on which the swinging beam slides, when the camera is in position; thus the image of an object will appear to run along the scale, as the swinging bar is moved from side to side. The ground glass can be brought approximately into focus by means of the already-mentioned movement to and fro on the swinging beam, but for accurate adjustment a slow motion arrangement is attached to the movable part itself; the handle H gives the required motion, and there is a scale S, called the 'focus scale,' by means of which these small movements can be accurately measured. On the lens-holder there is a movement, corresponding to the swing-back of an ordinary camera, by which the lens can be made to revolve vertically round a horizontal axis, without, of course, any corresponding movement of the ground glass; there is a vertical arc, V, by means of which we can read off the

vertical angles through which the lens is rotated. An arrangement is also supplied by means of which the lens can be moved backwards and forwards *on* the movable stand, thus allowing the position of the lens to be so adjusted that the horizontal axis can be made to pass through any point in its axis.

9. *Effective Aperture of Stops.*

Number engraved on stop.	Effective aperture. Inches.	*f*/number.	C.I. No.
No.......	....	....	....
No.......	....	....	....
No.......	....	....	....
No.......	....	....	....
No.......	....	....	....
No.......	....	....	....
No.......	....	....	....

"The effective aperture of one or more of the various stops supplied with the lens is found by a well-known method. The image of a very distant object is first brought into focus on the ground glass of the testing camera; a collimator, which has itself been previously focussed on a distant object, may be used instead of the distant object; the ground glass is then taken out and exactly replaced by a tin plate with a small hole at the centre; this hole, which should be very small, will, therefore, be at the principal focus of the lens. The room being darkened, a gas-burner is placed behind the small hole, and thus parallel rays, in the form of a cylinder, are made to issue from the lens towards the front. A piece of ground glass, with a graduated scale engraved on it, is now held in front of the lens, and the diameter of the illuminated disc, or section of the cylinder as seen on the glass, is directly measured off as any stop is inserted in its place. Thus is found the

effective aperture of the largest stop, as recorded in the Kew Certificate of examination. The ratio of the effective aperture to the diameter is the same for all stops of the same lens, and the effective aperture of the other stops is either measured as above, or calculated from the ratio thus found. As the rays are parallel when emerging from the lens, it is evident that, if the stop is in front of all the lenses, the effective aperture will be the same as the diameter of the stop itself.

"10. *Angle of Cone of Illumination with Largest Stop* = —— °, *giving a Circular Image on the Plate of* —— *inches diameter. Angle of Cone outside which the Aperture begins to be eclipsed with Stop C.I. No.* —— = —— °, *giving a Circle on the Plate of* —— *inches diameter.*

Diagonal of Plate = —— *inches, requiring a Field of* ——° (= 2 ϕ).

Stop C.I. No. —— *is the Largest Stop the whole of the Opening in which can be seen from the whole of the Plate.*"

The meanings of the terms used here have been explained in § 59; it should be carefully noticed that the angles in question are those between two extreme pencils on opposite sides of the axis, or the *whole* vertical angles of the cones.

"The outer cone, which we have called the 'cone of illumination,' gives the extreme angle of the field of the lens without regard to definition, and is what is known to French authors as the *champ de visibilité*. To find the angle of the cone of illumination, the lens is placed in the testing camera, and the observer looks through the small hole in a sheet of tin plate with which the ground glass has been replaced, as in the last test; the lens-holder is made to revolve about its horizontal axis, and as the axis of the lens moves away from zero, first in one direction and then in the other, the positions at

which all light appears to be cut off are noted ; the angle between these two positions as read on the vertical arc, V, gives the angle of the cone of illumination."

Before making the test, the axis of rotation should be made to pass through the nodal point of emergence, in the manner explained in § 105.

"The angle of the inner cone, that is, of the cone outside which the opening of the stop is partially eclipsed by the mounting of the lens, etc., is measured in the same way as above described for the outer cone, and with the same precautions. When looking through the small hole, the positions on each side of zero at which the aperture begins to be shut off, and beyond which it no longer appears as a perfect ellipse, are easily seen, and the angle between these two positions as measured on the vertical arc gives the angle required. The angles of these two cones are generally given when the observation is made with the largest stop supplied with the lens.

"In order to facilitate the consideration of the covering power of the lens, the diameters of the circles which these cones make by cutting the photographic plate, when the focus is adjusted for distant objects, are given in the Certificate of Examination. Having found the principal focal length in the manner to be described immediately, the size of these circles can readily be ascertained by a simple graphical method, which is hardly worth describing in detail.

"In connection with this test it may be convenient to adopt the use of the term *angle of field under examination* (denoted in this paper by 2ϕ), to signify the angle subtended at the nodal point of emergence by a diagonal of the plate, or the greatest angular distance which could be included in the photograph, supposing the focus to be taken on a distant object."

The angle is found either graphically or by the method of § 59 (end), and the result is entered on the certificate of examination.

As the lens is to be examined as to its behaviour with a plate of given size, the converse test is necessary; the size of the largest stop must be found which will include the given plate in its inner cone.

"If the illumination of the field is not to fall off rapidly towards the edges of the plate, for the normal use of the lens we should employ a stop which covers (or nearly covers) the plate of the given size with its inner cone; that is to say, we should use a stop not larger than the largest stop the whole of the opening in which can be seen from the whole of the plate. In order to find the largest stop which fulfils the above conditions, the lens is revolved about the horizontal axis until the vertical arc reads half the angle of field under examination, and then the different stops are put in one by one until the largest one is found which is seen not to be eclipsed when the observation is made through the hole in the tin plate. The number of this stop is recorded in the certificate.

"11. *Principal focal length = —— in.*
Back focus, or length from the principal focus to the nearest point on the surface of the lenses = —— in."

The principal focal length is found with the testing camera as follows:—The nodal point of emergence being on the axis of rotation, the swinging beam is brought approximately to a central position. The two iron stops, T and T′, are fixed so as to allow the beam to swing on either side of zero, through an angle whose tangent is 1/4 or $14^\circ\ 2'$. A distant object is focussed on the ground glass, and the testing camera is arranged so that when the beam is approximately in a central position, the image of some well-defined object seen through a hole in the window shutters, appears on the central line of the ground glass.

When this adjustment is made, the line joining F, the centre of the ground glass, to the centre of the lens, will,

if produced, pass through the distant mark. When the swinging beam is moved from side to side, the image appears to run along the ground glass; its position is noted when the beam is in contact with the stops T and T′. We shall see that twice the distance between the points so noted on the ground glass is equal to the principal focal length of the lens.

Suppose, to begin with, that the lens remains still, but that the position of the object is varied from B to

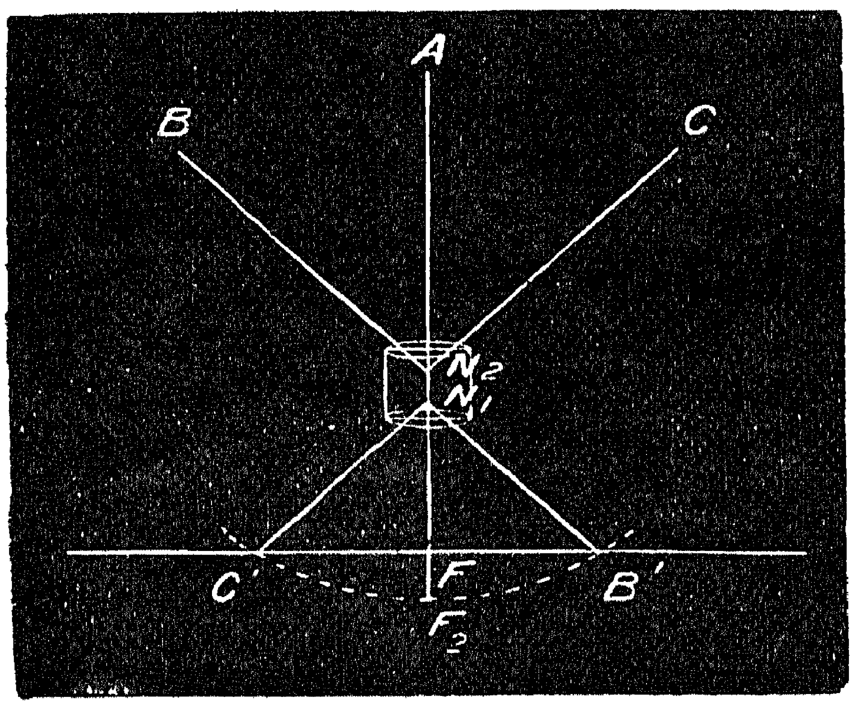

FIG. 73.

C (Fig. 73), so that the image on the ground glass moves from B′ to C′. If N_1, N_2 are the nodal points and the angle $C' N_1 F = 14° 2'$.

Hence $C' F = N_1 F \tan C' N_1 F = N_1 F \tan 14° 2' = N_1 F/4$

$$\therefore N_1 F = 4 C' F = 2 C' B'$$

and $N_1 F$ is the principal focal length.

If the lens is revolved and the object is kept stationary the result will be the same, for the motion of N_2 is too small to affect the size of the angle $C' N_1 F$.

Hence we have proved that the principal focal length

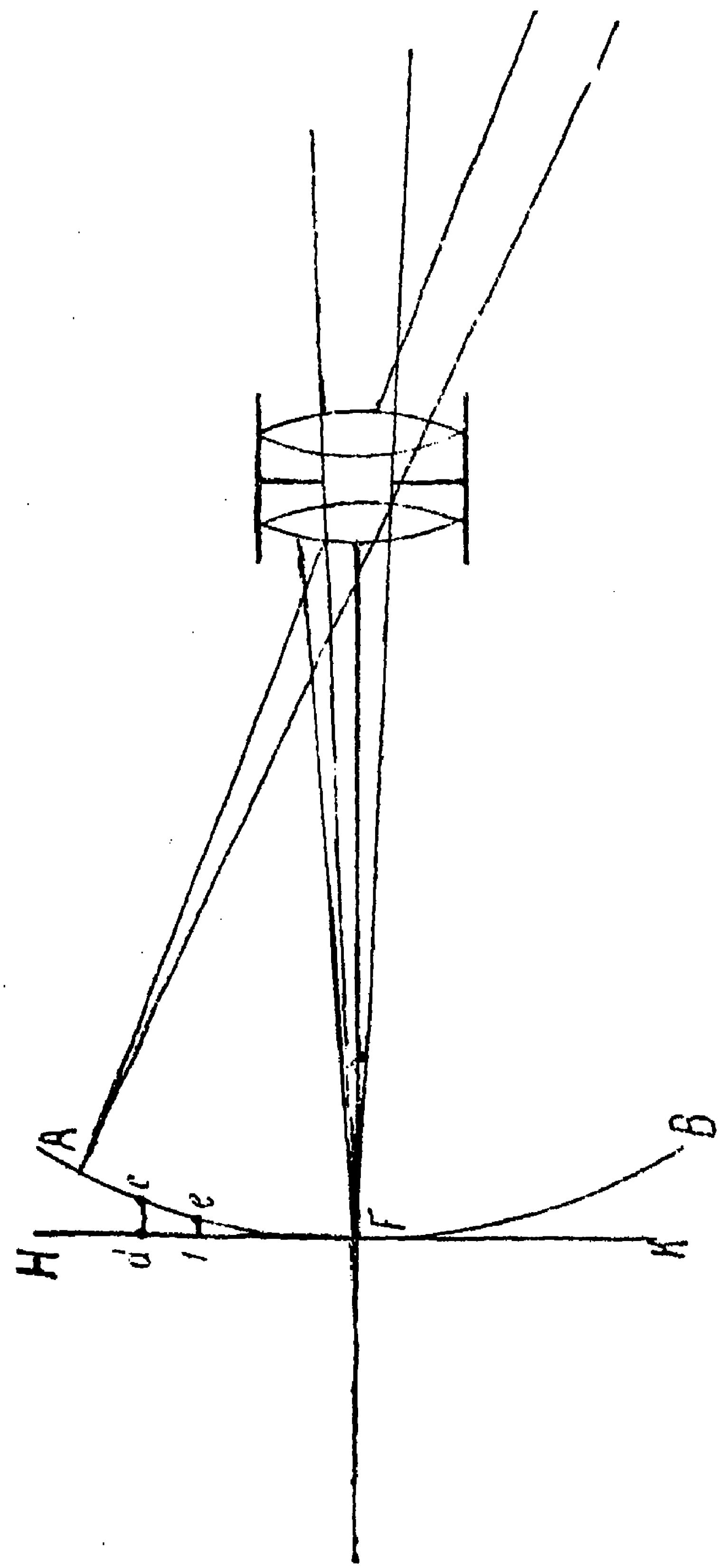

FIG. 74.

is equal to twice the length which the image travels along the scale on the ground glass.

Major Darwin devotes several pages to the discussion

of the accuracy of this method, and concludes that if worked with reasonable care it is as reliable as any of the other methods, and it has the advantage of being fairly rapid.

"12. *Curvature of the Field, or of the Principal Focal Surface. After focussing the plate at its centre, movement necessary to bring it into focus for an image —— inches from its centre = —— inches.*
Ditto for an object —— inches from its centre = —— inches.
Ditto for an object —— in. from its centre = —— in."

Curvature has been explained in § 75; the only question to be considered is the mode in which it is to be measured.

Let (Fig. 74) A F B be the section of the principal focal surface by a plane through the axis, and H F K the section of a plate touching A F B at F, the principal focus. Then the picture will be sharply focussed at the centre, but at points like f and d it will be out of focus; if, for instance, we wish to focus sharply at d, we must move the plate forward a distance $d\,c$, till d is on A F B.

The curvature can therefore be given conveniently for practical purposes by stating the distances like $c\,d$, $f\,e$ through which the plate must be moved in order to focus sharply at points such as d and f.

Another method of measuring the curvature of a curve like A F B, often used in mathematical work, is to give the radius of the circle which will most nearly fit the curve at the point F; the curve A F B is not as a rule exactly circular, but a small portion near F is very approximately so and we can calculate its radius. It can be shown that[1] if r is the radius

$$r = \frac{(f\,\mathrm{F})^2}{2\,f\,e} \text{ (very approximately)}$$

where f is a point near to F.

[1] Williamson's *Differential Calculus*, Ed. 4, p. 291.

Example.—Take the case given in the certificate quoted (heading 12).

Here f F = 1·5 inches, f e = ·02 inch.

$\therefore r = (1{\cdot}5)^2/{\cdot}04 = 56{\cdot}25$ inches = 4 ft. 8·25 in.

Hence the section of the portion of the principal focal surface near F is very approximately a portion of a circle of radius nearly 5 feet.

[In calculating *radius of curvature* the measures for a point as near as possible to the axis should be used.]

"The following is the method of finding the curvature of the principal focal surface. The image of a distant object (or of the collimating telescope) is thrown on that point on the ground glass where the axis of the lens cuts it, the focus is accurately adjusted, and the focus scale is read off. The swinging beam is then moved so that the image comes successively to positions at convenient intervals from the centre of the plate, and on each occasion the focus is adjusted afresh, and the focus scale read off. By subtracting the central reading from these outer readings, the results recorded on the Certificate of Examination are obtained.

"13. *Definition at the Centre with the Largest Stop, —— C.I. Stop, No. —— gives —— definition over the whole of a —— inch by —— inch plate.*

"The system by which the defining power is measured consists in ascertaining what is the thinnest black line of which the image is just visible, the test being conducted in the following manner. The test object consists of a thin straight strip of steel, about 0·1 inch wide, and about an inch long; it is capable of being rotated about an axis in the direction of its greatest length, thus, if seen against a bright background, making it appear as a black line of varying width; when presented edgewise to the objective, it is so thin that the image becomes invisible; and there is an arc so graduated that the angle subtended by the two edges of

the strip at the lens can be at once read off, thus giving a measure of the apparent thickness of the line. The test-object is placed as far as possible from the lens in a darkened room (at Kew the accommodation in this respect leaves much to be desired), and beyond it is a ground glass screen illuminated by a lamp.

"In order to test the defining power of a lens in the centre of its field, the focus is first very carefully adjusted on the ground glass, and the test-object is then slowly revolved from the edgewise position, where its image is invisible, until the first appearance of a dark line can be seen against the bright background; the angular width of the line is read off, and is noted as a measure of the defining power of the lens in the centre of its field. The light of the lamp is regulated so that the image of the line can be seen as soon as possible.

"Besides measuring the defining power where the axis of the lens cuts the focal surface, an observation is also made at a point representing the extreme corner of the plate of the size for which the lens is being examined, that is, at a distance from the centre equal to half the diagonal of the plate. As the object of this second test is to measure the general definition over the whole plate, the focus is taken at a position half-way between the point of observation and the axis of the lens, this being the method generally adopted by practical photographers when desirous of getting the best general focus. It is necessary, moreover, that the test-object should be so arranged that the steel strip makes an angle of 45° with the horizon; for, since the diffusion of the image near the margin may be due to astigmatism, a false impression of the defining power will be obtained if the image of the dark line coincides in direction with either of the focal lines; whereas if it bisects the angle between them, as will then be the case, there is no error in the result from this cause. The test is not, however, conducted in quite the same way as in the first instance; the test-object is set at a known angle, and

the stops are slipped in one after another, beginning with the largest and going on to smaller ones, until the image of the black line on the bright ground is first just visible; the C.I. No. of the stop with which the lens gives definition up to a known standard at the extreme corner of the plate is thus ascertained, and, as it may fairly be assumed that the definition will be no worse than this at any other part of the plate, it follows that the defining power over the whole plate comes up to or exceeds the standard selected.

"14. *Distortion. Deflection or sag in the image of a straight line which, if there were no distortion, would run from corner to corner along the longest side of a —— by —— plate = 0·—— inch.*"

The question of distortion has been treated at length in Chapter III.; it should be noted that the quantity given in the Kew certificate is not the same as that which we have taken as the measure of the distortion. The quantity we have used is the displacement of the point in question from the position it should occupy according to the simple theory, and it is this that is found with the tourniquet. But in the Kew test the quantity found is the amount by which the image of a straight line sags between its ends; which is practically useful in giving an idea of the curvature of the image.

"The following is the method adopted at Kew of measuring the distortion produced in the image by the lens under examination. Let Fig. 75 be a vertical section through the testing camera; G G representing the ground glass; F the principal focus; and N_1 the horizontal axis, which passes through the nodal point of emergence, the adjustment for that purpose having already been made for test No. 10. The lens-holder carrying the lens is first turned in either direction through an angle β, such that C' F, or F N_1 tan β, or f tan β is equal to half the *shortest* side of the plate for

which the lens is being tested. (The *horizontal* movement of the swinging beam in the testing camera gives an easy means of determining the angle β; a distant object is first brought to focus at the centre of the ground glass, and then the swinging beam is revolved about the axis A (see Fig. 72) until the image has moved along the graduated scale a distance equal to

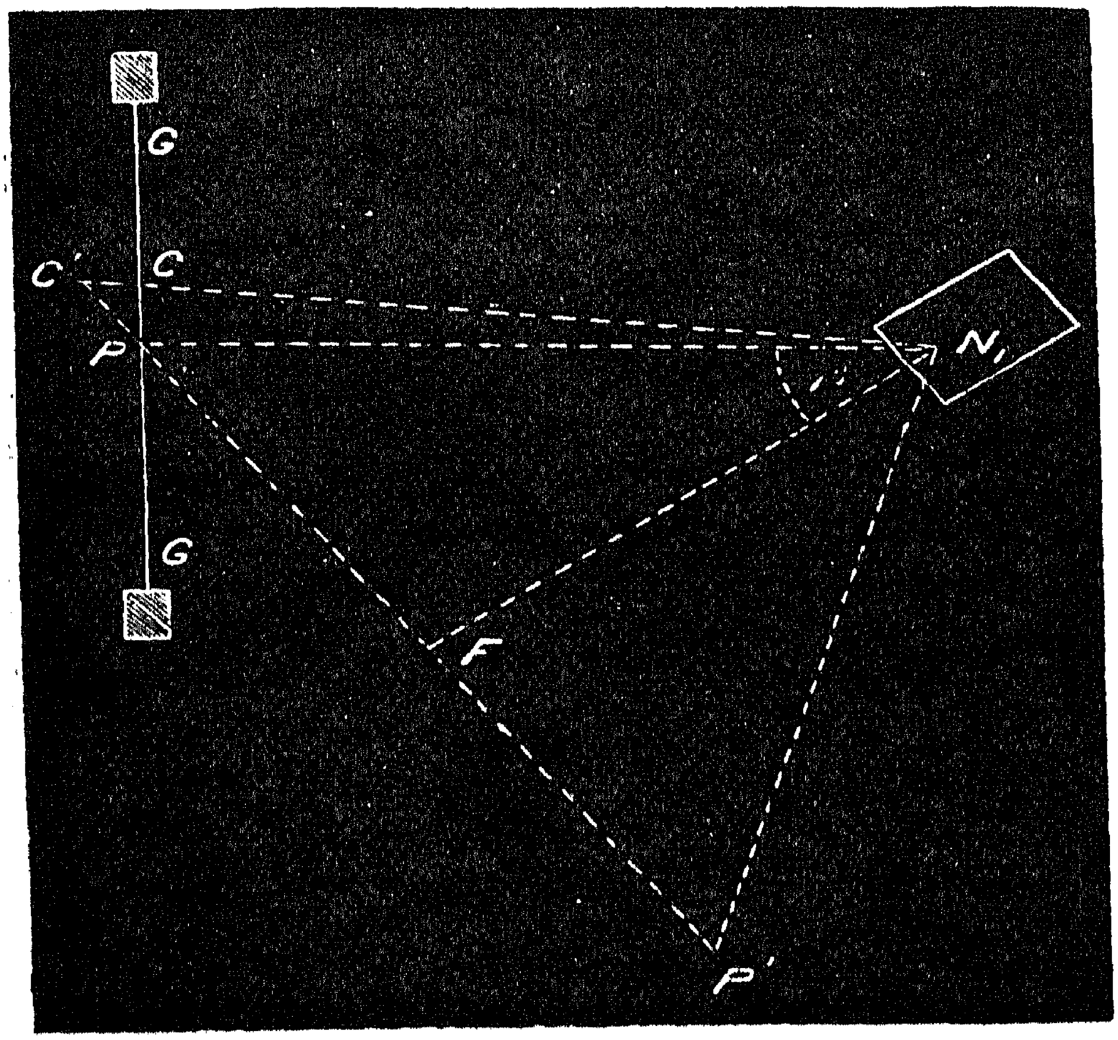

FIG. 75.

half the shortest side of the plate; the beam is thus made to move through the angle β, which can be read off with sufficient accuracy on B C, the top of the wooden stool, which is graduated for that purpose. After this adjustment has been made the ground glass is brought into focus by observing the image of a distant object at a point P, a little below C, the line engraved on the

glass ; under these circumstances, if the principal focal surface is a plane, and if the lens were being used in the ordinary manner, P P′ would be the position occupied by the photographic plate, the section shown being taken across the centre of the plate parallel to its shortest side. The small distance P C is carefully measured ; this length is then multiplied by secant β, thus obtaining C′ P, which we will call a. The swinging beam is now revolved about the pivot in either direction, so that the image moves along the scale on the ground glass a distance equal to half the *longest* side of the plate for which the lens is being examined ; the sketch in Fig. 75 is still more or less applicable, C′ P′

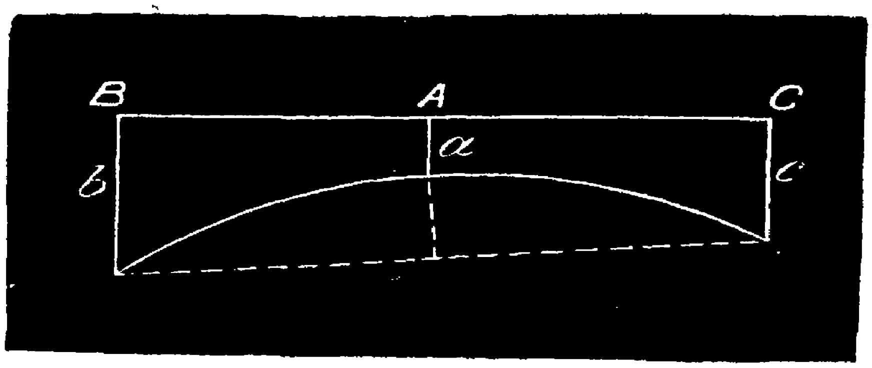

FIG. 76.

still representing a section across where the photographic plate ought to be, but this time at the end of the plate, not at its centre (F, therefore, no longer represents the principal focus) ; in fact, what has been done is to make the image describe what, neglecting distortion, would be a straight line from the centre to the corner along the longest edge of the plate : after this movement has been made, the length of C′ P is again obtained by measurement and calculation, and this time let the result be called b ; the operation is repeated when the swinging beam is revolved to an equal angle on the other side of zero, and a third length, c, is thus obtained. In Fig. 76, let B A C be equal in length to the longest side of the plate, and let a, b, and c be the lengths just ob-

tained; then the curve $b\,a\,c$ will evidently represent the image of a straight line thrown by the lens under examination along the edge of the longest side of the plate. Since the image travels along a line very nearly parallel to the engraved line on the ground glass, B A C will be nearly parallel to the chord of the curve, and $\frac{b+c}{2} - a$, which is the length recorded in the Kew certificate, will be a very close approximation to the sagitta or sag of the curve.

"15. *Achromatism. After Focussing in the Centre of the Field in White Light, the Movement necessary to bring the Plate into Focus in Blue Light (dominant wave-length* 4420), = 0·—— *inch. Ditto in Red Light (dominant wave-length* 6250), = 0·—— *inch.*"[1]

The test in this case is very similar to that described in § 110, but it is made by eye and not photographically.

"First the focus is carefully adjusted in daylight on a suitable object placed as far away as possible in the room, and then the focus scale is read off. After this, a sheet of blue glass, the colour of which has a dominant wave-length of 4420, is placed behind the object and close in front of a small opening in the shutter through which all the light enters the room; the focus is re-adjusted, the focus scale read off again, and the difference in reading to that observed in white light is noted."

The calculation to find the change in the principal focal length is very similar to that of § 110; here it is v that is varied and u that remains constant. If the symbols have the same meaning as before it can be shown in a similar manner that—

$$a = \left(\frac{f}{v}\right)^2 x$$

[1] For the unit in which wave-lengths are measured, the tenth metre, see § 6.

It is the calculated value of *a* which is entered on the certificate.

A similar process is then performed with a sheet of red glass the colour of which has a dominant wave-length of 6250.

"It may be observed that either the principal focal length or the position of the nodal point of emergence may vary as different coloured lights pass

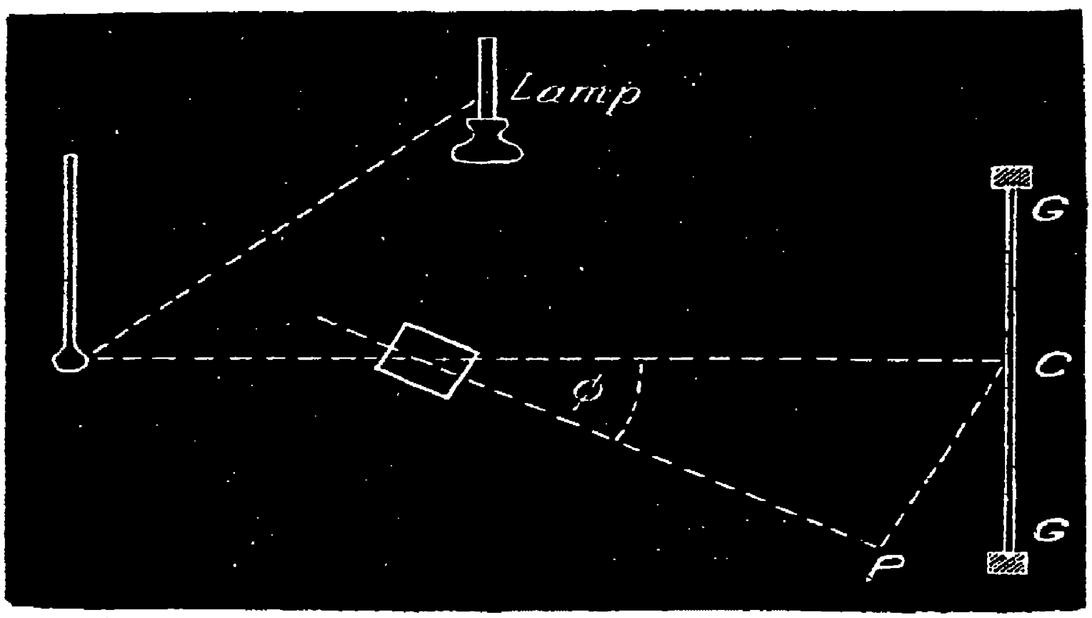

FIG. 77.

through a lens. It would not be difficult to investigate these two sources of error separately, but the results would be of little or no practical value.

"16. *Astigmatism. Approximate Diameter of the Disc of Diffusion in the Image of a Point, with stop C.I. No. —— at —— inches from the centre of plate =* 0·—— *inch.*

"The following is the method of examination for astigmatism:—The room is darkened, and in front of the lens is placed a thermometer bulb, thus obtaining, by means of the reflection of the light of a small lamp,

a fine point of light. The lens-holder of the testing camera is revolved upwards or downwards about the horizontal axis so that the axis of the lens makes an angle, ϕ, with the path of the rays coming from the thermometer bulb; the angle ϕ is such that the point of observation represents the extreme corner of the plate of the size for which the lens is being examined; that is to say, if, in Fig. 77, G G represents the position of the ground glass, then C P is equal to half the diagonal of the plate; this angle has already been found for previous tests. If the lens shows any astigmatism, the image of the point of light can be made to appear, first as a fine vertical line, and then, as the focus is lengthened, as a fine horizontal line. The focal scale is read off at each of these positions, and the difference, γ, between the two readings gives a measure of the astigmatism."

Major Darwin then shows how to calculate the size of the patch of light in the image caused by this astigmatism.

"17. *Illumination of the Field. The figures indicate the relative intensity at different parts of the plate.*

With C.I. Stop No. .			With C.I. Stop No. .	
At the centre.........	100	:	Ditto..................	100
At in. from the centre		:	Ditto..................	
At in. from the centre		:	Ditto..................	

"The intensity of illumination of the field is always greatest near the axis of the lens, and falls off more or less rapidly towards the edges of the plate. The lens should therefore be examined with the view of ascertaining if this inequality of illumination is greater than that which experience shows must be tolerated under given circumstances. The apparatus employed for conducting this test is shown in Fig. 78, the method being devised by Captain Abney. There is a fixed lamp L,

the position of which is not changed during the observations; F represents a paper screen, placed in that position in order to give a practically uniform source of light; O is the lens, which is fixed in a frame, not shown in the sketch, revolving about the pivot N; by means of a suitable adjustment, this axis, N, is made to pass through the nodal point of emergence of the lens. At S there is a sheet of cardboard with a small hole in the centre at H, and this screen, hole and all, is covered with thin white paper on the side away from the lens; the distance between H and N is always

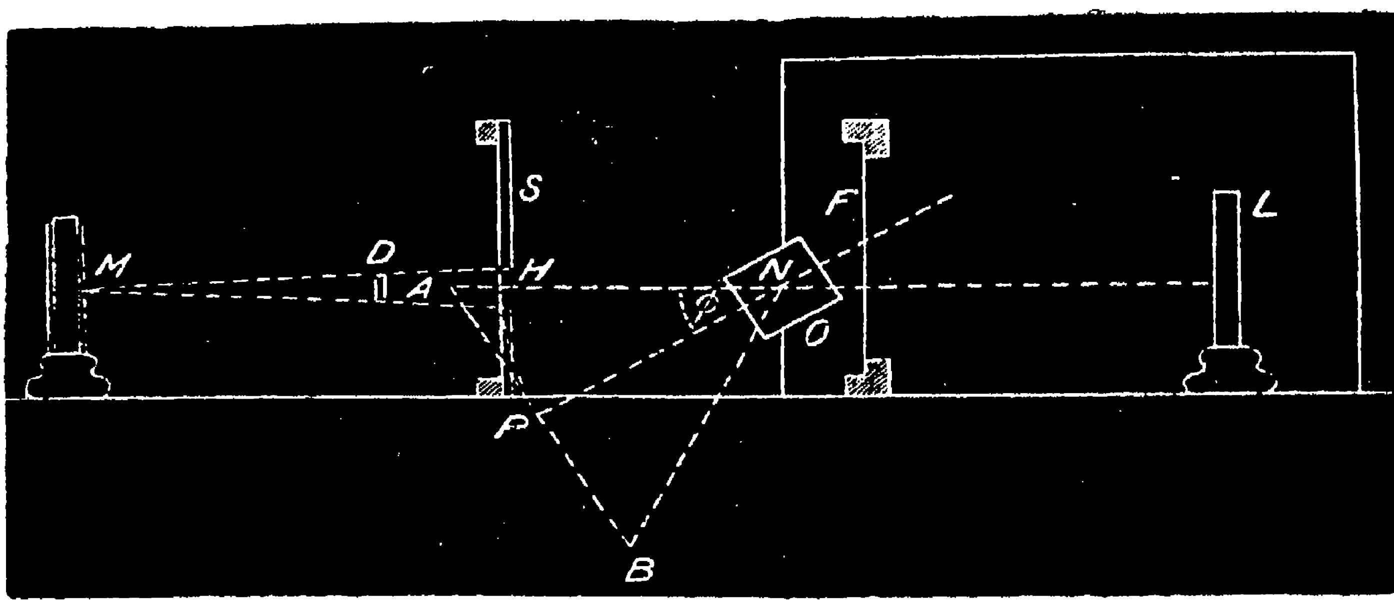

Fig. 78.

made equal to the principal focal length of the lens; the bar D is made to cast a shadow from the movable lamp M on the paper just over the hole in the cardboard; thus, in this shadow, the paper is illuminated entirely by transmitted light from the lens, whilst the paper round it is illuminated entirely by the light of the movable lamp.

"An observation is made in the following manner:— The lens is first placed in such a position that its axis passes through the hole H; the lamp M is then moved backwards or forwards until the transmitted illumin-

ation of the paper at H is made to match as nearly as possible the reflected illumination of the paper round it; the distance between S and M is then noted. The lens is now placed in the position shown in Fig. 78, where A B represents the length of the diagonal of the plate for which the lens is being examined, and where the angle ϕ is half the angle of field under examination. The balance of light is readjusted by a movement of the lamp, and the distance M S is read off a second time. By finding the inverse ratio of the squares of these two readings, we obtain the ratio between the illuminations at P and H, the lens being in the position shown in the sketch, and the object being supposed to be equally illuminated in both cases. But what is wanted is the ratio between the illuminations on the plate at P and A; this is found with perfect accuracy by multiplying the ratio of the illumination at P and H, as above obtained from the observations, by $cos^3\ \phi$, and this result is that which is entered in the Certificate of Examination."

The reason for multiplying by $cos^3\ \phi$ is as follows:—

The difference between the illuminations at H and A is due to two causes, first the different distances of H and A from N, and secondly the obliquity of the plate A B to the incident light.

Let I_H and I_P be the illuminations at H and P, and let I_A be the actual illumination of the plate as inclined, I_a what the illumination would be if the plate were at right angles to the incident light; then (§ 15, *a*) we have seen that

$$I_A = I_a \cos \phi$$

Also by the law of the inverse squares (§ 15)—

$$\frac{I_a}{I_A} = \left(\frac{N\ H}{N\ A}\right)^2 = \left(\frac{N\ P}{N\ A}\right)^2 = \cos^2 \phi, \therefore I_a = I_H \cos^2 \phi$$

combining the two results,

$$I_A = I_a \cos \phi = I_H \cos^3 \phi$$

$$\therefore \frac{I_A}{I_P} = \frac{I_H}{I_P} \cos^3 \phi$$

which is the required result.

The extracts from Major Darwin's paper may be terminated by the concluding paragraph in the discussion of the last test.

"In connection with this test it may be mentioned that the most serious omission in the Kew examination is, that there is nothing to show the actinic transparency of the glass. A slight yellow tinge in the lenses, which would not be noticed by the eye, might yet be sufficient to seriously affect the rapidity of the objective. But

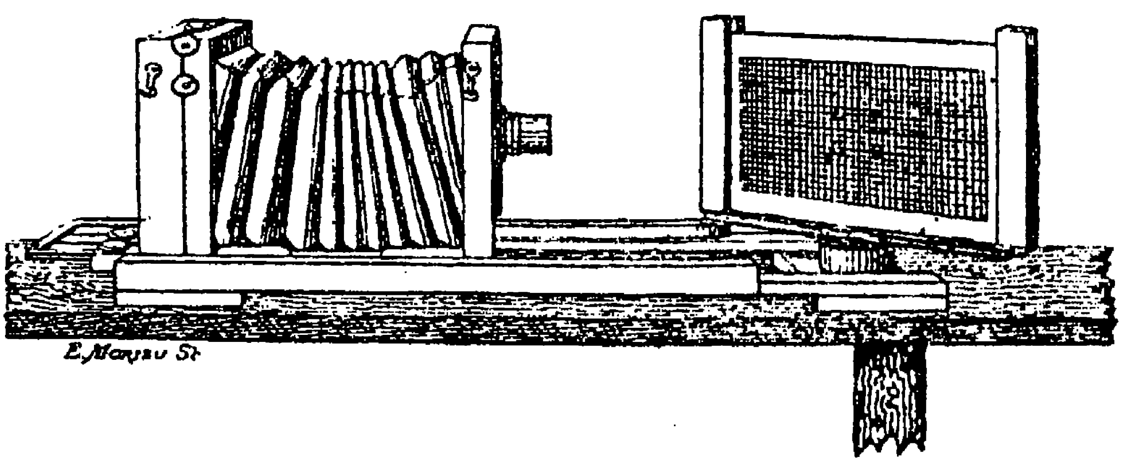

FIG. 79.

no test could be devised to investigate this point which did not introduce photographic methods, and, as already stated, the consideration of expense put such operations out of consideration for the present. I should like, if possible, to have introduced some test which would have at the same time indicated the actual rapidity of the lens, and also the actual falling off of density towards the margin of the photograph; with the aid of photography this would not have been difficult, and a plan of this kind would have been adopted, but for the cost. This subject is, however, still under consideration by Captain Abney."

116. Rapid Test of a Lens.—In the *Traité Encyclo-*

pédique de Photographie[1] is given a rapid method, due to M. Baille-Lemaire, by which a photographic test can be made; the information furnished is not so extensive or reliable as that given by the methods described above, but this method has the advantage over the former that it is within the power of any photographer to execute it. The arrangement is shown in Fig. 79; a screen ruled with horizontal and vertical lines which divide it into squares is placed so that the

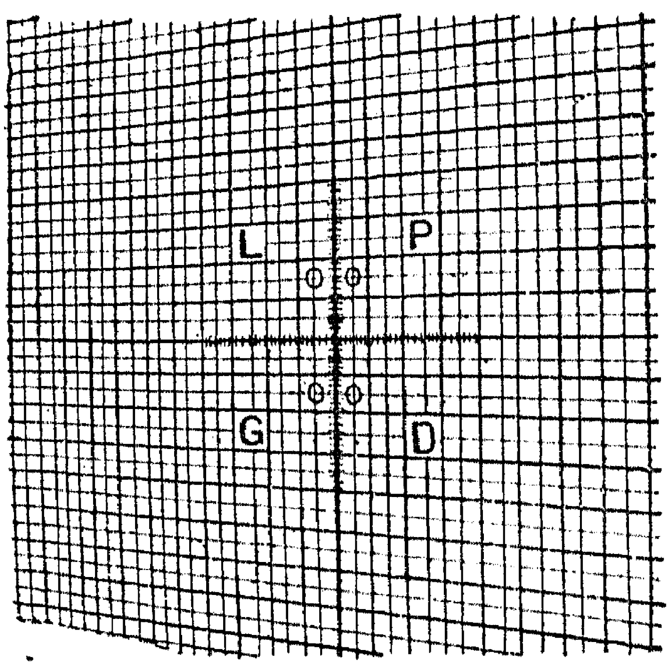

FIG. 80.

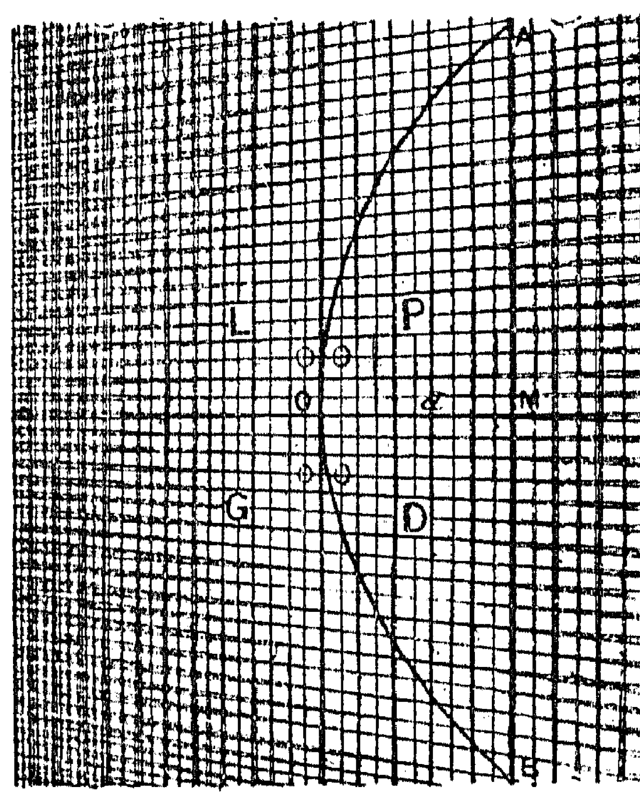

FIG. 81.

axis of the lens passes through its centre, and its plane is inclined at an angle of 45° to this axis. The centre vertical line of the screen is then focussed sharply and a photograph taken: the appearance of the screen is shown in Fig. 80, and its photograph will look like Fig. 81: the letters L, P, G, D are used to enable the corresponding portions of the original and the photograph to be easily identified.

[1] *Premier Supplément*, par C. Fabre, p. 125, Ed. 1892. Gauthier-Villars, Paris.

The *principal focal length*, disregarding the nodal points, can be found roughly by measuring the corresponding distances of object and image from the centre of the lens and calculating in the usual way. The *achromatism* is tested by examining if the vertical line which is sharpest in the photograph is the same as that which was focussed; if it is not so, an estimate can be made of the distance between the foci for visual and actinic rays, by noting the sharpest line, finding the difference between the distances of the two lines from the lens, and calculating as in § 110.

The *curvature of the field* can be judged by joining the sharpest points in the photograph by a continuous curve, as A O B in Fig. 81; this curve is *roughly* a section of the principal focal surface by a plane, inclined at an angle of 45° to the axis of the lens, and passing through the vertex of the surface.

The *depth of focus* can be examined by taking photographs of the screen while the ground glass is moved slightly and the screen kept still. The lens can be examined for *distortion* by placing the screen at right angles to the axis of the lens and examining if the lines in the photograph are curved or not.

ties of illumination of the plates in the cases to be compared.

119. The Illumination of the Plate.—In comparing intensities of illumination we shall suppose that the object in each case is of the same brightness, and that it consists of a plane of uniform brightness placed at some definite distance from the lens and at right angles to the axis; this will simplify our ideas and will not alter the result. Let us consider the illumination of a small circular area of the plate having its centre where the axis of the lens cuts the plate; if we know the illumination in this region and the relative illuminations as found in the Kew test, No. 17 (§ 115), we can find the illumination at any other part of the plate.

Let (Fig. 82) A B be the lens, C D the aperture in the stop (the lens drawn is a simple one with the stop in front, but the theory will apply to lenses of all kinds), X Y the plane object, $x\,y$ the ground glass on which the image is focussed. Suppose the lens thin and its centre o, let f be its focal length, u and v the distances of object and image respectively from o, and z the distance of the stop from o; in the case of a compound lens u and v will be measured as usual from the nodal points, and e will be the distance of the stop from the nodal point of incidence. Let $e\,f$ be the radius of the circular disc whose illumination is considered, and E F the radius of the corresponding disc on the object; we shall for convenience denote the discs by (E F), ($e\,f$). Let d be the diameter of the aperture of the stop. We must now inquire how the illumination of ($e\,f$) is affected by the sizes of the various quantities; let us see what effect the variation of the various quantities will have.

All the light that reaches ($e\,f$) comes from (E F), but all the light from (E F) does not reach ($e\,f$). The quantity of light from (E F) that reaches ($e\,f$) is the whole quantity that gets through the aperture C D; this depends on two things, the illumination at C D produced by (E F), and the area of the aperture C D. If

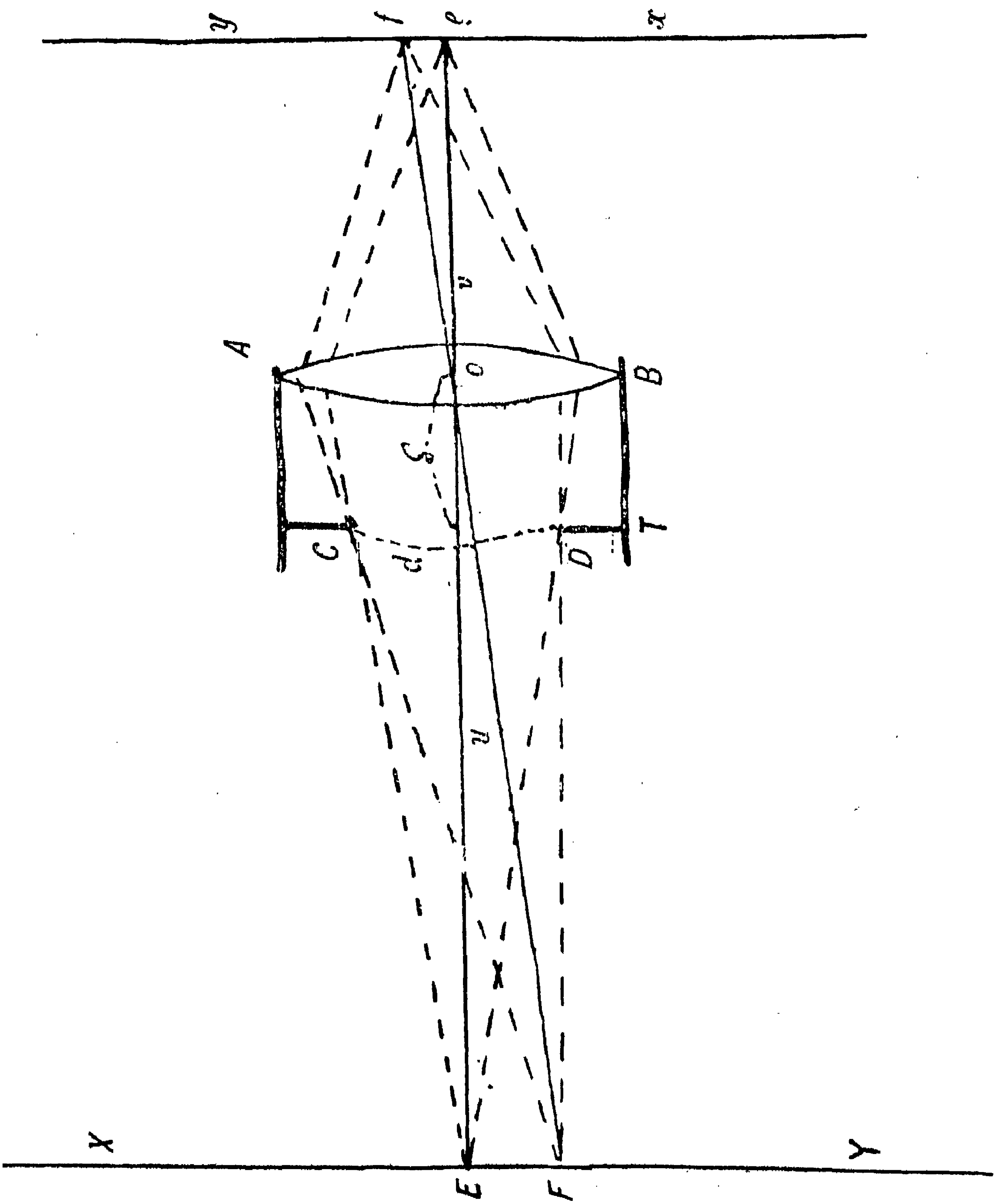

FIG. 82.

the distance of (E F) from C D, and hence the illumination, remains constant, then the illumination of ($e f$) is

proportional to the area of aperture C D, but this area is $\pi d^2/_4$ and is therefore proportional to d^2; the illumination of (ef) therefore varies as d^2.

Next suppose that the aperture C D remains unchanged, and the distances of object and image are varied, the consequent change in the illumination of (ef) is due to two causes.

(1) The distance of X Y from C D varies, and consequently the illumination at C D varies. We have seen (§ 14) that the illumination of an area, due to a source of light, varies inversely as the square of the distance; hence, on this account, the illumination varies as $1/(u - e)^2$.

(2) The area of the object to which the area (ef) owes its illumination varies as the distances u and v vary. Let A be the area of (E F), A′ that of (ef), then by similar triangles F E O, $f e O$—

$$\frac{\mathrm{A}}{\mathrm{A}'} = \frac{\pi \cdot \mathrm{E\,F}^2}{\pi \cdot ef^2} = \frac{u^2}{v^2} \text{ or } \mathrm{A} = \frac{u^2}{v^2}\mathrm{A}'$$

Since A remains (by hypothesis) unchanged, its illumination will be proportional to A, so that from this cause the illumination varies as A or as u^2/v^2.

Putting together these two effects of varying u and v, we see that, taking both of them into account, the illumination of (ef) varies as

$$\frac{u^2}{v^2} \times \frac{1}{(u - e)^2}$$

Now, it is shown in works on Algebra that if a quantity A depends on two others a and b, and if it varies as a when b is constant, and as b when a is constant, then if both vary, A varies as a and b jointly or $\mathrm{A} \propto a\,b$. So here if I be the illumination of (ef) and both the aperture C D and the distances u and v are varied together, then

$$\mathrm{I} \propto d^2 \times \frac{u^2}{v^2} \times \frac{1}{(u - e)^2}$$

Now the time of exposure varies inversely as the ilumination, for the greater the illumination the shorter is the time that is required for a given quantity of light to fall on the plate; hence if T be the time of exposure

$$T \propto \frac{1}{d^2} \times \frac{v^2}{u^2} \times (u - e)^2$$

which may more conveniently be expressed by

$$T = \frac{\lambda}{d^2} \times \frac{v^2}{u^2} \times (u - e)^2$$

where λ is a number whose value depends on the illumination of X Y and the sensitiveness of the plate.

This expression is applicable to the case of objects at all distances, and it can, if required, be expressed entirely in terms of v, the distance of the image from the lens; in many cases e may be neglected in comparison with u, and the expression then simplifies to

$$T = \frac{\lambda}{d^2} \cdot \frac{v^2}{u^2} \cdot u^2 = \lambda \frac{v^2}{d^2}$$

120. Expression when the Object is Distant.—When, as in ordinary outdoor work, the object is distant, the distance v becomes equal to f, the principal focal length of the lens (and u being great, e can be neglected), which gives the relation

$$T = \lambda \left(\frac{f}{d}\right)^2$$

which is the formula usually given; expressed in words it tells us that the time of exposure is proportional to the square of the number got by dividing the focal length of the lens by the diameter of the aperture of the stop.

121. The Quantity λ.—As stated above, the quantity λ depends on the illumination of the object and the sensitiveness of the plate; the latter quantity is now usually given by the makers of the plates on a scale

which will be explained later (§ 126), but the former is one that must be determined by the judgment of the photographer. Various tables have been published giving methods of calculating the exposures necessary for various kinds of objects at different times of the year and at different times of day and with different states of the sky; these are the result of experiment, and it would be of no practical use to theorize about them. Actinometers have been invented which, by the rate of blackening of a piece of sensitized paper, enable some estimate of the brightness of the day to be made, but it is doubtful if they are of much practical use, for often not only the nature but the situation of an object has to be considered, and the proper course can after all be determined only by experience.

122. Sizes of Stops.—In naming the sizes of the apertures in the stops of a lens it is convenient to adopt a system in which the name will give a clue to the relative exposure required with the stop. Several systems have been proposed, but the one which has been longest established and seems likely to die hard is as follows:—

The stops are denoted by the ratio f/d of the focal length of the lens to the diameter of the stop; and if, for instance, $f/d = 10$, then the stop is called $f/10$.

It will be seen at once from § 120 that the times of exposure are proportional to the squares of these numbers; thus, for instance, the exposure with the stop $f/20$ will be four times as long as with stop $f/10$, and so on.

Example.—To take a certain photograph an exposure of ten seconds is required with the stop $f/12$; find the requisite exposure with stop $f/32$.

Let t be the time required in seconds, then

$$\frac{t}{10} = \left(\frac{32}{12}\right)^2 = \left(\frac{8}{3}\right)^2 = \frac{64}{9}, \therefore t = \frac{640}{9} = 71 \text{ secs.}$$

The disadvantage of this system is that to find the

required exposure some calculation is necessary, which though easy enough at home is troublesome in the field; it is more troublesome still if we wish to find the size of a stop required to make the exposure twice or three times as long, for then we should have to extract the square roots of 2 and 3.

123. Other Systems for Stops.—Many other systems have been proposed for naming stops, the principle of them being that the number denoting the stop shall give the required information about the exposure with very little calculation; the numbering sometimes increases and sometimes decreases as the necessary exposure increases.

The two most important systems are those of the Photographic Congress at Brussels in 1889, denoted in the Kew Certificates as the C.I. system, and that adopted by the Photographic Society of Great Britain and denoted U.S.N. (*Uniform System* Numbers).

In the C.I. system the stop $f/10$ is taken as the starting-point and called No. 1; the time of exposure with this stop is taken as the unit exposure; the remaining stops are numbered so that the greater the number of the stop the longer the exposure required; No. 2 is taken to require double the exposure of No. 1; No. 3 three times that of No. 1, and so on. The rule to find the C.I. number of a stop is to divide the square of the focal length by 100 times the square of the diameter of effective aperture of the stop.

In the U.S.N. system the stop $f/4$ is taken as the starting-point, and then the stops are numbered as in the C.I. system, so that the time of exposure is proportional to the number of the stop.

Zeiss has his own system of marking the stops of his lenses: he takes $f/100$ as the starting-point, and numbers his lenses so that the times of exposure are inversely proportional to the numbers of the stops.

The following table shows the connection between the three systems; the numbers given are enough to enable

an idea of the relative values to be formed at a glance.

The C.I. numbers are not worked out as closely as the U.S.N. system, the nearest whole number in the common series corresponding to whole numbers in the C.I. system being given.

TABLE SHOWING THE CONNECTION BETWEEN THE DIFFERENT SYSTEMS FOR NAMING STOPS.

Ratio f/d Common System	C.I. System (Approximate)	U.S.N.	Zeiss	Ratio f/d Common System	C.I. System (Approximate)	U.S.N.	Zeiss
$f/1$		1/16		$f/25$		39·06	16
$f/1·414$		1/8		$f/28$	8	49·00	
$f/2$		1/4		$f/30$		56·25	
$f/2·828$		1/2		$f/32$	10	64·00	
$f/3$		1/·562		$f/35$	12	76·56	
$f/4$		1·00		$f/36$		81·00	8
$f/4·5$		1·26	512	$f/39$	15	95·06	
$f/5$	1/4	1·56		$f/40$	16	100·00	
$f/5·656$		2·00		$f/45$	20	126·56	
$f/6$		2·25		$f/45·25$		128·00	
$f/6·3$		2·47	256	$f/49$	24	150·06	
$f/7$		3·06		$f/50$		156·25	4
$f/7·1$	1/2	3·07		$f/55$	30	189·06	
$f/8$		4·00		$f/56$		196·00	
$f/8·7$	3/4	4·88		$f/57$	32	203·06	
$f/8·8$		4·87		$f/60$		225·00	
$f/9$		5·06	128	$f/63$	40	248·06	
$f/10$	1	6·25		$f/64$		256·00	
$f/11$		7·56		$f/69$	48	297·56	
$f/11·31$		8·00		$f/70$		306·25	
$f/12$		9·00		$f/71$	50	315·06	2
$f/12·5$		9·80	64	$f/75$	56	351·56	
$f/14$	2	12·25		$f/77$	60	370·56	
$f/16$		16·00		$f/80$	64	400·00	
$f/17$	3	18·06		$f/88$		484·00	
$f/18·5$		21·45	32	$f/90$		506·25	
$f/20$	4	25·00		$f/90·50$		512·00	
$f/22$	5	30·25		$f/96$		576·00	
$f/22·62$		32·00		$f/100$		625·00	1
$f/24$	6	36·00					

124. Exposure with Dallmeyer's Telephotographic Lens.—The principle of this lens is that a certain portion of the image formed by the front converging lens is picked out and magnified by the diverging lens; thus the quantity of light which, in a given time, falls on any particular area of the magnified picture is the same as that which would have fallen, in the same time, on the corresponding portion of the picture formed by the front lens. Thus the intensities of illumination in the two cases are inversely proportional to the areas of the corresponding pictures. For instance, let the ratio of the *linear* dimensions of the two pictures (or as Dallmeyer calls it the magnification) be $3\frac{1}{2}$, then the ratio of the two areas is $(3\frac{1}{2})^2$ or $49/4$; thus the light which would fall on a given area in the picture formed by the front lens alone has to cover an area $49/4$ times as great in the magnified picture.

Hence it is clear that the exposure for the magnified picture will be $49/4$ times as long as for the picture that would be formed by the converging combination alone.

Reasoning in this way we see generally that the time of exposure required with the telephotographic lens can be got from that required with the front lens alone by multiplying by the square of the linear magnification; where "linear magnification" has the meaning given to it above.

125. Transparency.—In the preceding sections it has been assumed that the glass of a lens causes no loss of light either by absorption or reflection, which is by no means the case. It is not often that the glass is seriously coloured, but in every lens light is lost by reflection and scattering at every surface (§ 10), and objectives with few surfaces let through more light than do complicated objectives.

Hence in the expression for the time of exposure in § 119, the quantity λ will depend on the nature and construction of the lens as well as on the other quantities

enumerated in § 121. The expression $T = \lambda v^2/d^2$ cannot therefore be relied on to compare the times of exposure with different lenses; but it will in most cases give a very fair idea of a required exposure. If two or three lenses are being constantly used, the operator will in a short time be able to form a fair idea of the relative speeds, and to find out how to modify the exposures found from the formula.

It is not hard to make a photographic examination of the relative powers of lenses by photographing the same object with constant illumination, trying various exposures with the different lenses; the experiment can be made with an ordinary camera and slide. The object should be a uniformly illuminated object of some size, such as a white wall on a fairly bright day; focus the object, and find as closely as possible the proper exposure with one of the lenses to be examined. In the final test the *same* plate must be used with the different lenses, to avoid differences in the development; this can be done without trouble.

When using the lens for which the proper exposure is known, draw the slide to expose only one-third of the plate, and use a small stop to make the exposure required as long as possible. Now take the slide to the dark room and turn the plate round, end for end, so that if the slide is now drawn it is the unexposed portion which is first exposed. Then fix the second lens to the camera and use the stop corresponding to that used in the first case, so that if the lenses were similar the exposures required should be equal; suppose the second lens is slower than the first. Draw the slide to uncover nearly all the unused portion of the plate, and expose for the same time as with the first lens; then push in the slide to hide a strip of the exposed plate, and expose for a short time longer, say half a second; again push in the slide to hide another strip of the plate, and expose for another half-second; and so on till three or four exposures have been made.

The effect is that different strips of the plate are exposed for different times, and can easily be recognized on development. Develop the plate and then compare the densities of the strips taken with the second lens with that of the part taken with the first lens. The strips of density equal to that of the first part will show what exposure with the second lens is equal to that with the first. From the result obtained the correction to be made in calculations of relative exposures can be calculated.

If a very exact determination is required, the relative illumination at different parts of the plate, as found by the Kew test, must be taken into consideration. The exposures should be made as long as possible to make the method capable of giving good results, for it is impossible to give a shorter exposure by hand than half a second, with any certainty; and if the exposure with the first lens was only one or two seconds, an error of even one-fourth second may make a large difference in the result.

Example.—The speeds of two lenses are to be compared, the stop $f/40$ is used with each; the exposure with the first lens is 12 seconds, and that with the second lens is found by experiment to be $10\frac{1}{2}$ seconds.

Here, since corresponding stops were used, the exposures required in the two cases should have been equal, had the lenses been quite similar; the longer exposure required with the second lens is due to the larger loss of light. Equal effects are produced by equal quantities of light, so that the ratio of the illumination produced by the second lens to that produced by the first is $12/10\frac{1}{2}$ or $24/21$ or $\cdot87$.

Hence the exposures required with the first lens are only ·87 of the corresponding exposures with the second lens.

126. Sensitometers and Sensitometer Numbers.—To compare the sensitiveness of different plates, we require evidently a constant source of illumination, and a means

of exposing the plate to certain definite portions of this illumination. The sensitometer most generally known and used is that of Warneke. The scale of the instrument consists of a plate of glass composed of twenty-five different pieces, tinted so as to be of constantly increasing opacity; on each piece of glass is placed an opaque number. This scale is placed in a special holder, in contact with the plate to be tested, a sheet of black paper being placed behind the plate.

The source of light employed is a phosphorescent plate of sulphate of calcium; to excite the phosphorescence a magnesium ribbon about 3 cm. long, ·015 cm. thick, and ·2 cm. breadth is burned as near as possible to the plate. Immediately the magnesium is burned, 60 seconds are counted, and during the 30 seconds which follow, the luminous plate is placed on the scale; the plate is then developed.

When developed the plate shows the numbers on the squares, the tints of which get fainter and fainter; the last number visible represents the sensitiveness, and is given as the sensitometer number.

In comparing two plates care must be taken to develop them under the same conditions and with identical proportions of fresh solution. Since the Warneke sensitometer was devised the sensitiveness of plates has been increased, and the original scale is not long enough; either the scale is extended, or makers make an estimate from what is shown by the old scale. In Fig. 83 is shown the result of the test on a Sandell II. plate; the sensitiveness is estimated at 28.

Hurter and Driffield[1] have published a careful investigation on the action of light on sensitive plates, and have devised a method for the estimation of sensitiveness; their paper is well worthy of careful study. It is impossible to give here more than a short abstract of the part which concerns the subject in hand.

[1] *Journal of the Society of Chemical Industry.* No. 5. Vol. ix. May 31, 1890.

They take as the density a quantity which is proportional to the amount of silver reduced by the action of the light and of the developer, per unit area of the sensitive film; this differs from Abney's definition (§ 19). They have devised a special form of photometer, with the scales arranged to read off the density directly. A series of experiments was made in which portions of the same plate were exposed for various times and then developed; the densities were then

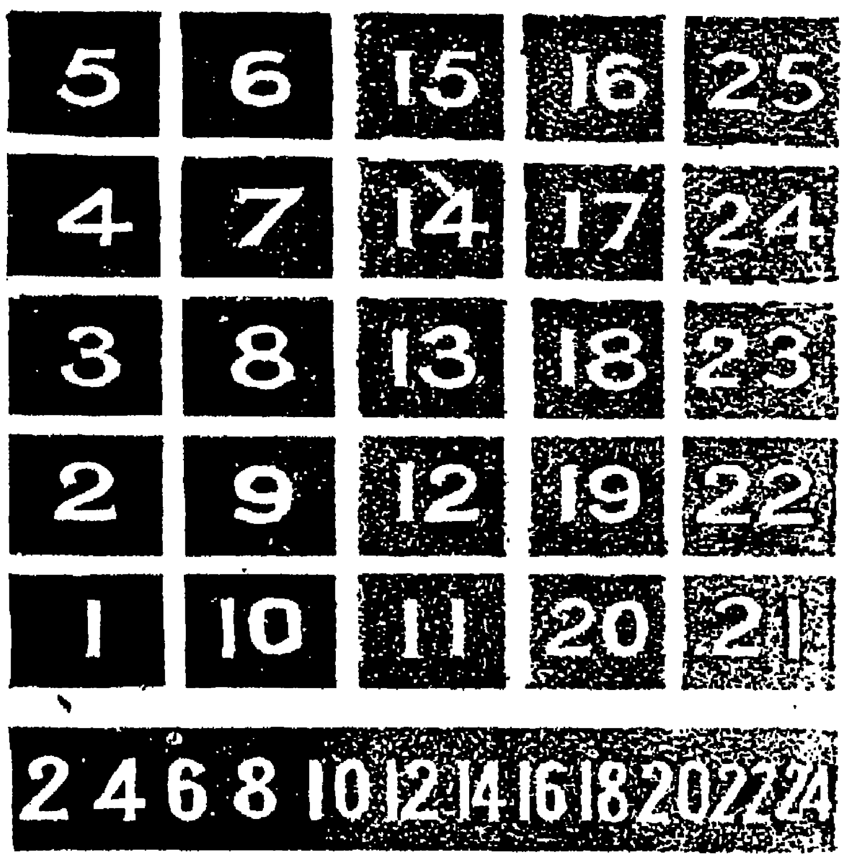

FIG. 83.

measured. It was found that the whole time of exposure might be divided into four periods: first came the period of under-exposure, during which the density increased very slowly with the exposure, but was nearly proportional to it; during the second period the density increased much more rapidly with the exposure, and the contrasts produced were in consequence sharp, this was called the *period of correct representation;* in the third period the density again increases slowly

with the exposure up to a maximum, this is the period of over-exposure; finally the last period is that during which the density diminishes with the exposure, and reversal takes place.

It was found both from theory and experiment that if the exposure lies in the second period, the connection between the density D, obtained with a given time of exposure of t seconds, is of the form

$$D = k \log \left(\frac{I\, t}{i}\right) \quad . \quad . \quad . \quad (a)$$

where I is the intensity of the light incident on the plate, and k and i are constants depending on the nature of the plate.

If two experiments are made we can from these calculate the values of k and i, and then use the formula to determine the densities produced by other exposures.

The intensity I of the light incident on the plate is measured in terms of that produced by a standard candle, placed at a distance of 1 metre from the plate; the exposure E, or the total quantity of light which falls on the plate, is proportional to the product of I into the time of exposure, so that we may put $E = I\, t$, and write the above relation

$$D = k \log \left(\frac{E}{c}\right) \quad . \quad . \quad . \quad (b)$$

If the time is measured in seconds, the unit of exposure here used is called a candle-metre second, or c.m.s.

The quantity i was called the inertia of the plate. If two plates of inertias i and i_1 are to be impressed with the same density by exposure to light, whose intensity is the same in each case, for times t and t_1 then will

$$\frac{I\, t}{i} = \frac{I\, t_1}{i_1} \text{ or } \frac{t}{i} = \frac{t_1}{i_1}$$

for then only would D have the same value for each.

This means that to produce similar effects on two plates, the times of exposure must be proportional to their inertias, and hence a knowledge of the inertia of a plate will enable us to form an estimate of its rapidity.

In order to find the quantity i for a plate: "We give to the plate at least two exposures falling within the period of correct representation and develop. We then measure the densities exclusive of fog. We thus obtain two equations connecting the two densities D_1 and D_2 with the two known exposures E_1 and E_2, viz.:

$$D_1 = k \log \frac{E_1}{i} \text{ and } D_2 = k \log \frac{E_2}{i}$$

from which we obtain by elimination

$$\log i = \frac{D_2 \log E_1 - D_1 \log E_2}{D_2 - D_1} \quad "$$

Hence the value of $\log i$ can be calculated and the value of i found.

In practice the central portion only of the plate should be used, as the film is liable to be of unequal thickness at the margin. In order to ensure at least two exposures falling within the period of correct representation, eight exposures of 2·5, 5, 10, 20, 40, 80, 160, and 320 c.m.s. (candle-metre second units) are given; a strip of plate is left unexposed, but is developed in order to make allowance for any fogging that may occur. Too great density is avoided, but a decided deposit is obtained for the lower exposures.

Example.—With a certain plate the following measures were made:—

Exposures	2·5	5	10	20	40	80	160	
Densities	·085	·175	·250	·460	·755	1·01	1·27	
Differences	·09	·075	·210	·295	·255	·260		

On looking at the differences between the densities for the various exposures we see that the exposures

2·5, 5, and 10 c.m.s. lie in the first period, and that exposures 20 to 160 c.m.s. lie within the period of correct representation. Choosing exposures 20 and 160 for calculation we get

$$log\ i = \frac{1{\cdot}27 \times log\ 20 - {\cdot}460 \times log\ 160}{1{\cdot}27 - {\cdot}460} = {\cdot}787$$

Hence $i = 6{\cdot}12$.

The speed of the plate is the inverse of the quantity i, for the greater the speed the shorter should be the exposure.

The quantity which is quoted by Hurter and Driffield as the speed is the value of $34/i$; for instance, for three grades of plates of a certain make called ordinary, rapid, and extra rapid, the values of i were found to be 2, 1·4, and ·56; the speeds are 17, 24, and 60 respectively.

The calculation of relative exposures with these numbers is made in the same way as with those of the Warneke system.

Example.—With the ordinary plate just mentioned the time of exposure required was 5 seconds; find the time of exposure required with the extra rapid plate; let t be the time of exposure required, then—

$$\frac{t}{5} = \frac{17}{60} \text{ or } t = 1{\cdot}42 \text{ sec.}$$

Hurter and Driffield at the beginning of their paper *assume* that with a given thickness of film, the proportion of incident light that is stopped is proportional to the quantity of silver precipitated per unit area. For instance, that if a certain quantity of silver cuts off one quarter of the incident light, then double the quantity of silver will cut off one half of the light, the thickness remaining constant. The correctness of this assumption is open to doubt; in cases where the quantity of silver is small, so that the particles are not crowded, it is most likely true, but if there is a large quantity of silver

already present, so that nearly all the light is intercepted, then if more silver be introduced it is very likely that some of it will merely lie behind that already present and not add to the opacity.

On the other hand, the silver is reduced in the film by the action of light, and hence all the silver present will probably be in such a position as to intercept light. So that though the assumption may not be true in general, yet in the special case of photography it is probably trustworthy.

127. Exposure for Objects in Motion.—When photographing objects in motion, there are considerations other than those of the sensitiveness of the plate that have to be attended to in estimating the exposure; for if the exposure be too long the object will have moved over a sensible distance on the plate and blurring will result.

All rapid exposures, though commonly called instantaneous, last for a definite time, and during that time the image of the moving object is moving across the plate; if the line traced by any point of the image exceeds a certain length it can be seen as a line, but if less than that length it will be to the eye indistinguishable as a point.

It is impossible to state this length exactly, but it may roughly be taken to be 1/100 inch; if the distance from which the picture is to be viewed is large, it may be made greater with safety.

To enable an estimate of the largest possible exposure to be made, we must find the speed at which the image traverses the plate, when we know the velocity of the object.

First let the object be moving parallel to the plate (Fig. 84), let A B be the distance moved by a point of the object in *one* second, and $a\,b$ the corresponding distance moved by the image on the plate. Let u and v be the distances of object and image from the lens, and let V be the velocity of the object in inches per second,

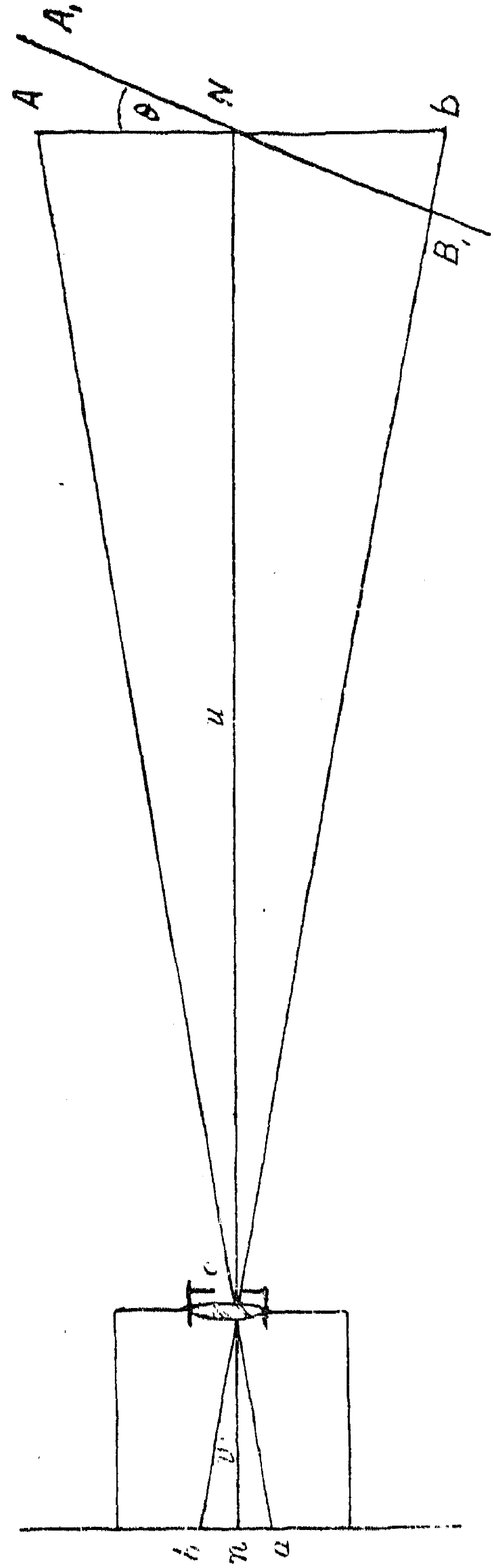

FIG. 84.

then A B = V inches; let f be the principal foca
length of the lens.

Then by similar triangles—

$$\frac{ab}{AB} = \frac{on}{ON} = \frac{v}{u}, \therefore a\,b = \frac{v}{u}\,A\,B = \frac{v}{u}\,.\,V \text{ inches.}$$

$$\text{but}\ \frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \therefore \frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{u+f}{uf}$$

$$\therefore ab = \frac{f}{u+f}.\ V \text{ inches} \quad . \quad . \quad . \quad (a)$$

Example.—The object is a train moving at the rate of 30 miles an hour, and it is distant 60 feet; the focal length of the lens is 6 inches.

Here 30 miles an hour = 44 feet per second.

$$\therefore V = 44 \times 12 \text{ inches per second}, u = 60, f = -6$$

$$\therefore ab = -\frac{1/2}{60 - 1/2} \times 44 \times 12 = -\frac{44 \times 12}{119} = -4\cdot4 \text{ inches.}$$

Hence, in this case the image would move 4·4 inches on the plate in one second, and to get a sharp picture the exposure must not be much more than one five-hundredth of a second.

The negative sign means that object and image move in opposite directions.

The accompanying table (p. 270), calculated by Mr. Henry Tolman, shows the number of inches through which the image moves on the ground glass, in one second, when the object is moving with various velocities and is 30, 60, or 120 feet distant from the camera; the focal length of the lens being 6 inches.

The distances moved through with lenses of different focal lengths may be found approximately from this table, for the distance is nearly proportional to the focal length, as an examination of expression (a) above will show.

If the motion is inclined at an angle θ to A B (Fig. 54) the result will not be quite the same as in the former case. Let the object move along A_1 B_1 inclined at

θ to A B, with velocity V inches per second, the resolved part of its velocity parallel to A B is shown to be V $\cos\,\theta$; we must use this in place of V and the relation becomes

$$ab = \frac{f}{u+f}.V \cos\,\theta$$

LENS 6 IN. EQUIV. FOCUS, GROUND GLASS AT PRINCIPAL FOCUS OF LENS.

Miles per Hour.	Feet per Second.	Distance on Ground Glass in inches with Object 30 ft. away, in one second.	Same with Object 60 ft. away.	Same with Object 120 ft. away.
1	$1\frac{1}{2}$	·29	·15	·073
2	3	·59	·29	·147
3	$4\frac{1}{2}$	·88	·44	·220
4	6	1·17	·59	·293
5	$7\frac{1}{2}$	1·47	·73	·367
6	9	1·76	·88	·440
7	$10\frac{1}{2}$	2·05	1·03	·513
8	12	2·35	1·17	·587
9	13	2·64	1·32	·660
10	$14\frac{1}{2}$	2·93	1·47	·733
11	16	3·23	1·61	·807
12	$17\frac{1}{2}$	3·52	1·76	·880
13	19	3·81	1·91	·953
14	$20\frac{1}{2}$	4·11	2·05	1·027
15	22	4·40	2·20	1·100
20	29	5·87	2·93	1·467
25	37	7·33	3·67	1·833
30	44	8·80	4·40	2·200
35	51	10·27	5·13	2·567
40	59	11·73	5·97	2·933
45	66	13·30	6·60	3·300
50	73	14·67	7·33	3·667
55	80	16·13	8·06	4·033
60	88	17·60	8·80	4·400
75	110	22·00	11·00	5·500
100	147	29·33	14·67	7·333
125	183	36·67	18·33	9·167
150	220	44·40	22·00	11·000

Example.—Take the same data as in the last example, except that the motion is inclined at an angle of 60° to the former direction, then

$a\,b = -4{\cdot}4 \cos 60^\circ = -4{\cdot}4 \times {\cdot}5 = -2{\cdot}2$ in.

or the motion now is only half of what it was before.

128. Object moving Towards or Away from the Camera.—Such a case as this occurs when an express train is photographed from a bridge under which it passes; the image then remains fairly still on the plate, but grows larger or smaller as the object advances or recedes.

Using the same figure as before (Fig. 84) let A B be the object receding from the lens; let A N = x inches, and let A B recede z inches in one second. We have seen that

$$b\,n = \frac{f}{u+f} \cdot \text{A N} = \frac{f}{u+f} . x \text{ inches.}$$

Let $b_1 n$ be the length of $b\,n$ after one second, then, since the object is now distant $u + z$ inches

$$b_1\,n = \frac{f}{u+z+f} . x \text{ inches.}$$

The difference between $b_1\,u$ and $b\,u$ is

$$\frac{f\,x}{u+f} - \frac{f\,x}{u+z+f} = \frac{f\,z}{(u+f)\,(u+f+z)}\,x$$

or if z can be neglected on comparison with $u + f$ in the denominator, then

$$\text{Distance moved by } b = \frac{f\,z}{(u+f)^2}\,x \quad . \quad . \quad . \quad (b)$$

Example.—A railway engine, breadth 5 feet, moving at 30 miles an hour, is at a distance of 200 feet from the camera; the focal length of the lens is 6 inches.

Here we may neglect f compared with u and get

$$\text{Distance moved} = \frac{f\,z}{u^2}\,x$$

Now $f = -6$, $z = 44 \times 12$ inches, $x = 30$ inches, $u = 200 \times 12$ inches.

$$\therefore \text{ Distance moved} = - \frac{6 \times 44 \times 12}{(200 \times 12)^2} \times 30 =$$

$-$ ·0165 inch.

Hence the edge of the image of the train moves at the rate of ·0165 inch per second, and the negative sign means that the size of the image is decreasing as it should do.

On comparing the expressions we notice that in (b) there is the square of $(u + f)$ in the denominator, while in (a) the first power occurs; since $(u + f)$ is fairly large compared with the other quantities involved, we see that the motion on the plate will be much less when the object advances directly towards the camera than when it is moving parallel to the plate.

Shutters.

129.—When a short exposure has to be made some mechanical device is required to uncover and cover the lens, the hand cannot make an exposure much under a quarter of a second. Many forms of shutters have been designed and are well known to practical photographers; we do not propose to give descriptions of the various forms, but to enumerate two or three classes in which many shutters can be placed, and to examine the principles which apply to their use.

130.—Duration of Exposure.—Many shutters which are worked by springs are marked by the makers to indicate the times of exposure with given adjustments; as the numbers given are not always reliable it is well to be able to test them, which can be done without much trouble.

(1) If the time of exposure to be tested is a fairly large fraction of a second, the help of a friend, a fair-sized roll of white paper, and a good light are all that is

required. Place the friend 20 or 30 feet in front of the camera, holding the roll of paper in one hand; arrange the picture so that the shoulder of the hand holding the paper is in the centre of the picture and the whole of the roll of paper is visible when the longest side of the plate is horizontal. If the paper is then whirled round at arm's length it will during the whole of its path be within the limits of the picture. Then let the paper be whirled round so that one revolution is made in one second, and at the same time take a photograph with the shutter set at the speed to be tested.

It is not hard with a little practice to whirl the arm round once a second, but if this proves inconvenient the time of one revolution can easily be measured by taking the time of 10 or more revolutions; and the necessary changes can easily be introduced into the calculations.

When the photograph is developed the image of the roll of paper will not appear sharp, but spread out into the sector of a circle, owing to the angle through which the paper has moved during the exposure. The angle moved through can be measured with a protractor, and the time of exposure calculated from this.

Example.—The paper is found to be whirled round once in 1·2 seconds, and from the photograph it is found to have moved through an angle of 15° during the exposure.

Here the roll of paper revolves through 360° in 1·2 seconds, or through 1° in 1·2 360 seconds, or through

15° in $\dfrac{15 \times 1{\cdot}2}{360}$ seconds.

$$\text{Hence time of exposure} = \frac{15 \times 1{\cdot}2}{360} = \frac{1}{20} \text{ sec.}$$

Hence the time of exposure is one-twentieth of a second.

(2) For short exposures the first method is not

reliable, and another requiring more apparatus must be adopted. A disc, whirling uniformly at a fair speed, is required; it may be whirled either by clock-work or by hand, and it should revolve at least three or four times a second. An electromotor, if available, is very suitable, for when the driving current is kept constant the speed keeps very nearly constant. But a hand arrangement, with a large wheel driving a small one by means of a belt, can with practice be driven at a very nearly constant speed.

Cover the disc, which should be made as large as is convenient, with black or dark paper, and on this paste a small sector of white paper. First make an exposure when the disc is at rest, to show the actual size of the image of the sector; then whirl the disc at a known rate and make an exposure with the shutter to be tested.

On development, the photograph of the sector when still will be found to be sharp, while that taken when the sector was moving will be spread out. By comparison of the two images and the aid of a protractor the angle moved through by the disc during the time of exposure can be found.

The number of revolutions made in one second by the disc should be ascertained directly and not from the speed of the multiplying wheel, for there is always some slip when a belt is used; the most convenient way is to make a small blunt projection on the axle or pulley which carries the disc, and to feel this with the finger. Since this projection is near the axis of rotation, its speed is small and it will not hurt the finger; the number of revolutions in a given time, 30 seconds for instance, should be counted, and the time of one revolution calculated from this.

Example.—The disc makes 107 revolutions in 30 seconds; and the sector is found to move through an angle of 23° during the exposure.

Since the disc goes through 360° in one revolution the exposure was 23/360 of the time taken by the disc

to revolve once. The disc revolves 107 times in 30 seconds, hence it revolves once in 30/107 seconds; therefore time of exposure $= \frac{30}{107} \times \frac{23}{360} = \cdot 02$ second (nearly), or the exposure was about one-fiftieth of a second.

In experimental work the light of an electric spark has been used instead of a shutter; the exposure is very much less than that possible with the quickest shutter. In this case the revolving disc can still be used, but it must be revolved at a much greater speed, and the time of revolution must be estimated by the aid of a tuning-fork.

131. Efficiency of a Shutter.—When using a shutter the effect on the plate is not correctly measured by the total interval that elapses between the time when the shutter begins to open and that when it is finally shut, or in other words we must not reckon the exposure as if the total aperture were unclosed from the beginning to the end of the exposure. A little reflection will show that the exposure may roughly be divided into three periods, the first that during which the shutter is opening, the second that during which the full aperture is open, the third that during which the shutter is closing; with some shutters the second period is absent. During the first and third periods the whole aperture is not unclosed, and consequently the illumination of the plate is not then as great as during the time when the aperture is quite open.

What we want to know, for practical purposes, is what exposure, with the whole aperture unclosed, is equivalent to that actually given; we shall call this the *equivalent* exposure, and the time between the first opening and the last closing, the *nominal* exposure.

The ratio $\frac{\text{equivalent exposure}}{\text{nominal exposure}}$ is called the *efficiency* of a shutter.

Let us take the simplest case possible, though it is

not one realized in practice; imagine the aperture to be square, and let this be uncovered and covered by the sliding in front of it, with uniform speed, of a panel pierced with a square hole equal in size to the aperture, the hole being so arranged as to exactly coincide with the aperture in one position. Consideration will show that every part of the aperture is uncovered for a time equal to half the time of the total exposure, and the aperture is of the same breadth from top to bottom; hence the total exposure is the same as would have been given with the whole aperture uncovered for half the time. Thus the efficiency is one half.

We have here assumed the speed of the shutter to be uniform; this is not at all likely to be the case, and except when the speed is uniform the efficiencies are very hard to calculate, even in simple cases, such as when the shutter falls under gravity; and in most cases the mode of motion is unknown. But a consideration of the efficiency of different forms of shutters with uniform speed is instructive, as it gives some rough indication of the efficiency in practice.

132. Efficiencies at Uniform Speed. The forms of shutters considered are shown in Fig. [illegible]; the calculations are best made by the aid of the Integral Calculus, and are very troublesome by elementary methods, hence the results only are stated; the verification of these results will provide an exercise for mathematical readers. An approximate estimate can be made in some cases by a method to be shown in the next section. The aperture is taken to be circular in each case.

(*a*) When the sliding panel is square, so that the opening begins from one side, the efficiency is ·5000.

(*b*) When the edges of the sliding panels are straight and there are two of them opening from the centre, as drawn, and closing again to the centre, the efficiency is ·576.

(*c*) When there is a single sliding panel with a circular hole, the efficiency is ·424.

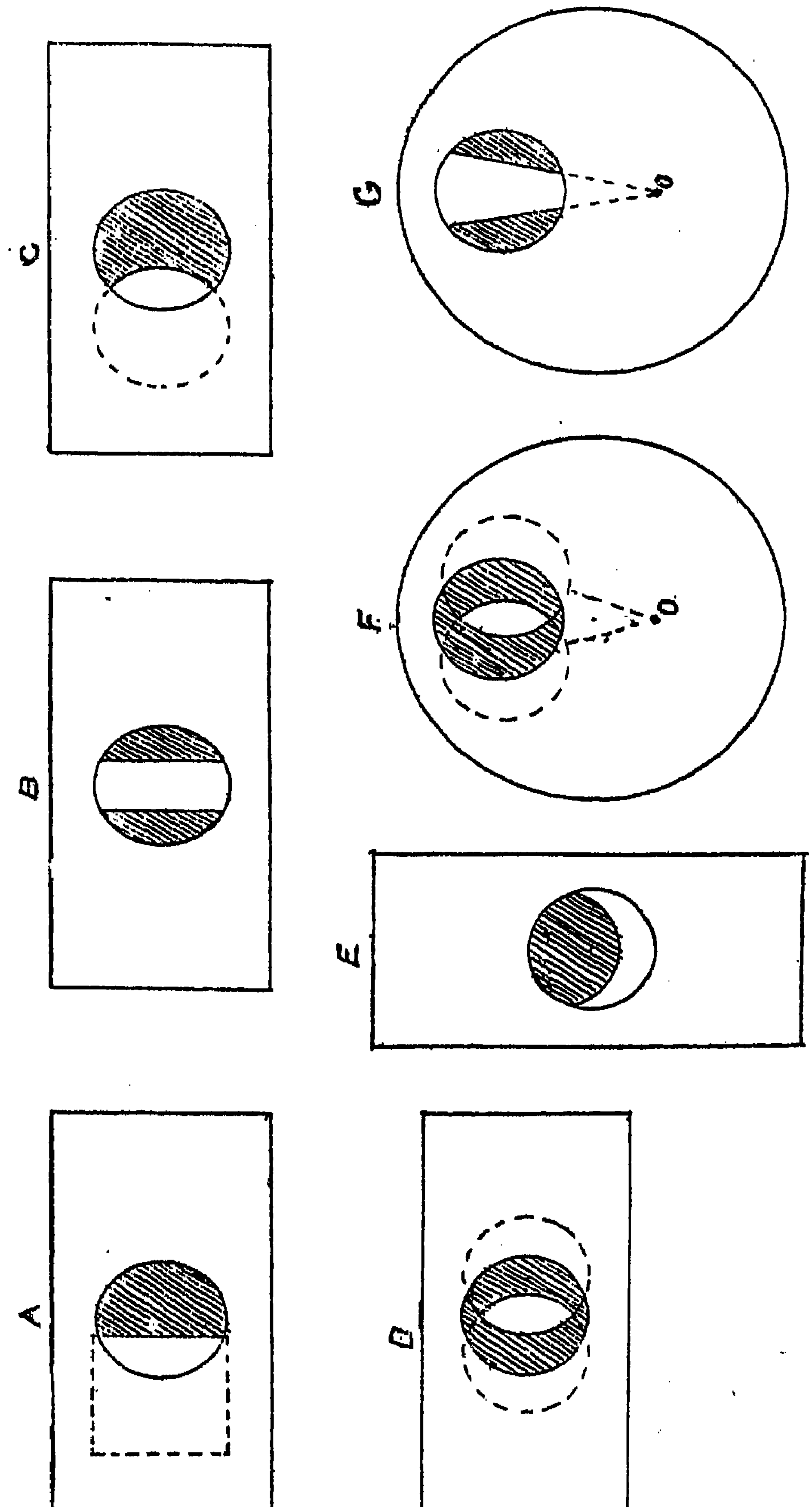

FIG. 85.

(*d*) When there are two sliding panels each with a circular hole, opening and closing at the centre, the efficiency is ·424, the same as in the last case.

(*e*) When the exposure is made by the rise and fall of a panel, with its end circular and equal to the radius of the aperture, the efficiency is ·576, the same as for (*b*).

(*f*) When there are two panels with circular holes which revolve about a pivot o; let z be the ratio of the radius of the aperture, to the distance of the pivot from the centre of the aperture.

The efficiency can be calculated for simple cases

when	$z = 0$	the efficiency	$= ·424$
,,	$z = 1/2$,,	,,	$= ·422$
,,	$z = 1$,,	,,	$= ·297$

When $z = 0$ the pivot is at an infinite distance and the motion of the panels becomes sliding motion as in (*f*); the result as we should expect is the same in the two cases.

(*g*) When the aperture is uncovered by two panels turning about a pivot fixed outside the aperture, if z have the same meaning as in the last case, then

when	$z = 0$	the efficiency	$= ·576$
,,	$z = 1/2$,,	,,	$= ·578$
,,	$z = 1$,,	,,	$= ·703$

In both (*f*) and (*g*) it should be noticed that $z = 1$ means that the pivot is in the circumference of the aperture.

133. Calculation of Efficiency in General.—The motion of a shutter is in general not uniform as assumed in the last article, but a method is given below (§ 134) by means of which the motion can be found, and from this an estimate of the efficiency can be made.

The quantity of light which during any short interval falls on the centre of the plate is proportional to the product of the interval into the average area uncovered during that interval; we can therefore make an estimate of the total effect of the exposure, by dividing up the

whole time of the exposure into short intervals, finding the average areas uncovered during each interval, and adding together the product of those areas into the corresponding intervals. If the quantity so found be divided by the total area the result will be the *equivalent* exposure.

Thus let T be the *nominal* exposure, let this be divided into a number of small equal intervals each equal to t, and let the average areas uncovered during each interval be, a_1, a_2, a_3, etc., and let A be the total area, then

$$\text{equivalent exposure} = \frac{a_1\,t + a_2\,t + a_3\,t + \text{etc.}}{\text{A}}$$

$$\text{and the efficiency} = \frac{a_1\,t + a_2\,t + a_3\,t + \text{etc.}}{\text{A T}}$$

Example.—The nominal exposure was 1/10 second.

for $\frac{1}{100}$ sec.	$\frac{1}{20}$	of the aperture was uncovered	
,, $\frac{1}{100}$	,,	$\frac{1}{10}$	,, ,, ,,
,, $\frac{1}{100}$	,,	$\frac{1}{2}$	,, ,, ,,
,, $\frac{4}{100}$	,,	the whole aperture was uncovered	
,, $\frac{1}{100}$	,,	$\frac{1}{2}$ of the aperture was uncovered	
,, $\frac{1}{100}$	,,	$\frac{1}{10}$	,, ,, ,,
,, $\frac{1}{100}$	,,	$\frac{1}{20}$	,, ,, ,,

Hence reckoning as above the efficiency is

$$\frac{\frac{1}{100}\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{2}\right)+\frac{4}{100}+\frac{1}{100}\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{2}\right)}{\frac{1}{10}} = \cdot 53$$

and the equivalent exposure is ·053 sec.

The following table (p. 280) has been calculated to facilitate the calculation of efficiency; it shows the fraction of the whole aperture that is unclosed at each tenth of the movement of the panel or panels required to fully unclose the aperture.

If other values besides these given are required the given values should be plotted on squared paper and the points found joined by a continuous curve in the manner familiar to engineers; intermediate values may then be read off, approximately, from the diagram.

TABLE SHOWING FRACTION OF APERTURE UNCLOSED FOR EACH TENTH OF MOVEMENT OF PANEL

Fraction of distance moved by panel		·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
Fraction of area unclosed for cases shown. See Fig. 85.	*a*	·0528	·1395	·2500	·3738	·5000	·6262	·7500	·8605	·9472	1·0000
	b, *c*	·1272	·2524	·3774	·5000	·6090	·7210	·8120	·8944	·9630	1·0000
	c, *d*	·0370	·1056	·1880	·2790	·3910	·5000	·6226	·7476	·8728	1·0000

134. Experimental Examination of Shutters.—Captain Abney has devised a method for examining the motion of shutters by means of which a diagram is drawn showing the position of the parts at any instant during the exposure.

The method is to place in the aperture of the shutter a piece of cardboard, in the middle of which is cut a slit at right angles to the direction of motion of the shutter; the image of this slit is thrown on a plate or film, which is moving in a direction at right angles to the slit. If the plate were at rest the effect of the exposure would be to produce a photograph of the slit, but when the plate moves this is stretched out into a band; the breadth of this band at any part of the exposure shows the breadth of the opening of the shutter. If a scale of times can be marked on the diagram we can by inspection find the state of the shutter at any time required.

The shutter aperture with slit is shown in Fig. 86, in which C C are the pieces of card containing the slit, and S S are the moving panels of the shutter, shown partly withdrawn.

The plate can be moved by hand, but this is not very convenient as it requires the use of a dark room; the more convenient arrangement is to roll a flexible sensi-

tive film round a drum which is made to revolve rapidly; this arrangement is shown in Fig. 87, in which the drum is arranged in a box made to fit the camera like a dark slide. The spindle and small pulley at the side are for driving the drum; the most convenient thing for this

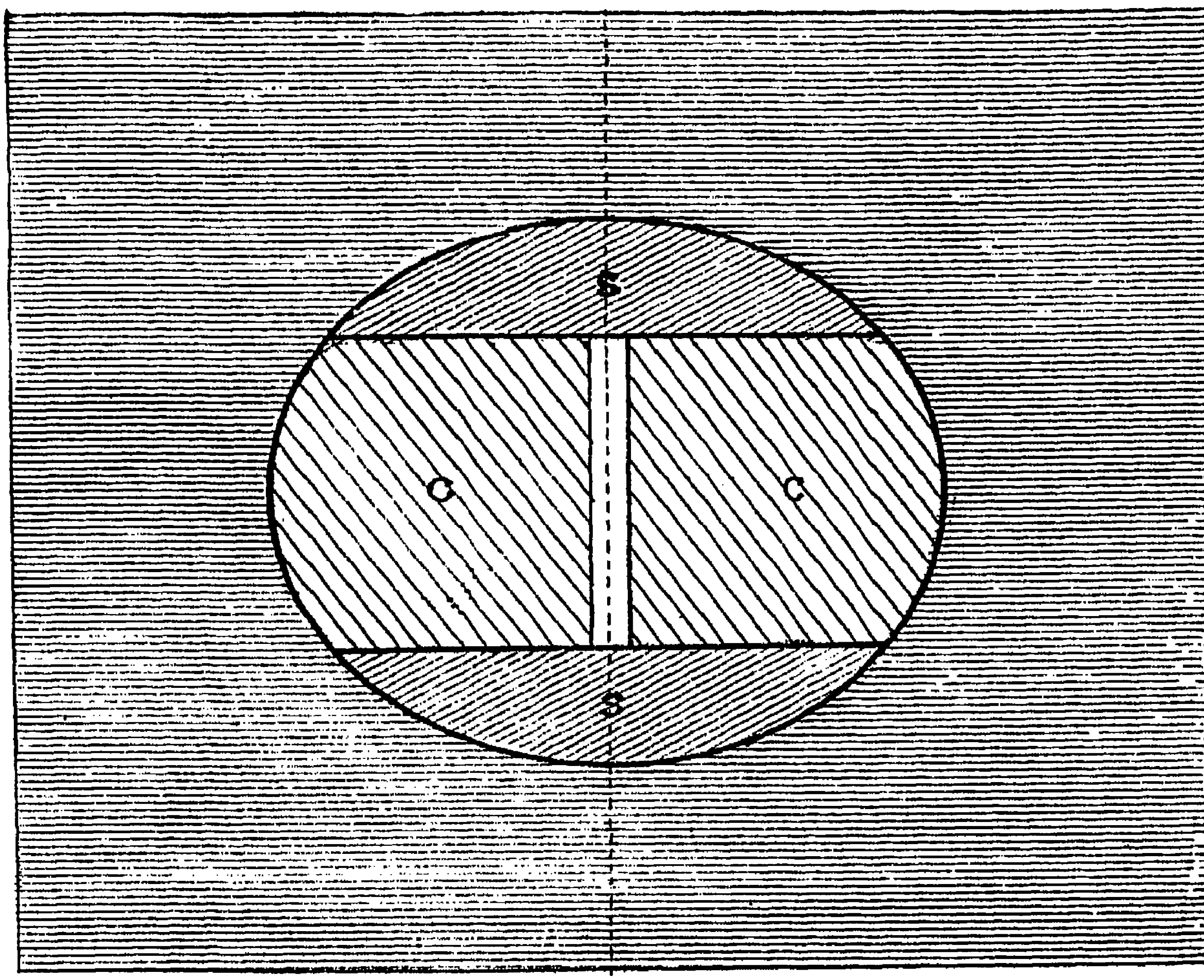

FIG. 86.

purpose is an electromotor which can be made to run at a very nearly constant rate.

The general arrangement of the apparatus is shown in Fig. 88; on the extreme left is an electric arc lamp which provides the necessary light: next is placed a lens which acts like the condenser of a lantern, throwing a beam of light on the shutter: next comes the lens to be tested, fitted with the cardboard slit (which is here

horizontal): next comes a wheel, to be explained below, for timing purposes: next is the camera with the box containing the revolving drum: at the end is the electromotor to drive the drum.

The wheel is so placed that the image of the slit can be obscured by the spokes; if the wheel revolves at a definite rate the light from the slit will be shut off at definite intervals. Lines are thus marked across the

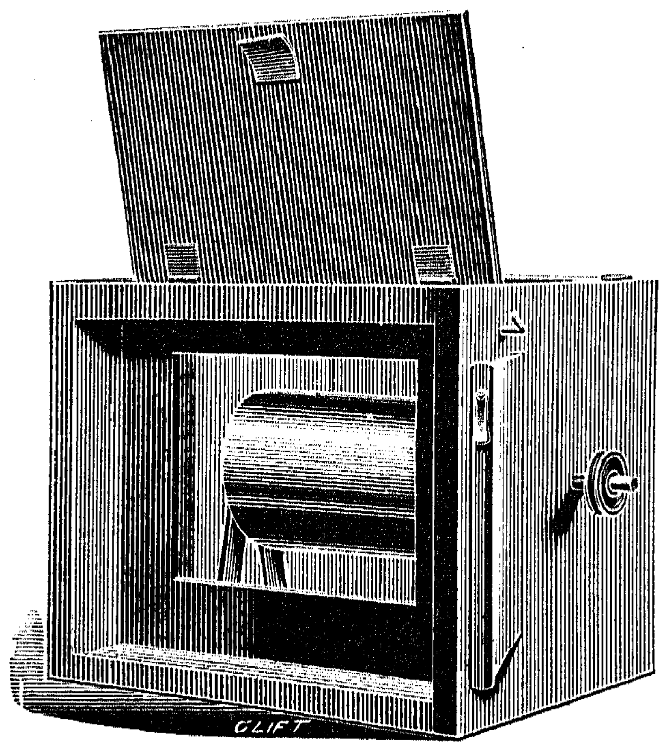

FIG. 87.

diagram, the distances between them representing equal intervals of time.

The holes round the rim are for measuring the speed of the wheel, air is blown through them as the wheel revolves, forming a syren; the pitch of the note can be found by comparison with a tuning-fork or other instrument of known pitch, and thus the number of holes which pass the air-jet in one second is known, and the speed of revolution can be reckoned.

For a fuller explanation of this process we must refer readers to some text-book on sound where the formation

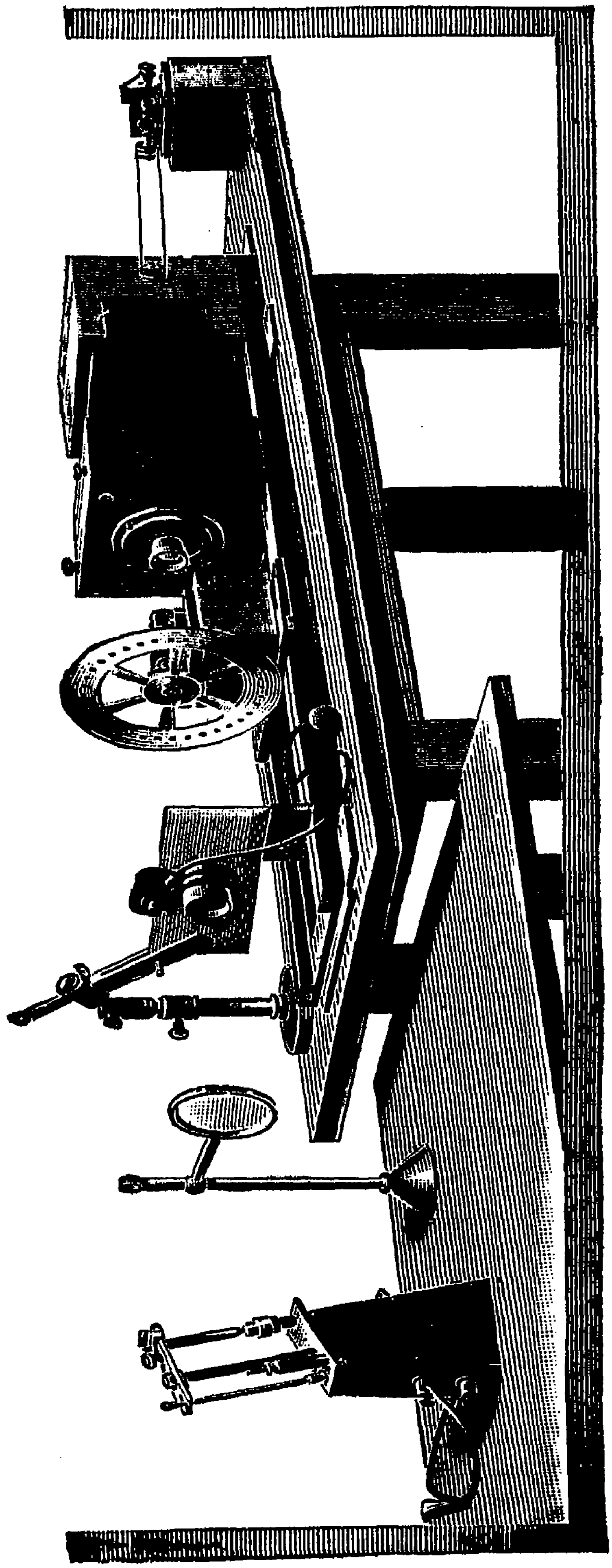

Fig. 88.

of notes of definite pitch and the action of the syren are explained; the following table gives the number of vibrations per second required to produce the notes of an octave beginning at middle C.

	Scientific scale.	Society of Arts scale.
C	512	528
C sharp	540	559
D	576	594
D sharp	600	622
E	640	660
F	683	704
F sharp	720	745
G	768	792
G sharp	800	837
A	853	880
A sharp	900	932
B	960	990
C	1024	1056

135. The Study of a Shutter Diagram.—Let us examine the diagram shown in Fig. 89, which is given by Abney. Here the direction of motion of the film was parallel to A B, and the slit was at right angles to this direction; the white lines across are due to the interruptions caused by the spokes of the revolving wheel.

The first thing to notice is that the interruptions are marked at equal distances along the film, showing that the drum revolved uniformly. Since the line A E B remains straight for a long time it is clear that the shutter was one in which the opening began at one end of the slit, as in Fig. 85 (*A*); the sliding of the panel is shown by the sloping line A C, and the full aperture is reached at E C. The portion between E C and B F represents the interval during which the aperture was fully unclosed; the straight line C D and the sloping

line B D show that the aperture was closed by a panel sliding over it in the same direction as the former one.

The diagram could have been given by a shutter like that in Fig. 85 (*A*), if the aperture in the sliding panel instead of being square were a rectangle, so that the aperture may remain fully uncovered for a finite time.

The line A C is straight, which shows that the speed of the opening panel was uniform, but B D is curved and convex towards C D, showing that the motion of the closing panel was retarded. To sum up, the opening took $2\frac{1}{2}$ intervals, the aperture was fully open for $3\frac{1}{2}$ intervals, and the closing took about $3\frac{3}{4}$ intervals. In this particular case the note given by the syren was E, which means that 640 holes passed the air-jet in one second; the wheel used had 6 spokes and 36 holes in

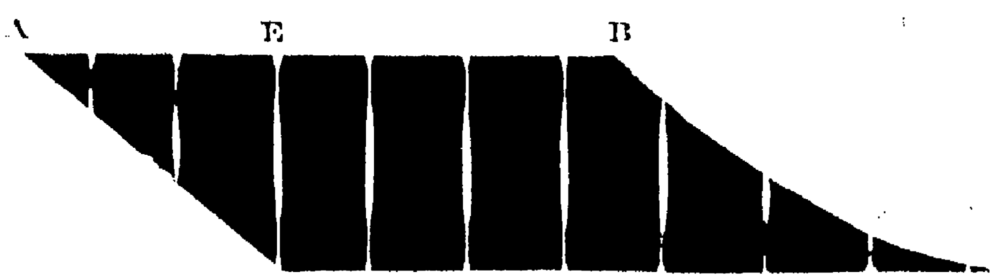

FIG. 89.

the rim, or 6 holes to each spoke, which makes the number of eclipses by the spokes to be 640/6, or 107 nearly in one second; thus the time between the successive eclipses is $1/107 = {\cdot}0093$ sec., or roughly one hundredth of a second. Calculating from this we find that the total time of exposure was ·095 sec., the opening took ·023 sec., the aperture was fully open for ·032 sec., and the closing occupied ·04 sec.; the times are given to thousandths of a second, but it is not likely that the method is accurate to this extent; it would be safer to give the results to the nearest hundredth of a second.

To calculate the efficiency we must remember that for this kind of shutter the value given in § 132 (*a*) was ·5; so if we regard the opening and shutting as

uniform, the time with full aperture to which the time of opening and shutting is equivalent, is half the actual

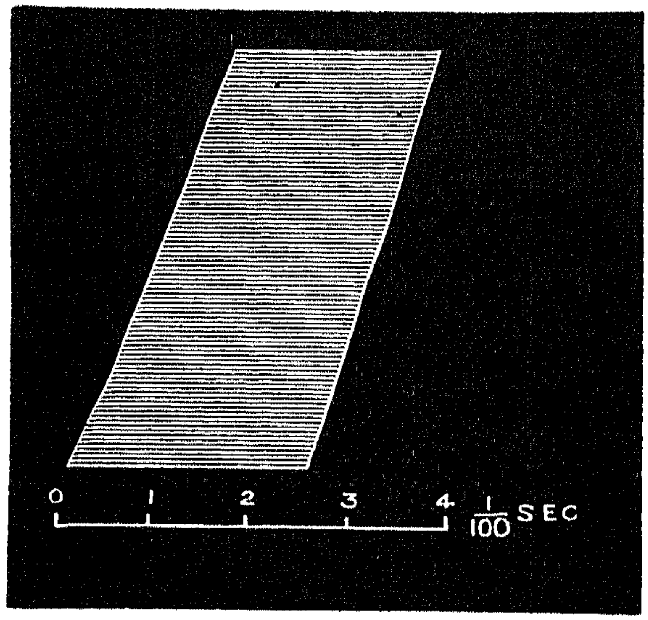

FIG. 90 (*a*).

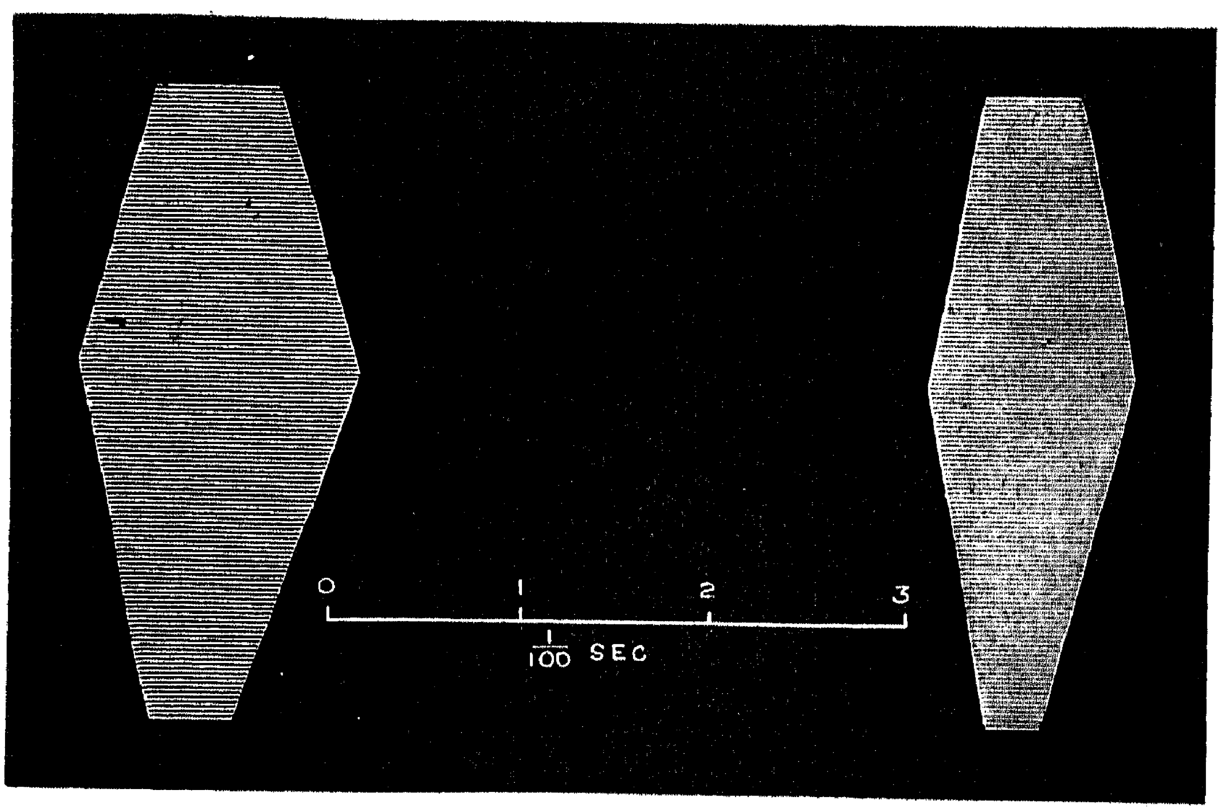

FIG. 90 (*b*).

time taken to open and close, that is, to half the sum of $2\frac{1}{2}$ and $3\frac{3}{4}$, or of $6\frac{1}{4}$ intervals; hence adding in the

time of full aperture, equivalent exposure $= 3\frac{1}{8} + 3\frac{1}{2}$ $= 6\frac{5}{8}$ intervals, but normal exposure $= 10\frac{1}{4}$ intervals, hence dividing the equivalent by the nominal exposure we get for the efficiency ·64.

136. Timing by the Speed of the Drum.—After

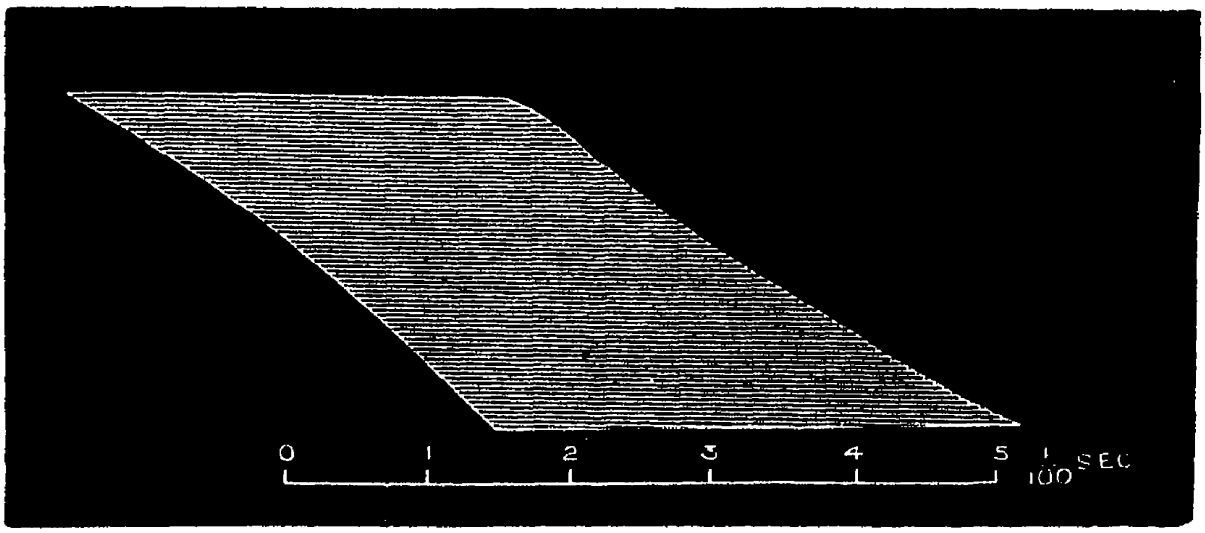

FIG. 90 (*c*).

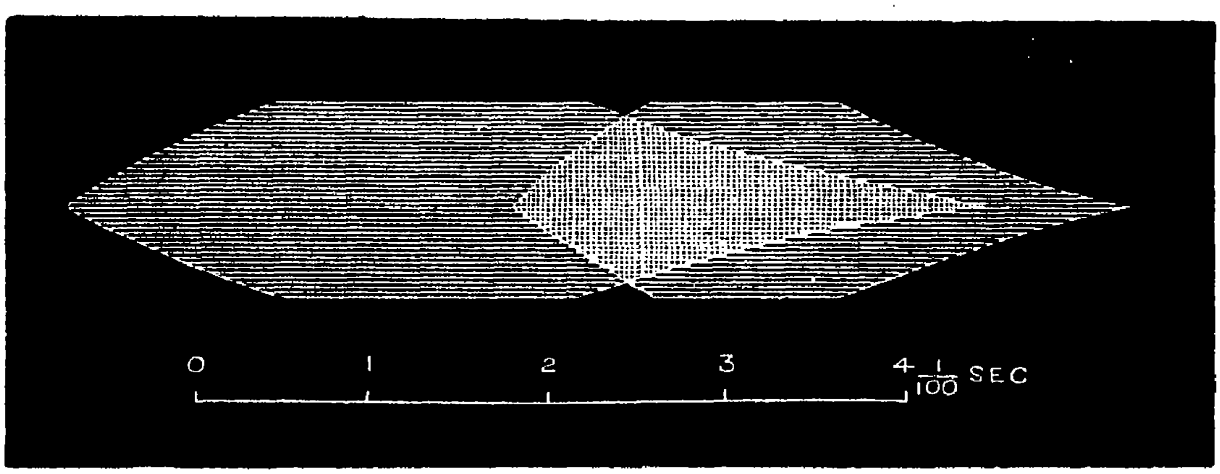

FIG. 90 (*d*).

several experiments Abney found that the motion of the drum kept so nearly uniform that the time of exposure could be estimated from its speed of rotation and from its diameter. To find the speed of rotation use was made of an old turnstile counter, which was

attached to the axle, and the number of revolutions in one minute were counted.

Example.—It is found that the drum makes 400 revolutions in one minute, and its diameter is 2 inches, the total length of the shutter diagram is $4\frac{1}{2}$ inches; find the nominal exposure.

The radius is 1 inch, hence the circumference is 2π inches, so that the film moves through a distance of $400 \times 2\pi$ inches in a minute, or $400 \times 2\pi/60$ inches in one second.

During the exposure the film moves through $4\frac{1}{2}$ inches, hence the duration of the exposure is

$$\frac{60}{400 \times 2\pi} \times 4\tfrac{1}{2} = \cdot 096 \text{ sec. about, or nearly one-tenth}$$

of a second.

In Fig. 90 are shown some of Abney's diagrams;

(*a*) is that for a drop shutter (remember that the image of the slit is inverted, so that the bottom of the diagram corresponds to the top of the shutter).

(*b*) contains diagrams for Thornton and Picard's shutter.

(*c*) is a diagram for Hawkins' shutter.

(*d*) contains diagrams for a Key shutter.

137. Unequal Exposures at Different Parts of the Plate.—It is a matter of common experience that when a shutter is used the edges of the plate are often less exposed than the centre, to the detriment of the picture; a shutter diagram enables us to study the exposures of the different portions of the plate. To prevent confusion it should be remembered that we have reckoned the efficiencies only for the centre of the plate.

Let Fig. 91 represent a section through the axis of the lens; let the lens be as shown, and D E the section of the full aperture of the shutter in its proper position. Consider an oblique pencil, whose extreme rays cut the line D E at the point *a a*; this

pencil falls on the lower part of the plate; let a central pencil cut the line D E at *b b*, and an oblique pencil on the lower side of the axis, in *c c*. Suppose for example that the shutter opens and closes at the centre, then it is evident that the pencil *b b* will be admitted first, and the pencils *a a*, *c c* completely when the sliding panels are above the highest *a* and below the lowest *c*; also when the panels close the oblique pencils are cut off first. On both these accounts the oblique pencils are

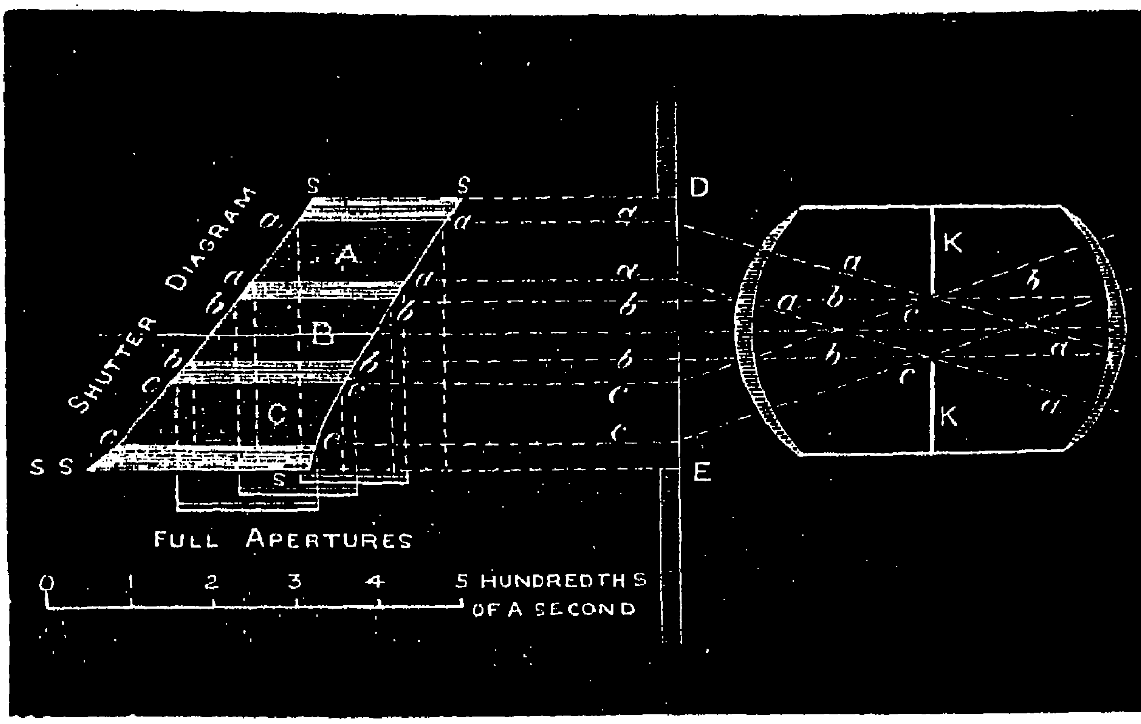

Fig. 91.

admitted for a considerably shorter time than the central one.

Now take the case of a shutter whose diagram is that of Fig. 90 (*a*), opening at one side, and let the diagram be placed on the section, as shown, so that the top and bottom lines S S pass through D and E. To understand the meaning of the diagram, it should be remembered that when it was made the film moved in the direction S S, and consequently the breadth of the opening of the shutter at any instant is given by the

breadth of the diagram measured perpendicular to S S, and at a point corresponding to the given instant.

From the points *a a*, *b b*, *c c* draw parallels to S S to cut the diagram; the darkly shaded parts of the diagram show the areas between these pairs of lines. To avoid mistake it must be remarked that the diagram shows the nature of motion of the shutter, but that the motion really takes place along D E. The shutter opens from one side as the lower horizontal line S S shows; while the panel moves across *c c*, as shown by the left-hand edge of the lowest dark portion, the lower oblique pencil is being admitted; then in succession the lower oblique pencil is admitted: the shutter remains fully open for a short time, and then another panel moves in the same direction as the former, cutting off the pencils in the order of their admission.

Let us now fix our attention on the pencil *a a* and trace its history: this pencil is admitted last; its admission begins when the panel reaches the position shown by the lower *a* on the left; the pencil is fully admitted when the upper *a* on the same side is reached: the pencil continues to be fully admitted till the closing panel reaches the position shown by the lower *a* on the right, and is completely cut off when the upper *a* on the same side is reached. We see that the time during which the pencil is fully admitted is given by the horizontal distance (parallel to S S), between the upper *a* on the left and the lower *a* on the right; vertical lines have been drawn through these points and joined by a horizontal line just below the diagram. Similar remarks apply to the pencils *b b* and *c c*, and similar constructions have been made to show the duration of the times of complete admission.

It is worth noticing that the portion *a a a a* of the shutter diagram may be regarded by itself as the complete diagram for the pencil *a a*, and the efficiency for the pencil might be calculated from it; similarly for the pencil *c c*. It should also be noticed that the portion

b b b b is all that applies to the central pencil, so that the efficiency calculated from the whole diagram would not be correct ; it is evident from this that with a small stop and the shutter at a considerable distance from the lens, the whole of the diagram may not apply to the central pencils. In this case we see that the times of total admission of the three pencils are nearly equal, and also, from the great similarity of the portions *a a a a*, *b b b b*, *c c c c*, that the efficiencies for each portion are nearly equal ; but the exposures do not take place quite simultaneously.

We have here considered the circumstances of portions of the plate in a line parallel to the slit, placed in the centre. Our conclusions will not in general hold good for portions in a line at right angles to this,

FIG. 92.

through the centre of the plate. This latter case can be investigated by taking a diagram with the slit in the shutter at right angles to its former position ; the discussion of the results will be similar to that given. A pair of diagrams, taken by Abney, with the slit in two directions at right angle, are given in Fig. 92.

138. Focal Plane Shutter.—This shutter differs widely from any of those we have considered, for these are either very near to the lens, or are at the diaphragm, while the focal plane shutter is close to the plate. The exposure is given by the rapid passage of a narrow slit across the plate ; this clearly lends itself to rapid exposures, which can be adjusted by altering the breadth of the slit (Fig. 93).

This shutter has however the disadvantage of ex-

posing the different parts of the plate at different times, the difference between the times at which the two extremities are exposed being in most cases greater than the time of exposure; this produces a distorting effect on the picture of a moving object. Suppose for instance that the mast of a rapidly moving ship is being photographed, and that the shutter slit travels from the bottom of the plate to the top. Then, allowing for the reversal of the picture on the plate, the image of the top of the mast will be admitted first, and that of the bottom of the mast after an interval three or four times the length of the exposure, and during this

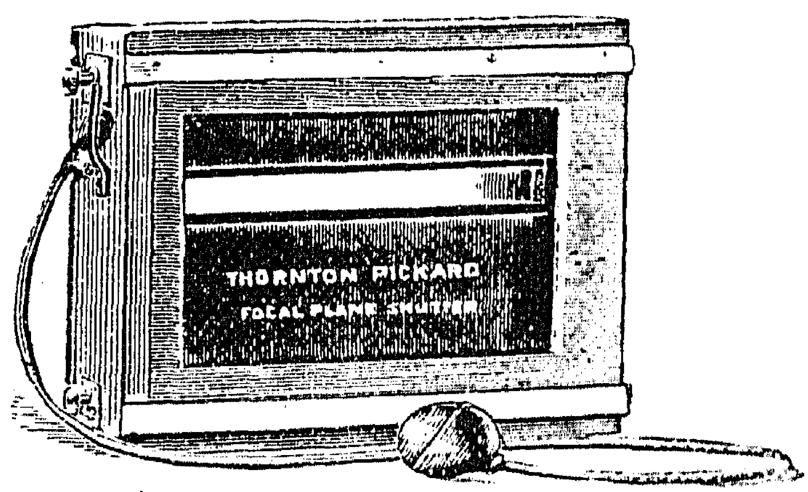

FIG. 93.

interval the ship will have moved; the intermediate portions will be exposed at intermediate times. In the photograph the base of the mast will be in advance of the top, and the mast will appear to slope backwards; in this particular case the sloping backwards may improve the picture, by giving the ship a rakish appearance, but it is not truthful. Besides this, cases, such as a man walking rapidly, can be imagined in which the effect would be disastrous.

139.—We have considered some typical forms of shutters, but there are many others which are to be found in makers' catalogues and photographic annuals;

the reader who is interested in the matter may also refer to the *Traité Encyclopédique de Photographie*,[1] vol. i. pp. 150—205, where much interesting information is given.

From the discussion of the question, it appears that an exposure made with a shutter is not so satisfactory as a slow one made by hand, and that in order to secure equal exposure of all parts of the plate, the form shown in Fig. 85 (*A*) is preferable to one like Fig. 85 (*B*) or (*D*), which open in the centre.

[1] By C. Fabre. Gauthier-Villars, Paris, 1890.

CHAPTER VII

ENLARGEMENT, REDUCTION, DEPTH OF FOCUS, AND HALATION

140. Introductory.—The difficulties of making large photographs directly are very great; the apparatus required is large, heavy, and inconvenient to carry about; a lens of large diameter must be employed, which is troublesome to make, and in consequence is expensive; the plates are, from their size, difficult to manipulate; and, lastly, the expenditure in materials is considerable, for large plates are expensive, and need large quantities of chemicals.

It is, therefore, much more convenient in many cases to take photographs, first on a small scale, and then, if the negatives are good, to make enlargements; a great saving results, both in the original cost of the apparatus and also in the current expenses, for the failure of a small negative is not so costly as that of a large one. It is not surprising, therefore, that enlarging is very popular. Reduction is required mainly to make lantern slides from negatives which are too large to admit of contact printing.

Many forms of enlarging and reducing apparatus are sold, differing little in principle, but many of them are of needless complexity, and most of them are very highly priced.

It is proposed first to give some account of the optical principles of enlarging, and then to show how those who cannot afford expensive apparatus can pro-

duce enlargements at a comparatively small cost by the aid of an ordinary camera and lens.

141. Optical Principles.—The general principles which apply to the production of pictures of a size different from that of the original have been explained in § 37; this section applies equally to reduction.

The essential parts of the enlarging or reducing apparatus are, the negative, the lens, the sensitive plate or paper which receives the image, and the source of light. The light shines through the negative, and an enlarged or reduced picture of it is formed by the lens on the paper or plate, and results in the formation of a positive picture.

Thus, in any enlarging or reducing apparatus, the requirements are, uniform illumination of the negative, and facilities for adjusting the relative distances of negative, lens, and sensitive receiving surface.

Similar remarks apply to the optical lantern, with the exception that, instead of a negative a positive is used, and a positive picture is thrown on the screen.

The illumination may be either direct sunlight, diffused daylight, or artificial light, provided the light is uniformly distributed over the negative. To distribute the light, a lens, called a condensing lens, is generally used, though this can be in some cases dispensed with.

We shall now study the action of the condensing lens, and for this purpose shall consider Woodward's apparatus, which was one of the earliest pieces of apparatus for enlarging.

141*a*. Woodward's Apparatus.—In the early days of photography, before the introduction of the very sensitive plates and papers we now employ, the production of an enlargement was a matter of difficulty. When enlarging on slow silver paper, in order to reduce the time of exposure within reasonable limits, it was necessary not only to use direct sunlight, but also to concentrate it as much as possible by means of a condensing lens.

An early apparatus was that of Woodward, shown in

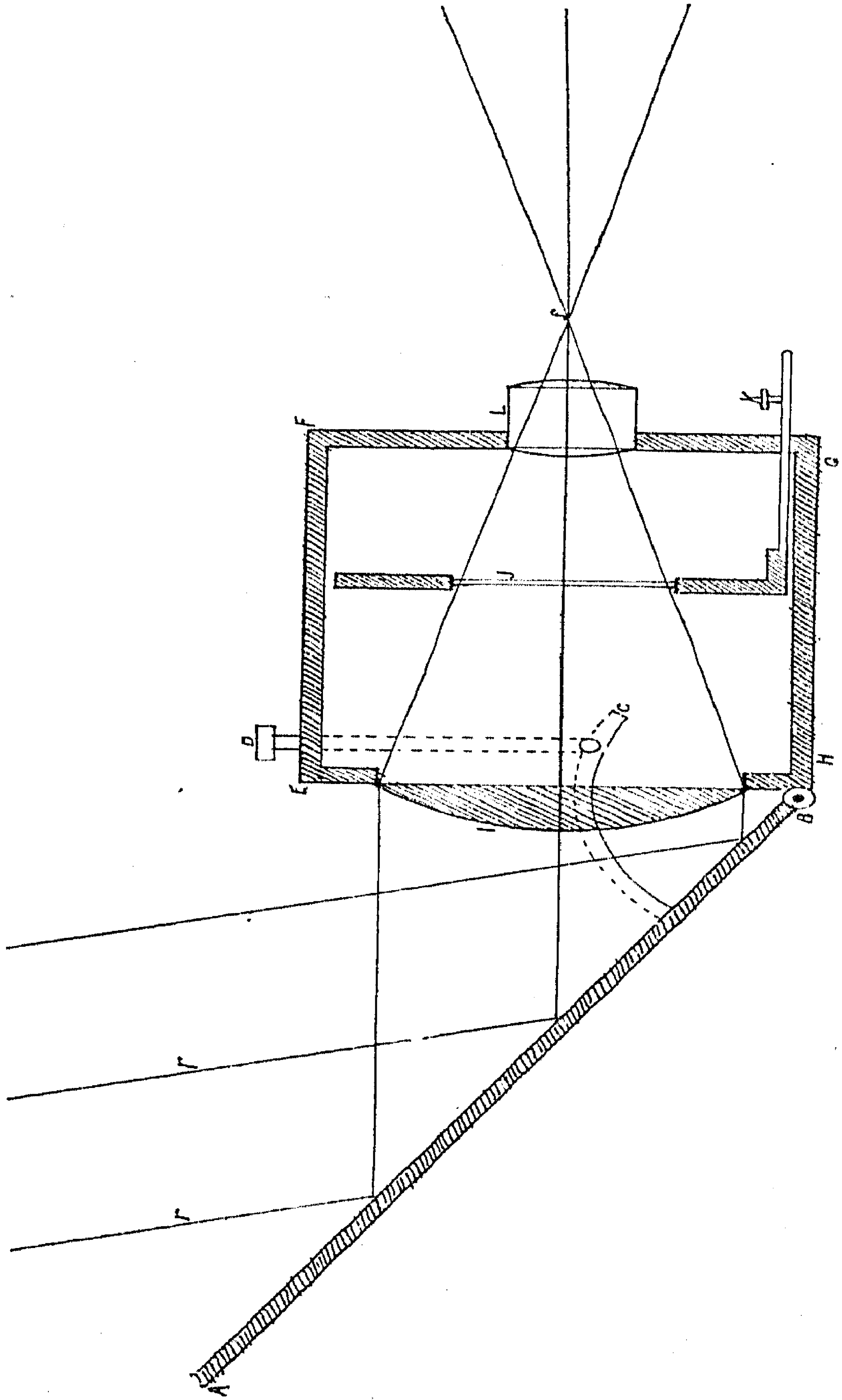

FIG. 94.

Fig. 94; here A B is a mirror to reflect the sunlight on to the condensing lens I; its position was regulated by means of the screws indicated at B and C. The lens I concentrates the light to a point *f* near the objective L, so that all the light falls on L, and none is interrupted by the mounting. J is the negative, carried in a holder which can be moved backwards or forwards, and fixed by means of the screw K; the enlarged picture is projected by the lens L on a screen in front, not shown in the picture. The whole arrangement is carried by a wooden box E F G H, and is placed in a window so that the box passes through a hole in the shutter, the part A B being outside, and carefully fitted so that no light can enter the room except through the lens.

There was at one time a lively discussion about this and other arrangements, which has now lost most of its interest for us; it will be found at length in Monkhoven's *Optics*; we shall here consider the theory only so far as it throws light on the action of modern apparatus.

Let us examine the manner in which the enlarged image is formed; it may be looked at from two different points of view.

I. Imagine first that the negative is removed, and that the light from I converges to the nodal point of incidence of the lens L, then (§ 44) the rays will pass undeviated, and on the screen behind will be formed a circle of light whose size depends on the distance of the screen from L. If now a grating, such as a piece of perforated zinc, be placed *anywhere* between I and L, it will cut off some of the rays, and its *shadow* will constitute an image on the screen. From this point of view the position of the object which intercepts the light is immaterial.

Even if the light from I does not converge to the nodal point of incidence to each ray in the incident cone, there will correspond one particular ray in the emergent cone, and a shadow image will result as before.

This depends on the supposition that one ray of light, and one only, passes through each point of the negative, so that a point in the negative does not send out a pencil of rays which can be converged by the lens L to a conjugate focus.

II. On the other hand, if any scattering takes place when light passes through the negative, each point of the negative that is not opaque will send out a small pencil of light, which will fall on the objective and be regularly refracted, and thus a regular image of the negative will be formed.

We should thus expect images of two different kinds; the first kind of image is a shadow image, formed at all distances, and the second an image by the regular refraction of small pencils coming from the different points of the negative; the best result evidently will be obtained when the screen is placed to receive the second image, for the two images will then coincide.

The supposition that one ray of light only passes through each point of the negative is never strictly true; even with the sun the incident rays are not quite all parallel, for the sun has a diameter which subtends at the earth an angle of half a degree, and hence every point on which sunlight falls receives a pencil of rays whose angle is half a degree. If a cloud passes over the sun so that the light is all diffused, the effect becomes very marked. Also, if artificial light is used, the source of light is always of finite size, and each point of the negative receives light from all points of the source, all the rays received forming a pencil. On this account, then, as well as because of scattering, every clear point of the negative sends a pencil of rays to the lens, and a regular image is formed.

It is not hard to see that, if the condensing lens exhibits any spherical aberration, it will help the formation of the regular image.

The conclusion to which we come is, that owing to the finite size of the source of light, to the scattering of

light at the negative, and to the spherical aberration of the condenser, the objective forms a regular image, and its action is, on the whole, the same as if it were forming an image of an object at the same distance as the negative illuminated by diffuse light.

Similar remarks will clearly apply to the optical lantern.

142. Concentration of the Light.—In Woodward's apparatus the condensing lens I is arranged to produce an intense illumination of the negative J; it can be seen from the figure that the light which falls on the whole area of I (which is larger than that of J) falls on the negative J, and so by increasing the size of the condenser, we can increase the quantity of light which falls on J. The necessity of intense illumination, when using silver paper, led to the employment of very large condensers, having a diameter of as much as 19 or 24 inches, but it was found very difficult to get a sharp image when they were used, and the negative was often broken by the intense heat which resulted.

To obtain the greatest illumination possible, all the light from the condenser should pass through the negative and fall on the objective, and the position of the negative must be that shown in Fig. 94.

If we have given the number of times the picture is enlarged, the size of the negative, the focal length of the objective, and the diameter of the condenser, we can calculate the focal length of the condenser required, supposing that the condenser concentrates light to the nodal point of incidence of the objective (§ 152).

143. Modern Arrangements.—Now that both plates and paper have been made extremely sensitive, it is no longer important to secure a very intense illumination, and a large condenser is no longer necessary. The negative is now placed close to the condenser, and, in consequence, the diameter of the condenser needs to be only slightly greater than the length of the diagonal of the negative; this very much reduces the size of the

apparatus. The remarks made above about the formation of the image will hold good here also.

The form of the condensing lens now usually employed

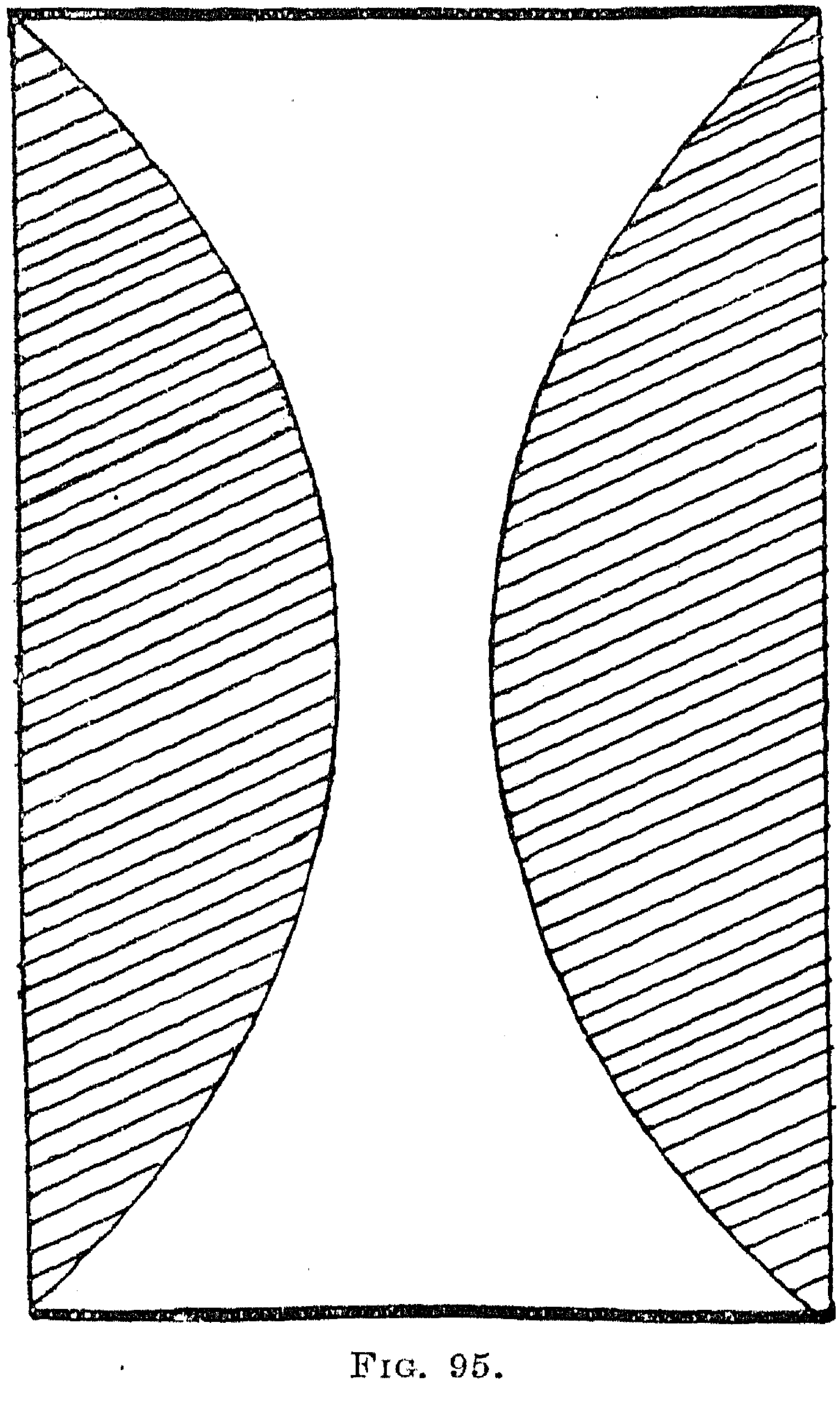

FIG. 95.

is shown in Fig. 95; it consists of two plane convex lenses placed with their convexities turned towards each other. The following are the dimensions of a condens-

ing lens used with an optical lantern to take slides of the standard size, $3\frac{1}{4}$ inches square:

Radii of curved surfaces = 3 inches, very nearly.

Thickness of lenses at centre = ·8 inch.

Diameter of lenses = 4 inches.

Distance between plane faces = 2 inches.

Approximate focal length of the combination = 3 inches.

The spherical aberration was found to be considerable.

Lenses of larger diameter have their dimensions proportional to those above.

The lenses should fit loosely in their mounting, otherwise they may be broken by the heat from the source of light.

144. Illumination without a Condenser.—A condenser may be dispensed with when intense illumination is not required, provided the negative can be uniformly illuminated by other means. The required illumination can in some cases be obtained by placing a sheet of ground glass between the negative and the source of light; the light coming from an opal or uniformly frosted globe of a lamp might in some cases be suitable. It is impossible to say exactly when such an arrangement will be suitable and when not—that can be decided only by experiment; but those who cannot afford expensive apparatus will find something of the kind well worthy of a trial.

When using daylight a condenser can easily be dispensed with, if outside the window is fixed a board covered uniformly with white paper or cloth, having one side horizontal with the plane of the board inclined at an angle of 45° to the vertical; the white surface will then be illuminated by the sky, and will act as a uniform luminous background for the negative. The board must of course be large enough to illuminate the whole picture, or unequal printing in the enlargement will result.

145. Daylight Enlarging Apparatus.—As it may prove of interest, a description will be given of the method of enlarging by daylight, using the camera in which the negative was taken, when a room whose

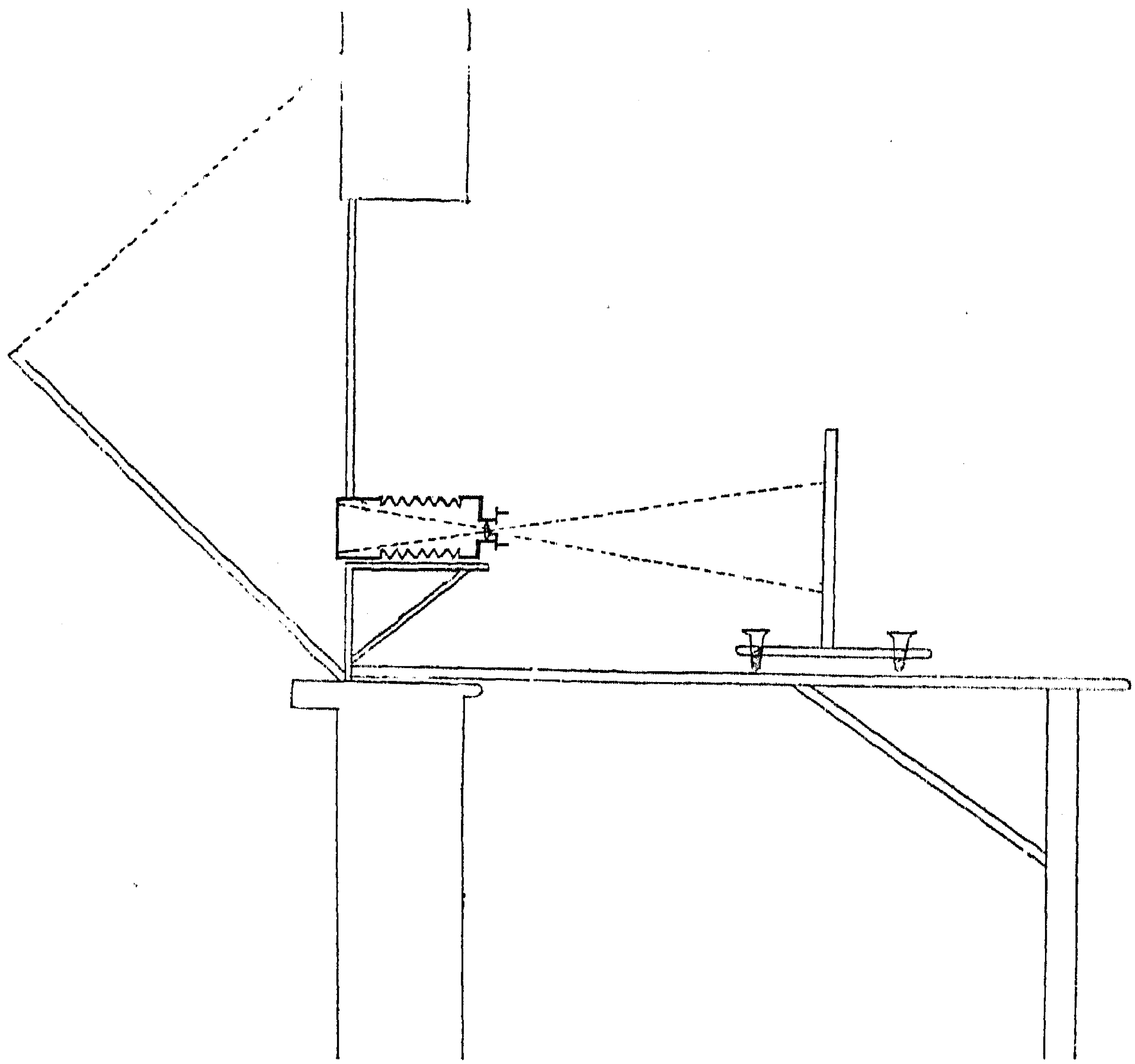

FIG. 96.

window can be blocked with a shutter is available. A section of the arrangement is shown in Fig. 96.

Outside the window is placed the board covered with white paper to act as the source of light; just inside the window (the framework of which is not shown) is fixed the shutter with a shelf to support the camera; the ground glass is replaced by the negative to be

enlarged, and the end of the camera passes through a hole in the shutter; a cloth carefully placed round the camera will make the arrangement light-tight. Below the camera is a bench on which can slide the screen to carry the sensitive paper.

Another hole must be made in the shutter, and covered with a medium that admits safe light only so that there may be light enough to make the necessary adjustments; the development can then be conveniently carried out in the enlarging room.

The routine will be, the ground glass is replaced by the negative, the camera is placed in position, and the image formed is thrown on the screen, which is at present covered with white paper only; the position of the screen and the extension of the camera necessary to obtain a picture of the size required are found by trial—this can be done without much trouble. The cap is now placed on the lens and the position of the screen is noted (as explained in § 148), so that the screen can be removed and replaced in its original position without trouble; the cap is then placed on the lens, the sensitive paper placed on the screen, the exposure is made, and the picture developed.

Some photographers have focussed the picture on the sensitive paper itself by using a cap to the lens in the centre of which is inserted a piece of yellow or canary glass, so that the light thrown on the paper is not photographically active; but there is no need for this if the screen can easily be replaced exactly, when removed.

146. Geometrical Constraints.—It is of importance that the movable screen which carries the sensitive paper should be easy of adjustment, so that it may be readily placed in position with its plane perpendicular to the axis of the lens; also it should be possible to remove it, to fasten the sensitive paper to it, and to replace it, *in the dark*, in the exact position it originally occupied.

Many pieces of apparatus sold leave much to be desired in these respects, the screen being fixed to a base which slides in grooves and wobbles unless the fitting is very good ; some even run on wheels, which offer every opportunity for shaking.

All unsteadiness may be avoided, ease of adjustment attained, and accurate fitting dispensed with by attention to the geometrical principles of the case ; these principles are well known and are applied to the construction of scientific apparatus.[1] It can be shown by geometry that a rigid body perfectly free is capable of six distinct and independent movements, and that any movement whatever can be effected by a combination of these elementary movements. The six movements are, three displacements parallel to three fixed directions at right angles, and three twists about axes at right angles. A perfectly free rigid body is thus said to have six degrees of freedom ; any constraint that is applied will reduce the number of possible elementary motions or degrees of freedom.

For instance, if one point of the body be fixed it loses three degrees of freedom, for the only elementary motions left are the three twists. If one more point is fixed two more degrees of freedom are destroyed, for the body can now rotate only about the line joining the two points ; if one more point, not in the same straight line with the other two, is fixed, the body will be completely fixed.

It is not, however, easy to fix a point of a body, it is more convenient to make various points of the body bear against a surface or surfaces. When a rigid body touches a smooth surface at one point one degree of freedom is lost; for instance, a sphere touching a smooth plane cannot move in a direction perpendicular to the plane, but it can twist about any three directions at right angles, and it can be displaced in any two

[1] Thomson and Tait's *Natural Philosophy*. Ed. 1879. Vol. i. pp. 150—155.

directions at right angles parallel to the plane. We conclude then, that to completely fix a body it must bear against other bodies which are fixed, at six points ; and that for each bearing point less there remains one degree of freedom.

Take for example a three-legged stool in the middle of a level floor, one point of each leg is in contact with the ground ; it can be displaced in any two horizontal directions at right angles and can be twisted about a vertical axis, without being raised off the ground. Now let the stool be pushed against one of the side walls of the room, so that two of its legs are in contact with the side wall, then there are five bearing points and the stool can only be slid parallel to the wall if the bearing points keep all in contact. If now the stool be slid along till one of the legs touches a second wall, then there are six bearing points and the position of the stool is completely determined. If the stool be removed from this position it can be exactly replaced by bringing all the six bearing points in contact with the walls and the floor, an operation which can be performed equally well in light or in darkness.

We see then that six bearing points only are required to completely determine the position of a body ; if there are to be more points of contact their positions cannot be independent, and if there is not to be shaking or wobbling, the fitting must be good. For instance, with a four-legged stool, since three points of contact are enough to rest a body steadily on a plane, all four legs will not be in contact with the floor unless they are made of the proper length. In other words, *any* three-legged stool will rest steadily in contact with a plane without wobbling (provided of course that the line of action of the resultant weight falls within the triangle formed by the points of contact), but if there are more than three legs, careful fitting is required to make the stool rest steadily on all the legs.

But we want not only to be able to fix a body

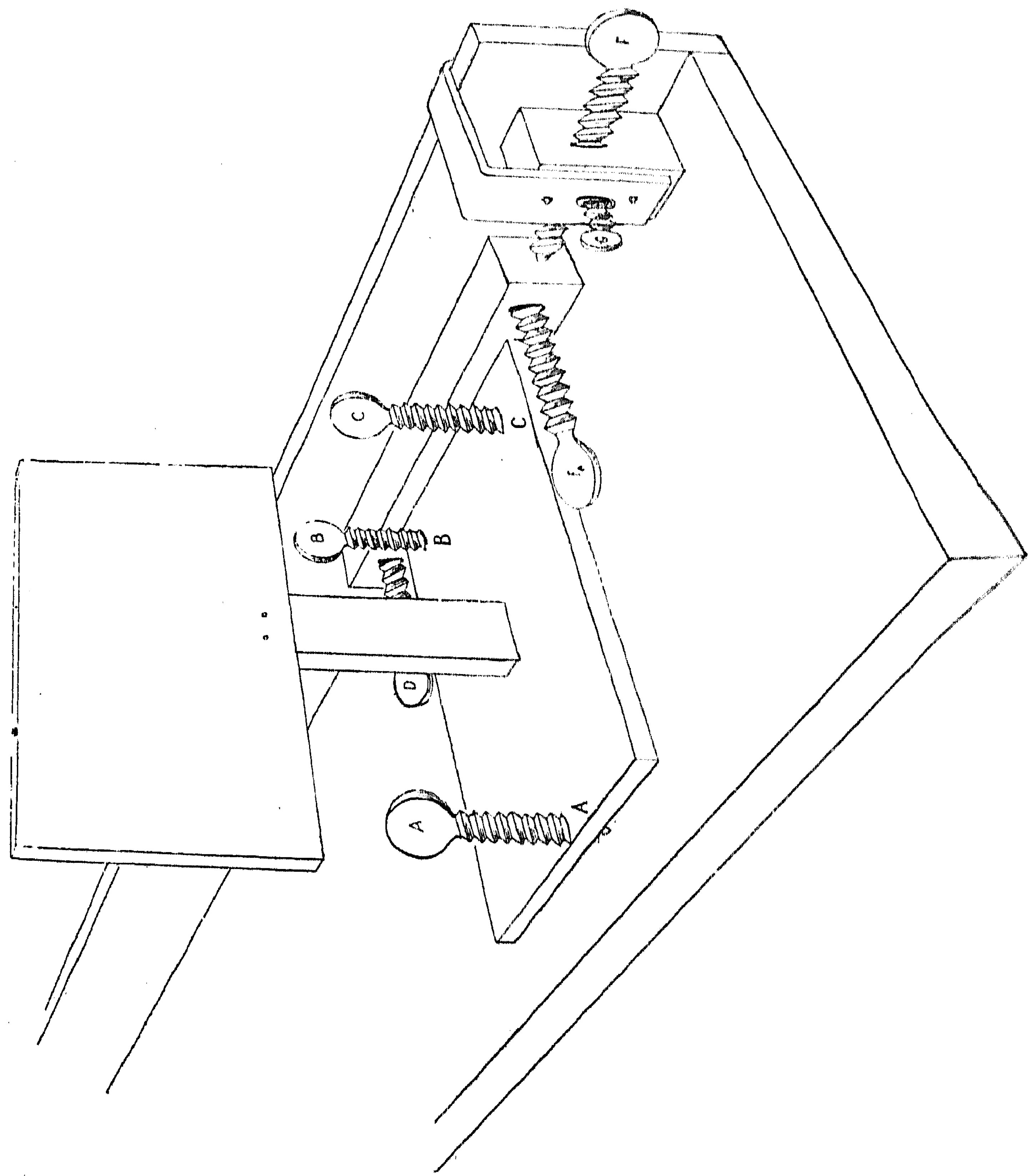

FIG. 97.

definitely, but also to effect the necessary adjustments; this can be done by varying the positions of the six bearing points relative to the body by means of screws.

147. Application to Movable Screen.—To obtain the required movable screen we must adopt the principle of the three-legged stool with the modifications necessary to suit it to our case. The arrangement is shown in Fig. 97; the base board carrying the screen is supported on three screws, A, B, C, with round ends, which rest on the table. Along one side of the table is fixed a straight board parallel to the required direction of motion; to the base board are fixed two horizontal screws D, E which bear against the raised edge.

If all five screws are kept in contact the screen will have only one degree of freedom, *i. e.* it can slide parallel to the straight raised edge. To completely determine the position of the screen one more bearing point is wanted; to supply this there is a sixth screw F, which works in a block of wood which can be clamped to the side of the straight board as shown, the screw being parallel to the direction of sliding. The screen can now be moved till the base comes into contact with the screw F, and there being now six points in contact the position is completely determinate. If the screen be removed it can be replaced, even in the dark, by bringing all the six bearing points again into contact.

If desired, the screws, as shown, can be dispensed with, and metal screws with round heads used, placed so that the round heads form the bearing points.

148. The Adjustments of the Movable Screen.—The screws are used to adjust the position of the screen; there are three of them, A, B, C, resting on the horizontal plane; if A be turned the screen is turned about a horizontal axis perpendicular to the plane of the sensitive paper; if B or C be turned the screen is inclined forwards or backwards, and by means of D and E it can be twisted about a vertical axis. Lastly, when the position for sharp focus is found the sixth screw can be clamped so that it just bears against the base, and a small final adjustment can be made by

turning the screw. Thus every possible adjustment can be easily made.

We have now shown how geometrical principles properly applied prove of great use in designing a proper sliding screen ; the same principles can be applied to many kinds of apparatus, with the result in most cases of greatly cheapening and steadying them.

149. Enlarging with a Box.—When a room is not available daylight enlarging can be conducted, though not so conveniently, by means of a box ; the design is shown in Fig. 98, from which it will be seen that the ordinary camera is again used.

The general arrangement does not need much explanation. The ground glass is replaced by the negative and the sensitive paper is placed on the panel indicated by the dotted lines ; the box is then held up to a window or lamp so that the light falls on the negative, and the sliding shutter inside the box is withdrawn for the time of exposure. The camera is fixed to a ledge which can be folded up against the box when not in use ; the panel carrying the sensitive paper is held in grooves in the sides of the box, its distance from the lens depending on the size to which it is required to enlarge.

The box is rendered light-tight, not by a close-fitting lid which might warp, but by overlapping edges and an inside board resting on a ledge which extends all round the box ; there are then five corners which must be turned by any light which penetrates to the interior. The whole of the inside of the box, the lid, and the inner board are painted a dead black, to prevent the reflection of stray light.

Light is prevented from entering at the lens hole, round the sides of the lens mounting, by a flange which just overlaps the mounting and is painted dead black. Some difficulty may be found in focussing the picture properly, but when once it is found the trouble may be avoided by marking on the tail-board of the camera

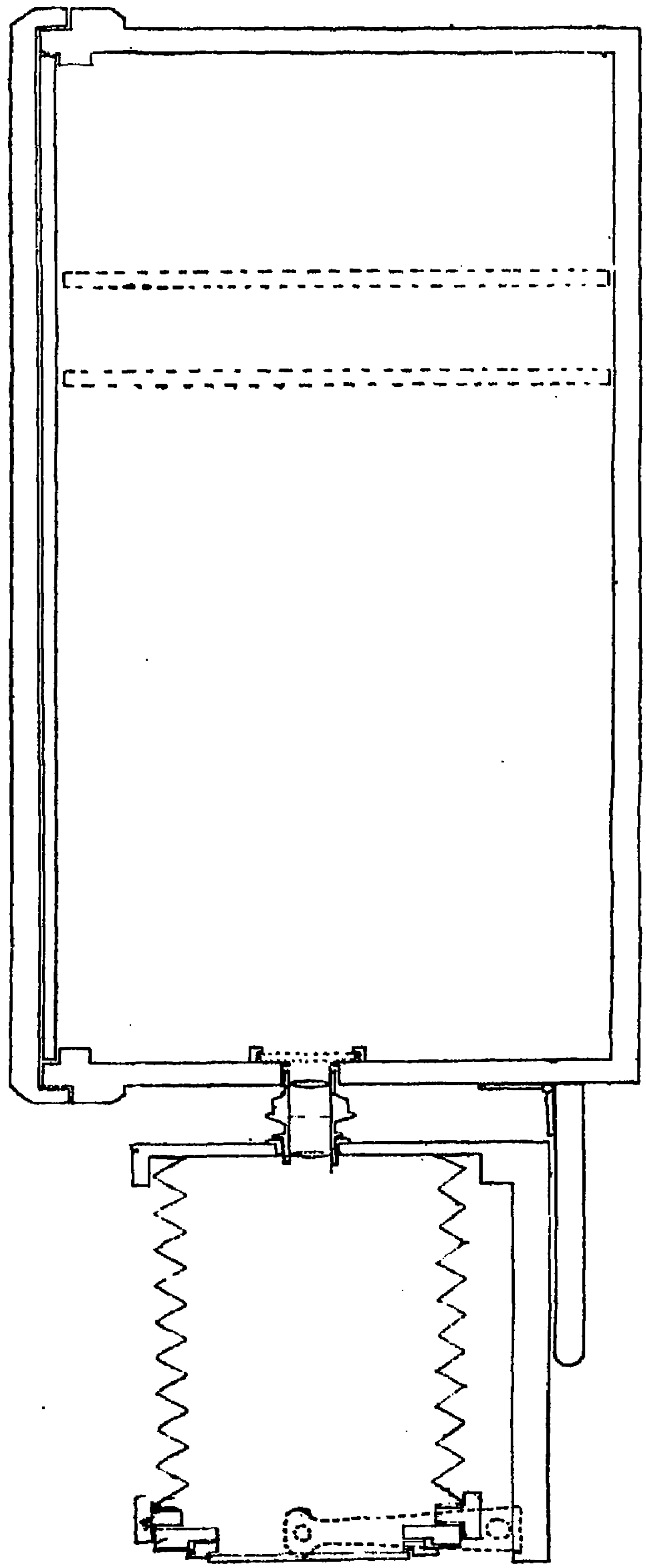

FIG. 98.

the positions of the negative corresponding to various positions of the panel.

The enlarging box described has the convenience that the whole outfit can be packed inside it for travelling.

150. Reducing Apparatus.—It is often required to reduce a picture to form a lantern slide; the optical principles of reduction are exactly the same as those of enlarging, and the relative distances can be calculated in a similar manner. The enlarging box described in the last article may be used for reducing, provided the camera can be extended enough and a board carrying the lantern plate can be placed at a suitable distance.

When designing apparatus for enlarging or reducing, care should be taken to use the correct focal length of the lens, for an error of even half-an-inch will make a great deal of difference in the relative positions of negative and enlargement. The focal lengths given by makers in their catalogues are often the back focus, and not the true focal length.

151. Distances of Negative and Enlargement from the Lens.—The distances of the negative and the enlargement from the nodal points of the lens can be calculated from the principles explained in Chapter II. If u and v be the distances of the negative and enlargement, f the focal length of the lens, and n the linear magnification required, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (1) \qquad \frac{v}{u} = -n \quad (2)$$

the negative sign being used in (2) as the picture is inverted.

Solving for v and u we get

$$u = -(1 + n)f/n, \quad v = (1 + n)f$$

Example.—With a lens of 8 inches focal length it is required to enlarge from a quarter plate negative to five times the linear dimensions.

Here $f = -8$ inches, $n = 5$

$$\therefore u = (1 + 5) \times 8/5 = 48/5 = 9{\cdot}6 \text{ inches},$$

and $v = (1 + 5) \times 8 = -48$ inches;

or the negative must be placed at a distance of 9·6 inches from the nodal point of incidence, the paper to receive the enlarged picture at a distance of 48 inches from the nodal point of emergence. The table on next page gives the relative distances for lenses of various focal lengths; in the vertical column on the left is given the principal focal length of the lens, and along the top the linear ratio of enlargement. The distances required are read off in the usual manner.

The results will clearly hold good in whatever units the lengths are measured as long as all three are at one time in terms of the same unit.

This table can also be used for reduction, for we have only to interchange the places of the negative and sensitive surface; for instance, in the example above, if the negative be placed at a distance of 340 inches and the sensitive surface at a distance of 17·9 inches, the picture will be reduced to one-nineteenth of its original size.

Example.—The focal length of the lens is 17 inches and it is required to enlarge 19 times.

At the row containing 17 and the column containing 19 *t* we find the numbers 340, 17·9, which mean that the negative must be distant 17·9 inches and the enlargement 340 inches from their respective nodal points.

152. Times of Exposure when Enlarging or Reducing.—We have seen in §119 that if T be the time of exposure, u the distance of the object, and e that of the stop from the nodal point of incidence, d the diameter of the aperture in the stop, then

$$T \propto \left(\frac{u - e}{d}\right)^2$$

If e is small enough to be neglected then

$$T \propto u^2/d^2$$

Let T_1 be the exposure with distance u_1 and diameter d_1, then we get

$$\frac{T}{T_1} = \frac{u^2}{{u_1}^2} \times \frac{d^2{}_1}{d^2}$$

TABLE FOR ENLARGEMENT AND REDUCTION.

Principal Focal Length of the Lens.	Distances of the Negative and the Sensitive Paper from the Nodal Points of the Objective.																								
	1 *t.*	2 *t.*	3 *t.*	4 *t.*	5 *t.*	6 *t.*	7 *t.*	8 *t.*	9 *t.*	10 *t.*	11 *t.*	12 *t.*	13 *t.*	14 *t.*	15 *t.*	16 *t.*	17 *t.*	18 *t.*	19 *t.*	20 *t.*	21 *t.*	22 *t.*	23 *t.*	24 *t.*	25 *t.*
5.	10 10	15 7·5	20 6·7	25 6·2	30 6	35 5·8	40 5·7	45 5·6	50 5·6	55 5·5	60 5·4	65 5·4	70 5·4	75 5·4	80 5·3	85 5·3	90 5·3	95 5·3	100 5·3	105 5·3	110 5·2	115 5·2	120 5·2	125 5·2	130 5·2
6.	12 12	18 9	24 8	30 7·5	36 7·2	42 7	48 6·9	54 6·8	60 6·7	66 6·6	72 6·5	78 6·5	84 6·4	90 6·4	92 6·4	102 6·4	108 6·4	114 6·3	120 6·3	126 6·3	132 6·3	138 6·3	144 6·3	150 6·3	156 6·2
7.	14 14	21 10·5	28 9·3	35 8·8	42 8·4	49 8·2	56 8	63 7·9	70 7·8	77 7·7	84 7·6	91 7·6	98 7·5	105 7·5	112 7·5	119 7·4	126 7·4	133 7·4	140 7·4	147 7·4	154 7·3	161 7·3	168 7·3	175 7·3	182 7·3
8.	16 16	2 4 12	32 10·7	40 10	48 9·6	56 9·3	64 9·1	72 9	80 8·8	88 8·8	96 8·7	104 8·7	112 8·6	120 8·6	128 8·5	136 8·5	144 8·5	152 8·4	160 8·4	168 8·4	176 8·4	184 8·4	192 8·3	200 8·3	208 8·3
9.	18 18	27 13·5	36 12	45 11·3	54 10·8	63 10·5	72 10·3	81 10·1	90 9·9	99 9·9	108 9·8	117 9·8	126 9·7	135 9·6	144 9·6	153 9·6	162 9·5	171 9·5	180 9·5	189 9·5	198 9·4	207 9·4	216 9·4	225 9·4	234 9·4
10.	20 20	30 15	40 13·3	50 12·5	60 12	70 11·7	80 11·4	90 11·3	100 11·1	110 11	120 10·9	130 10·8	140 10·8	150 10·7	160 10·7	170 10·6	180 10·6	190 10·6	200 10·5	210 10·5	220 10·5	230 10·5	240 10·4	250 10·4	260 10·4
11.	22 22	33 16·5	44 14·7	55 13·8	66 13·2	77 12·8	88 12·6	99 12·4	110 12·2	121 12·1	132 12	143 11·9	154 11·8	165 11·8	176 11·7	187 11·7	198 11·6	209 11·6	220 11·6	231 11·6	242 11·5	253 11·5	264 11·5	275 11·5	286 11·4
12.	24 24	36 18	48 16	60 15	72 14·4	84 14	96 13·7	108 13·5	120 13·3	132 [illegible]	144 13·1	156 13	168 [illegible]	180 [illegible]	192 [illegible]	204 [illegible]	216 [illegible]	228 [illegible]	240 [illegible]	252 [illegible]	264 [illegible]	27 [illegible]	288 [illegible]	300 [illegible]	312 [illegible]

13.	26 26	39 19·5	52 17·3	65 16·3	78 15·6	91 15·1	104 14·9	117 14·6	130 14·4	143 14·3	156 14·2	169 14·1	182 14	195 13·9	208 13·9	221 13·8	234 13·8	247 13·7	260 13·7	273 13·7	286 13·6	299 13·6	312 13·6	325 13·5	338 13·5
14.	28 28	42 21	56 18·7	70 17·5	84 16·8	98 16·3	112 16	126 15·8	140 15·6	154 15·4	168 15·3	182 15·2	196 15·1	210 15	224 14·9	238 14·9	252 14·8	266 14·8	280 14·7	294 14·7	308 14·7	322 14·6	336 14·6	350 14·6	364 14·6
15.	30 30	45 22·5	60 20	75 18·8	90 18	105 17·5	120 17·1	135 16·9	150 16·7	165 16·5	180 16·4	195 16·3	210 16·2	225 16·1	240 16	255 15·9	270 15·9	285 15·8	300 15·8	315 15·8	330 15·7	345 15·7	360 15·7	375 15·6	390 15·6
16.	32 32	48 24	64 21·3	80 20	96 19·2	112 18·7	128 18·3	144 18	160 17·8	176 17·6	192 17·5	208 17·3	224 17·2	240 17·1	256 17·1	272 16·9	288 16·9	304 16·9	320 16·8	336 16·8	352 16·8	368 16·7	384 16·7	400 16·7	416 16·6
17.	34 34	51 25·5	68 22·7	85 21·3	102 20·4	119 19·8	136 19·4	153 19·1	170 18·9	187 18·7	204 18·5	221 18·4	238 18·3	255 18·2	272 18·1	289 18·1	306 18	323 17·9	340 17·9	357 17·9	374 17·8	391 17·8	408 17·7	425 17·7	442 17·7
18.	36 36	54 27	72 24	90 22·5	108 21·6	126 21	144 20·6	162 20·3	180 20	198 19·8	216 19·6	234 19·5	252 19·4	270 19·3	288 19·2	306 19·1	324 19·1	342 19	360 18·9	378 18·9	396 18·8	414 18·8	432 18·8	450 18·8	468 18·7
19.	38 38	57 28·5	76 25·3	95 23·8	114 22·8	133 22·2	152 21·7	171 21·4	190 21·1	209 20·9	228 20·7	247 20·6	266 20·5	285 20·4	304 20·3	323 20·2	342 20·1	361 20·1	380 20	399 20	418 19·9	437 19·8	456 19·8	475 19·8	494 19·8
20.	40 40	60 30	80 26·6	100 25	120 24	140 23·3	160 22·9	180 22·5	200 22·2	220 22	240 21·8	260 21·7	280 21·5	300 21·4	320 21·3	340 21·3	360 21·2	380 21·2	400 21·1	420 21	440 21	460 20·9	480 20·9	500 20·8	520 20·8
21.	42 42	63 31·5	84 28	105 26·3	126 25·2	147 24·5	168 24	189 23·7	210 23·3	231 23·1	252 22·9	273 22·8	294 22·6	315 22·5	336 22·4	357 22·3	378 22·2	399 22·2	420 22·1	441 22·1	462 22	483 21·9	504 21·9	525 21·9	546 21·8
22.	44 44	66 33	88 29·3	110 27·5	132 26·4	154 25·7	176 25·1	198 24·8	220 24·4	242 24·2	264 24	286 23·8	308 23·7	330 23·6	352 23·5	374 23·4	396 23·2	418 23·2	440 23·2	462 23·1	484 23	506 23	528 23	550 22·9	572 22·1
23.	46 46	69 34·5	92 30·7	115 28·8	138 27·6	161 26·7	184 26·3	207 25·9	230 25·6	253 25·3	276 25·1	299 24·9	322 24·8	345 24·6	368 24·5	391 24·4	414 24·3	437 24·3	460 24·2	483 24·2	506 24·1	529 24	552 24	575 24	598 23·9
24.	48 48	72 36	96 32	120 30	144 28·8	168 28	192 27·4	216 27	240 26·7	264 26·4	288 26·2	312 26	336 25·8	360 25·7	384 25·6	408 25·5	432 25·4	456 25·3	480 25·3	504 25·2	528 25·1	552 25·1	576 25	600 25	624 25
25.	50 50	75 37·5	100 33·3	125 31·3	150 30	175 29·2	200 28·6	225 28·1	250 27·8	275 27·5	300 27·3	325 27·1	350 26·9	375 26·8	400 26·7	425 26·6	450 26·5	475 26·4	500 26·3	525 26·3	550 26·2	575 26·1	600 26·1	625 26	650 26

Hence we can compare the exposures in two different cases.

The quantity u can be got from the table in the last section, and d can be found from the focal length of the lens and the number of the stop used.

Example.—When using a lens of 7 inches focal length and stop $f/20$ to enlarge five times, the exposure required was 40 seconds; find the exposure when a lens of 9 inches focal length is used with the stop $f/30$ to enlarge six times.

We find from the table that for the first case, $u_1 =$ 42 inches, and for the second case, $u = 63$ inches; also in the first case, $d_1 =$ focal length $/20 = 7/20$ inch, in the second case $d = 9/30 = 3/10$ inch, and T_1 the first exposure $= 40$ seconds.

$$\therefore T = \frac{u^2}{u_1{}^2} \times \frac{d_1{}^2}{d^2} \times T_1 = \left(\frac{63}{42}\right)^2 \times \left(\frac{7}{20} \times \frac{10}{3}\right)^2 \times 40$$

$= 122$ seconds, about.

153. Relative Positions of Condenser and Objective.—It has been stated in § 142 that if the full advantage is to be taken of the condenser to concentrate light on the negative, certain relations must exist between the focal lengths of the condenser and objective; we now proceed to investigate these relations. The results will be approximate only, for it would complicate the calculations too much to take account of the spherical aberration of the condenser and the size of the source, when artificial light is used; we shall suppose both the condenser and objective to be thin lenses.

In Fig. 99, S is the source of light, A B the condenser, C D the diagonal of the negative, G H J K the mounting of the objective, E F the lens equivalent to the objective, here supposed thin, L M N the points on which the common axis of the lenses meets the lenses and negative, X the point towards which the light is converged by the condenser, and Y the point to which

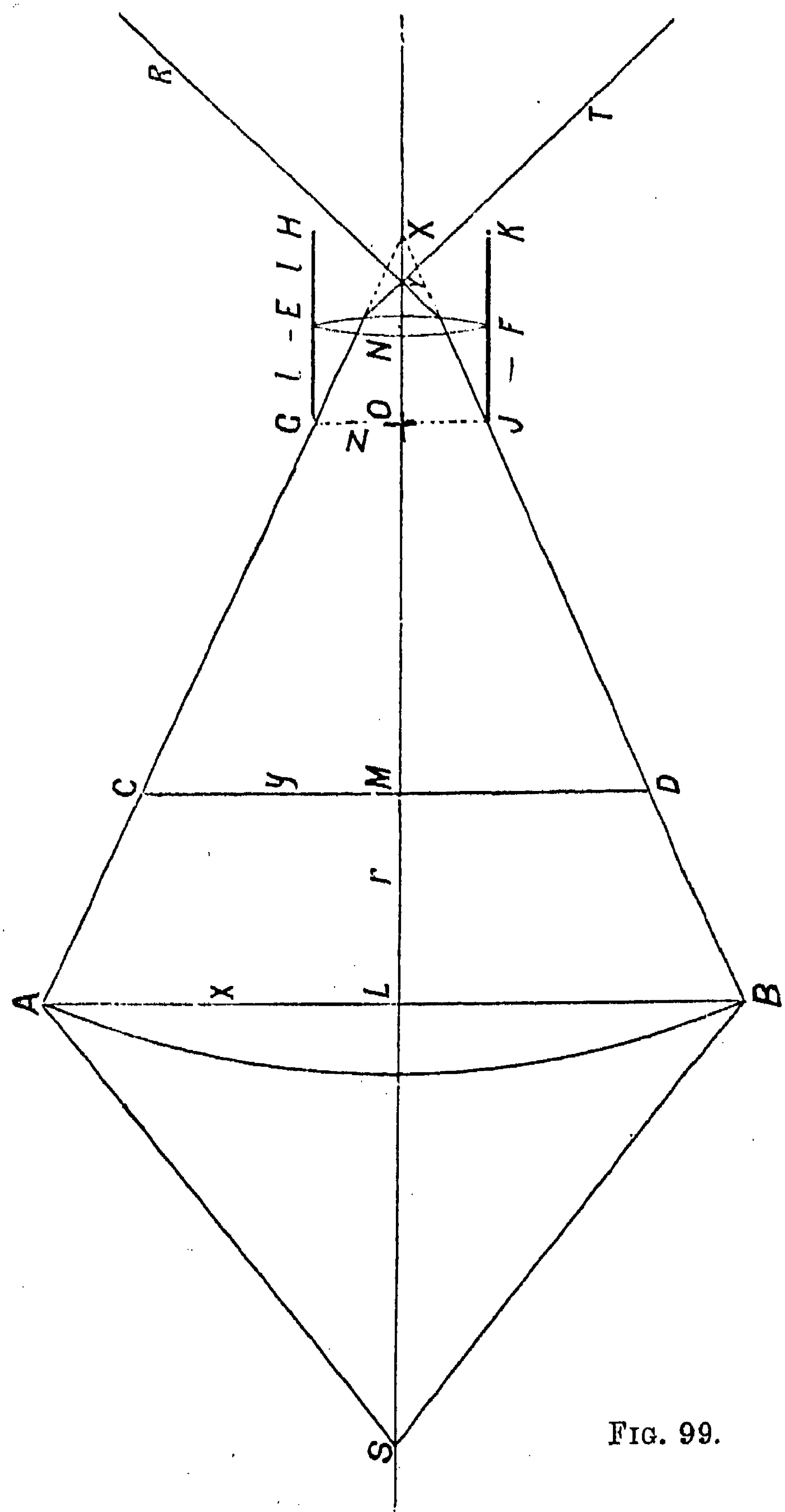

FIG. 99.

the light going towards X is converged by the objective, so that X and Y are conjugate foci.

Let u, v, be the distances of negative C D, and the enlargement from N.

Let U, V, be the distances of S and X from L.

,, $2l$,, ,, length G H of the mounting.

,, r ,, ,, distance L M.

,, $2x$ and $2z$ be the diameters of A B and E F.

,, $2y$ be the diagonal of the negative.

,, F and f be the principal focal lengths of the condenser and objective.

,, n be the ratio of linear magnification.

Then we can at once write down the following relations—

$$\frac{1}{\mathrm{V}} - \frac{1}{\mathrm{U}} = \frac{1}{\mathrm{F}} \ (1), \ \frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \ \frac{v}{u} = -n$$

From the last two we find as in the last section

$$u = -(1+n)f/n, \ v = (1+n)f \ . \ . \ . \ . \ (2)$$

If the negative is placed as in the figure so that its diagonal is just within the cone of rays, we have the triangles A L X, C M X similar, and hence, A l : C M = L X : M X

$$\text{or } x : y = \mathrm{V} : \mathrm{V} - r \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (3)$$

But if the cone of light is to just fit into the mounting as in the figure we must have from the similar triangles A L X, G O X

$$\mathrm{AL} : \mathrm{GO} = \mathrm{LX} : \mathrm{OX}$$

$$\text{or } x : z = \mathrm{V} : \mathrm{V} - v - r + l \ . \ . \ . \ . \ . \ (4)$$

for $\mathrm{OX} = \mathrm{ON} + \mathrm{NX} = \mathrm{ON} + \mathrm{LX} - \mathrm{LM} - \mathrm{MN} = l + \mathrm{V} - r - v$.

These four relations will enable us to calculate anything we wish.

(a) Take the case given in § 142 where sunlight is used and it is required to find the focal length of the condenser where f, x, y, n, l are given. Here, since the sun is the source of light, S is very distant and we get

from (1) $F = V$. From (3) and (4) it can be shown that

$$\frac{v-l}{V} = \frac{y-z}{x}$$

but $V = F$, and from (2) $v = (1+n)f$

$$\text{hence } \frac{(1+n)f}{F} = \frac{y-z}{x} \text{ or } F = \frac{x}{y-z} \cdot (1+n) \cdot f$$

(b) Next consider the case when the negative is placed close to the condenser, and let it be required to find the distance of S from the condenser when x, z, n, F, f are known.

Here $r = 0$ and we get from (4)

$$1 - \frac{v-l}{V} = \frac{z}{x} \quad \text{or} \quad \frac{1}{V} = \frac{x-z}{x} \cdot \frac{1}{v-l}$$

Combining this with (1) we get

$$\frac{1}{u} + \frac{1}{F} = \frac{x-z}{x}\,\frac{1}{v-l} \text{ or } \frac{1}{v} = \frac{x-z}{x}\,\frac{1}{(1+n)f-l} - \frac{1}{F}$$

from which we can find u the distance of S from the condenser.

Example.—Let the focal lengths of the condenser and objective be 3 inches and 6 inches; hence, $F = -3$, $f = -6$; also let $n = 5$, $x = 1{\cdot}5$ inches, $z = {\cdot}5$ inch, $l = 1{\cdot}5$ inches, then

$$\frac{1}{U} = \frac{1{\cdot}5 - {\cdot}5}{1{\cdot}5} \times \frac{1}{-36 - 1{\cdot}5} + \frac{1}{3} = {\cdot}3155$$

$$\therefore U = 3{\cdot}17 \text{ inches.}$$

Or the source of light S must be 3·17 inches from the condenser to give the best illumination.

It should be noticed that the nearer S is to the condenser the greater is the amount of light which falls on the condenser.

154. The Position of the Source of Light.—We can use Fig. 99 to explain the faults in illumination of the image, familiar to every one who has manipulated a

lantern, which are due to the wrong position of the source of light S.

If S approach the condenser X will recede from it, and some of the extreme rays of the cone will be cut off by the mounting of the objective. Also if S recede from the condenser X will approach it; this will cause Y the vertex of the emergent cone to move towards E F, and at the same time the angle of the cone will widen; on both these accounts, if S be moved too far the outer rays of the emergent cone will be intercepted by the mounting. In both these cases a dark ring will be formed round the edge of the disc on the screen.

If S be moved off the axis to one side it can easily be seen that some of the rays will be intercepted by the mounting, and a dark space will be formed on the disc, on the *same* side as that to which S was displaced.

155. Depth of Focus.—It has been pointed out (§ 90) that to obtain a sharp picture it is not necessary to have the light proceeding from a point in the object converging exactly to a point in the image; but that as long as the section of the refracted pencil by the plate does not exceed a certain definite size the patch of light formed will appear to the eye as a point.

This gives us some latitude in focussing, for the ground glass can be moved about, within certain limits, without the section of the pencil becoming too large to appear as a point; we shall call this permissible displacement, *depth of focus*.

Consider first the case when the object is at a great distance so that rays converge to F, the principal focus of the lens (Fig. 100). Let F be the principal focal length of the lens, $2e$ the greatest permissible breadth of the patch of light, and x the greatest distance from F to which the ground glass can be moved; then it is clear from the figure that

$$\frac{x}{\mathrm{F}} = \frac{e}{y} \quad \text{or} \quad x = e\frac{\mathrm{F}}{y}$$

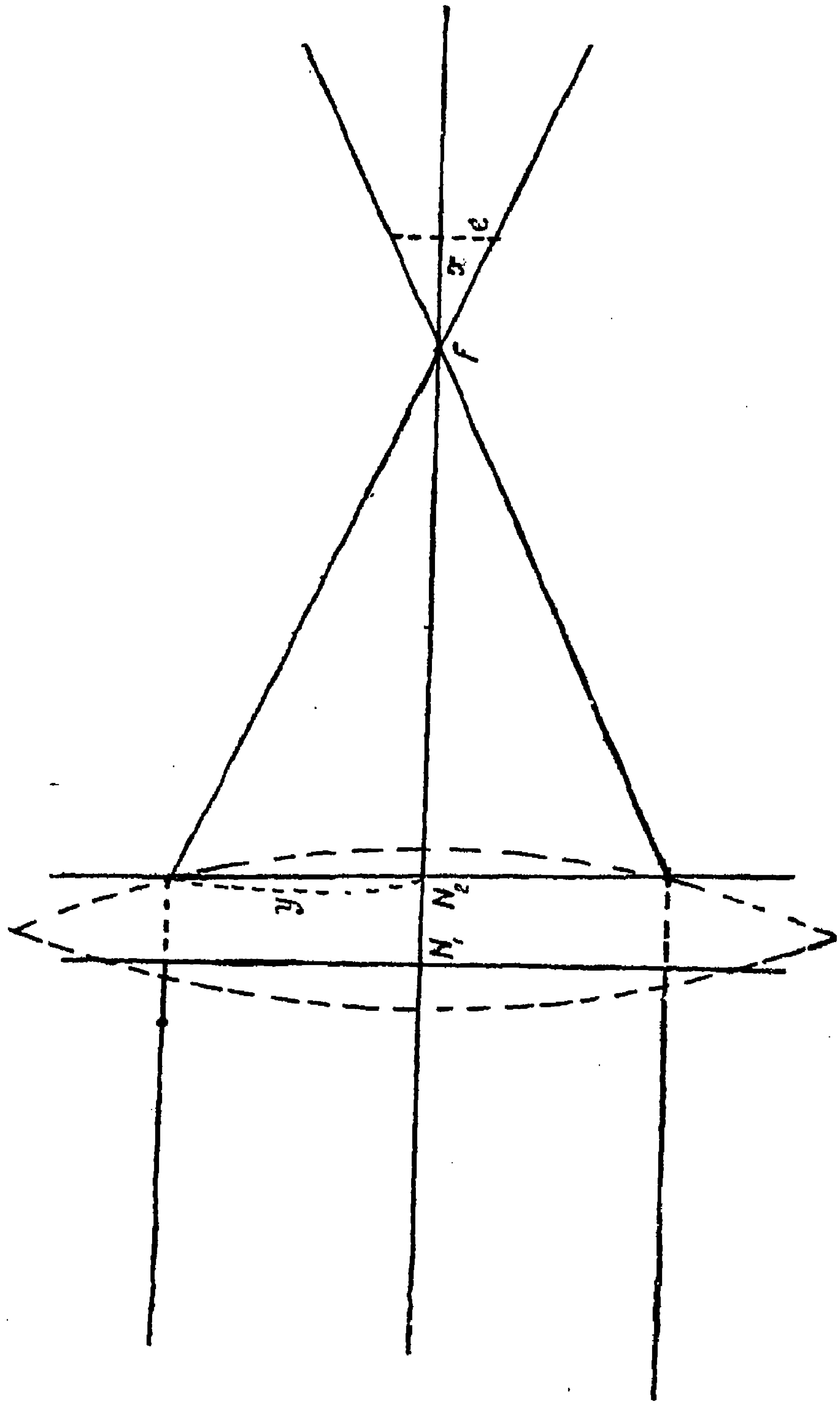

FIG. 100.

but the screen can be moved to an equal distance on the other side of F, so the depth of focus is

$$2\,x = 2\,e\,\mathrm{F}/y.$$

Next let the object be at a distance u, and the image at a distance v from the lens (Fig. 101); then it is clear from the figure that

$$\frac{x}{v} = \frac{e}{y} \quad \text{or} \quad x = e\,\frac{v}{y}$$

If the expression is required in terms of the distance of the object from the lens, we have

$$v = \frac{u\,\mathrm{F}}{u + \mathrm{F}} \;\therefore x = e \,.\, \frac{\mathrm{F}}{y} \,.\, \frac{u}{u + \mathrm{F}}.$$

Hence depth of focus $= 2\,x = 2\,e \,.\, \dfrac{\mathrm{F}}{y} \,.\, \dfrac{u}{u + \mathrm{F}}$

It should be noticed that x increases as y decreases, or the smaller the stop the greater the depth of focus.

Example.—The object is 20 feet distant, and the focal length of the lens is 6 inches, the radius of the aperture is ·5 inch, and $2\,e$ is taken as one-fiftieth of an inch—

$$\therefore \text{Depth of focus} = \frac{1}{50} \times \frac{-6}{\cdot 5} \times \frac{240}{246} = -\,\cdot 058 \text{ inch.}$$

No particular meaning can here be given to the negative sign, as we have not said in which direction the depth of focus is to be reckoned.

156. Depth of Field.—The displacement that can be given to a point on the axis of the lens, without the size of its image focussed when the point is placed in a given position becoming broader than $2\,e$, is called the depth of field; in this case it is the object which is moved while the ground glass remains steady.

The special interest of the question lies in its application to hand cameras, where it is required to know the least distance of the object which will give a sharp picture.

Let us consider how near to the lens an object originally focussed when at a great distance can be moved without spoiling the sharpness of the picture.

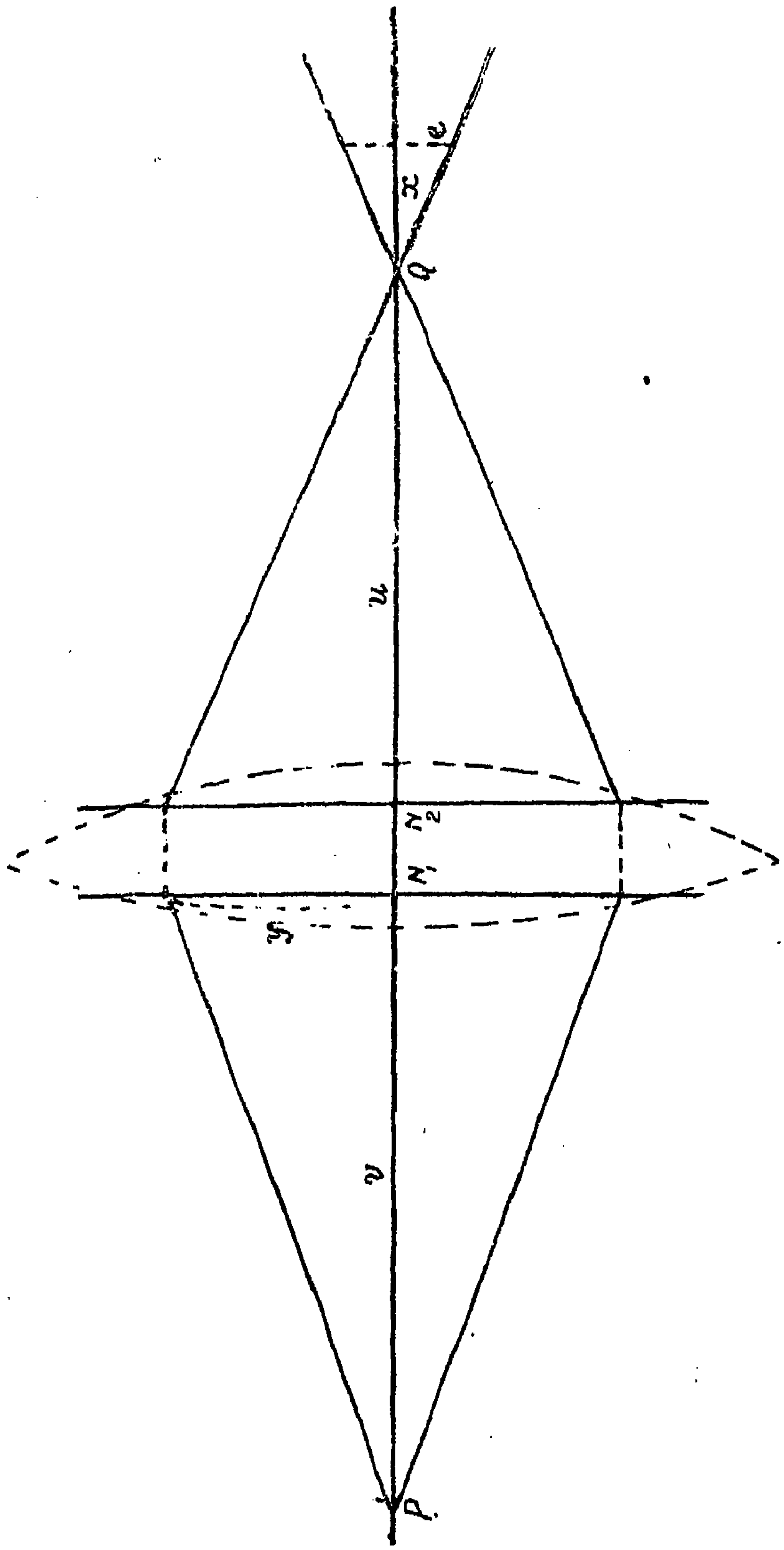

FIG. 101.

In Fig. 102, F is the principal focus and Q is the focus conjugate to P, the nearest point for which the

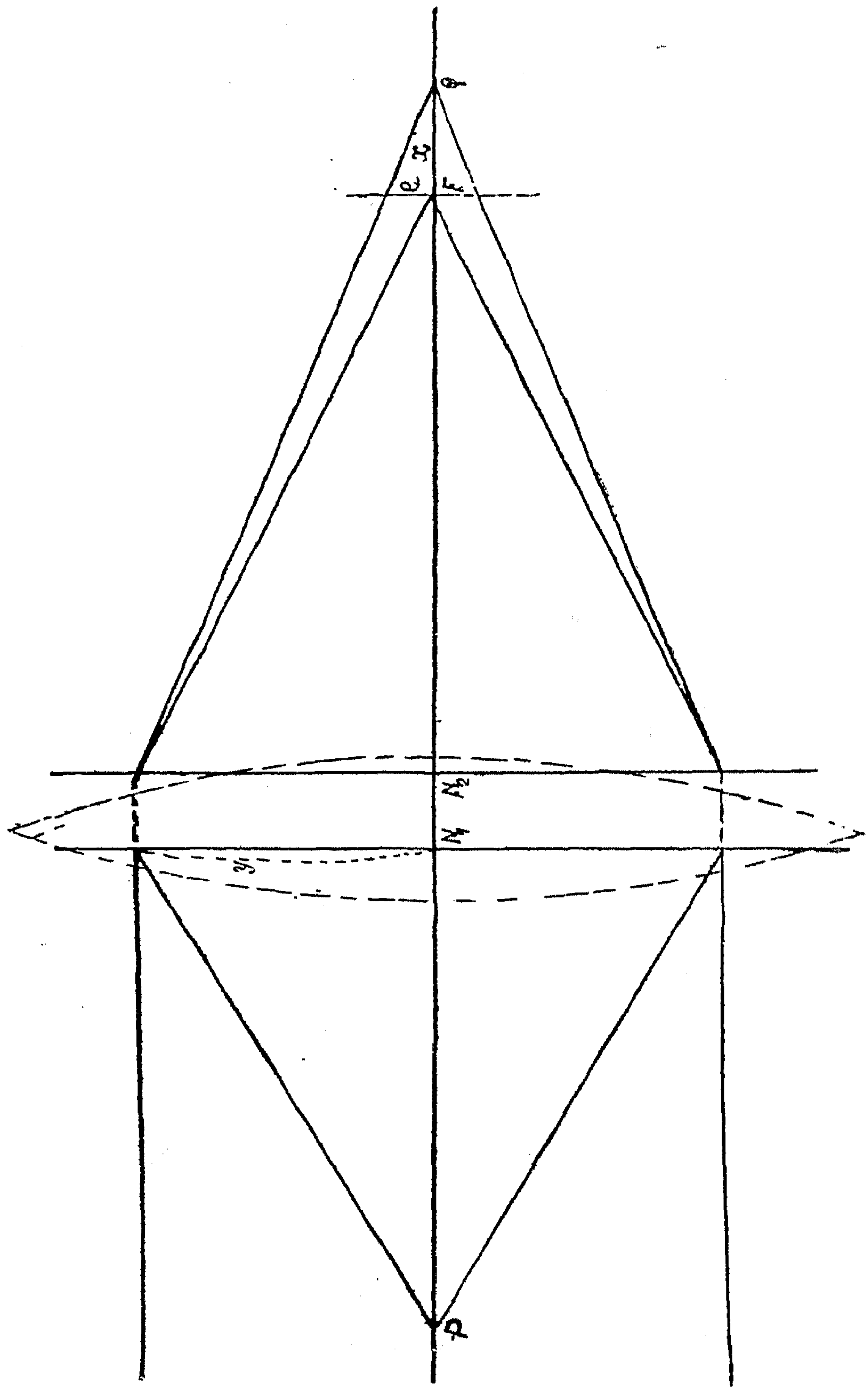

FIG. 102.

image is sharp; let F be the focal length of the lens, let $F Q = x$, and let u be the distance of P from N_1, the nodal point of emergence.

When the object is at P, the section of the pencil of rays by the ground glass at F is $2e$, hence we get from the figure

$$\frac{x}{e} = \frac{F + x}{y} \quad \text{or} \quad \frac{F + x}{x} = \frac{y}{e}$$

But since P and Q are conjugate foci

$$\frac{1}{F + x} - \frac{1}{u} = \frac{1}{F} \text{ or } \frac{1}{u} = \frac{1}{F + x} - \frac{1}{F} = - \frac{x}{F(F + x)}$$

$$\therefore u = - F \cdot \frac{F + x}{x} = - \frac{F y}{e}$$

which gives the distance of the object required.

The following table gives the values of u calculated from this expression for various lenses and apertures.

The focal length is given in centimetres, the value of $2e$ is taken to be one hundredth of a centimetre, and the results are given in metres.

Principal Focal Length in Centimetres.	Aperture.											
	$\frac{f}{5}$	$\frac{f}{10}$	$\frac{f}{15}$	$\frac{f}{20}$	$\frac{f}{25}$	$\frac{f}{30}$	$\frac{f}{35}$	$\frac{f}{40}$	$\frac{f}{45}$	$\frac{f}{50}$	$\frac{f}{55}$	$\frac{f}{60}$
5...........	2·5	1·3	0·9	0·7	0·5	0·5	0·4	0·4	0·3	0·3	0·3	0·3
10...........	10·0	5·0	3·4	2·5	2·0	1·7	1·5	1·3	1·2	1·0	0·9	0·9
15...........	22·5	11·3	7·5	5·7	4·5	3·8	3·3	2·9	2·5	2·3	2·1	1·9
20...........	40·0	20·0	13·4	10·0	8·0	6·7	5·8	5·0	4·5	4·0	3·7	3·4
25...........	62·5	31·3	20·9	15·7	12·5	10·5	9·0	7·9	7·0	6·3	5·7	5·3
30...........	90·0	45·0	30·0	22·5	18·0	15·0	12·9	11·3	10·0	9·0	8·2	7·5
35...........	122·5	61·3	40·9	30·7	22·5	20·5	17·5	15·4	13·6	12·3	11·2	10·3
40...........	160·0	80·0	53·4	40·0	32·0	26·7	22·9	20·0	17·8	16·0	14·6	13·4
45...........	202·5	101·3	67·5	50·7	40·5	33·8	29·0	25·4	22·5	20·3	18·4	16·9
50...........	250·0	125·0	83·4	62·5	50·0	41·7	35·8	31·3	27·8	25·0	22·8	20·9

For example, with a lens of 25 cm. focal length and a stop of $f/30$, all objects between infinity and a distance of 10·5 metres will be in focus at the same time.

157. Halation or Irradiation.—It sometimes happens that chemical action takes place in parts of the plate on which the light has not directly fallen, causing the

phenomenon known to practical photographers as *Halation.* Experience shows that halation depends to a great extent on the nature and thickness of the sensitive surface, and is specially prominent when the light is very bright, and the contrasts sharp. If the image of a bright point of light, such as the reflection from a drop of mercury, be photographed, not only is

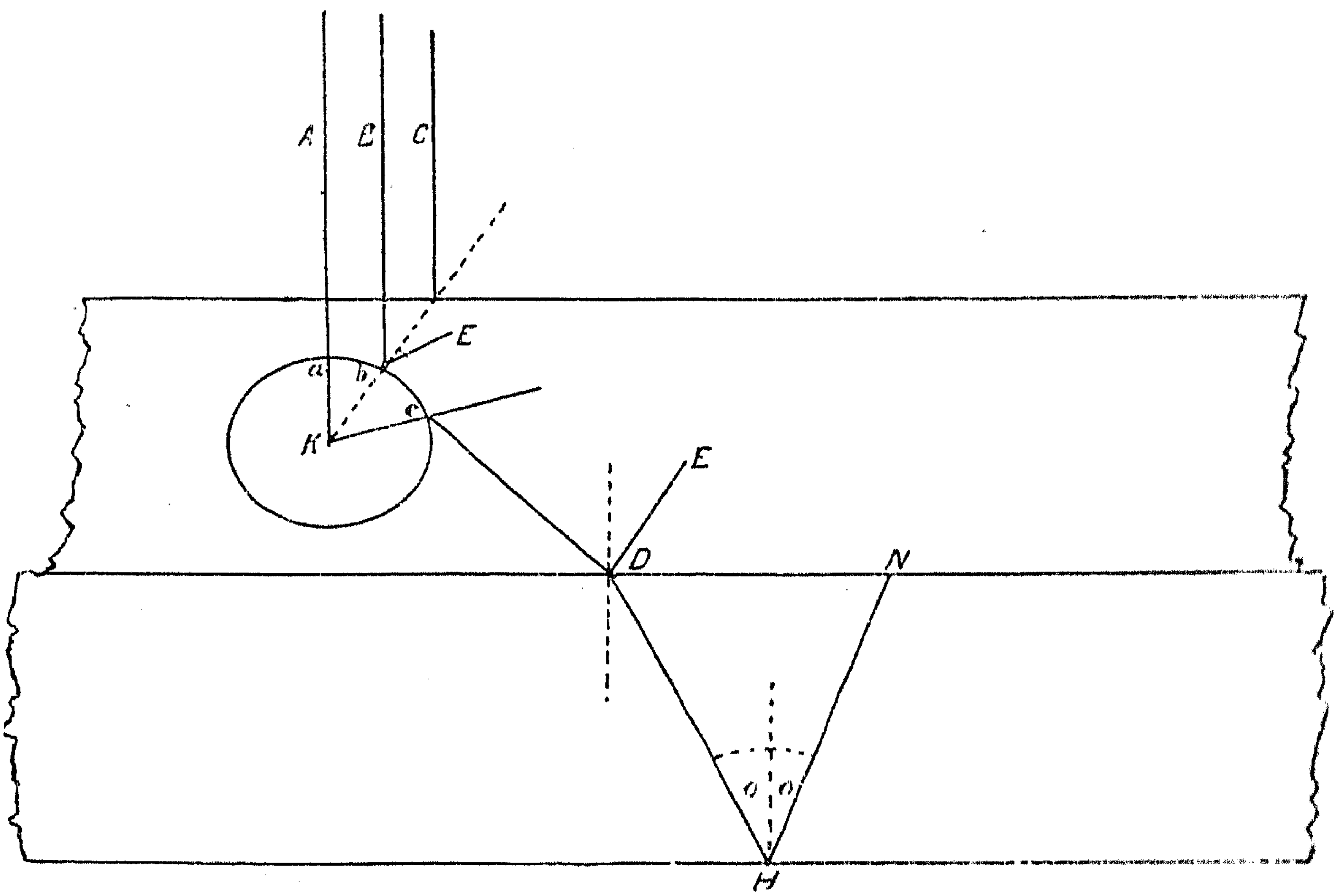

FIG. 103.

the ordinary image formed, but round it, and separated from it by a clear space, is a dark ring. The effect of a bright line of light is that got by superposing the effects of all the points composing the line, the result being that the line is broadened.

The phenomenon has been explained by Abney, as due to reflection of light which has passed through the film, at the back of the plate; the reflected light causing chemical action at points around the regular image.

To explain this more clearly, let the figure (Fig. 103) represent a magnified section of a piece of the sensitive film and of the supporting glass; and let K be a particle of silver bromide in the film.

Let A *a*, B *b*, C *c* be three rays of light which strike the particle; the ray A *a* which impinges on the top of the particle is either absorbed or directly reflected back; the ray B *b*, slightly to one side of A *a*, is reflected along *b* E and does not produce an effect at any great distance from the regular image; the ray C *c* which strikes the particle considerably to one side is reflected along *c* D and into the glass along D H to meet the back surface of the glass at H. At H part of the light is refracted and part reflected along H N, entering the film again and producing chemical action.

The amount of light reflected depends on the angle of incidence, increasing rapidly as the critical angle (§ 12) is approached, and afterwards total internal reflection takes place.

We thus see that when the ray D H has a small angle of incidence, very little light is reflected and no considerable action takes place at points near to the regular image, but as H recedes from K and the angle of incidence increases up to the critical angle, the amount of light reflected becomes large, a black ring is formed round the regular image.

A remedy proposed for the prevention of halation, is to paint the back of the plate black, but it is hard to see how this can help, since the light which does the harm never emerges. It would be much better to have the back of the plate ground to prevent any regular reflection. Several kinds of plates are now made in which halation is prevented, either by the thickness of the film or by an opaque layer between the film and the glass, either of which prevents light from passing through to the glass.

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